Boundary layer receptivity to acoustic waves interacting with wall roughness

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Ravello, ITALY
April 27–28, 2000

Receptivity mechanisms

A classification of fluid dynamics instabilities:

- Local inviscid: centrifugal instabilities (Görtler)
- Layer inviscid: jets and wakes
- Layer viscous: Tollmien–Schlichting waves
- Algebraic: can explain bypass transition

$\Rightarrow$ Different **excitation mechanisms** $\Leftarrow$

External perturbation:

- **acoustic** wave
- **vorticity** wave
- wall **vibration**

Receptivity mechanisms:

- **leading edge** receptivity
- **wall roughness** effect

Receptivity mechanisms

Possible approaches:

- **Asymptotic** theory (triple or double–deck): power series in Re^{-1} -
  e. g. Goldstein (1983, 1985), Goldstein and Hultgren (1989), Bodonyi, Welch, Duck and Tadjfar (1989), Bessiere (1999)
- **Orr–Sommerfeld** formulation: exact solution for parallel flow -
  e. g. Crouch (1992), Choudhari and Streett (1992), Nayfeh and Ashour (1994), Hill (1995)
- **PSE**: adjoint formulation for LPSE -
  e. g. Herbert (1997), Airiau and Bottaro (1998)
- **DNS**: complete Navier–Stokes equations -
  e. g. Casalis, Gouttenoire and Troff (1997),

Aims of the present research

Multiple–scales approach
extended to non–homogeneous case

- ☺ **Complex boundary layers** (real geometries)
- ☺ Corrections for **non–parallel** flow effects
- ☺ Fast tool for transition prediction to be integrated in an **industrial code** (wing design)

Multiple–scales non–homogeneous approach

$$\boldsymbol{H}(t)\,\frac{d\mathbf{x}(t)}{dt} + \boldsymbol{A}(t)\,\mathbf{x}(t) = \tilde{\epsilon}\mathbf{y}(t) \qquad T = \tilde{\epsilon}t$$

Quantities slowly varying with t

$$\mathbf{x}(T) = e^{\frac{\phi(T)}{\tilde{\epsilon}}}\left(\mathbf{f}_0(T) + \tilde{\epsilon}\mathbf{f}_1(T) + \tilde{\epsilon}^2\mathbf{f}_2(T) + \cdots\right)$$

Hierarchy of equations at different orders:

$$\begin{aligned}
\left(\frac{d\phi}{dT}\boldsymbol{H}(T)\mathbf{f}_0(T) + \boldsymbol{A}(T)\mathbf{f}_0(T)\right)e^{\frac{\phi(T)}{\tilde{\epsilon}}} &= 0\\
\tilde{\epsilon}\left(\frac{d\phi}{dT}\boldsymbol{H}(T)\mathbf{f}_1(T) + \frac{d\mathbf{f}_0}{dT} + \boldsymbol{A}(T)\,\mathbf{f}_1(T)\right)e^{\frac{\phi(T)}{\tilde{\epsilon}}} &= \tilde{\epsilon}\mathbf{y}(T)\\
\cdots &= \cdots\\
\tilde{\epsilon}^n\left(\frac{d\phi}{dT}\boldsymbol{H}(T)\mathbf{f}_n(T) + \frac{d\mathbf{f}_{n-1}}{dT} + \boldsymbol{A}(T)\,\mathbf{f}_n\right)e^{\frac{\phi(T)}{\tilde{\epsilon}}} &= 0
\end{aligned}$$

Multiple–scales non–homogeneous approach

0th order
Eigenvalue problem

$$[\boldsymbol{A}(T) + \lambda_k(T)\boldsymbol{H}(T)]\,\mathrm{f}_0(T) = 0$$

$$\Downarrow$$

$$\boxed{\lambda_k(T), \tilde{\mathbf{u}}_k(T)} \qquad \mathbf{f}_0(T) = c_k(T)\tilde{\mathbf{u}}_k(T)$$

1st order
Singular problem

$$[\boldsymbol{A}(T) + \lambda_k(T)\boldsymbol{H}(T)]\,\mathrm{f}_1(T) = -\frac{d\mathbf{f}_0}{dT} + \mathbf{y}(T)e^{-\frac{\phi(T)}{\tilde{\epsilon}}}$$

$$\Downarrow$$

Solvability condition

$$\boxed{\tilde{\mathbf{v}}_k(T) \cdot \left(-\frac{d\mathbf{f}_0}{dT} + \mathbf{y}(T)e^{-\frac{\phi(T)}{\tilde{\epsilon}}}\right) = 0}$$

Multiple–scales non–homogeneous approach

Solvability condition ($\mathbf{f}_0(T) = c_k(T)\tilde{\mathbf{u}}_k(T)$):

$$\tilde{\mathbf{v}}_k(T) \cdot \tilde{\mathbf{u}}_k(T)\frac{dc_k}{dT} + \tilde{\mathbf{v}}_k(T) \cdot \frac{d\tilde{\mathbf{u}}_k(T)}{dT}c_k = \tilde{\mathbf{v}}_k(T) \cdot \mathbf{y}(T)e^{-\frac{\phi(T)}{\tilde{\epsilon}}}$$

closed–form solution for $c_k(T) \Rightarrow c_k \cdot \tilde{\mathbf{u}}_k$ unique

$$\mathbf{x}(T_f) = c_k(T_f)\tilde{\mathbf{u}}_k(T_f)e^{\frac{\phi(T_f)}{\tilde{\epsilon}}} + \mathcal{O}(\tilde{\epsilon})$$

$$\Downarrow$$

$$\boxed{\mathbf{x}(T_f) = \tilde{\mathbf{u}}_k(T_f)\int_{T_0}^{T_f} \mathbf{r}(T) \cdot \mathbf{y}(T)\, dT + \mathcal{O}(\tilde{\epsilon})}$$

$\mathbf{r}$: **receptivity** vector

General receptivity formulation

$$
\begin{aligned}
\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} &= \tilde{\epsilon} \hat{S}^m \\
\frac{\partial \hat{u}}{\partial \hat{t}} + \hat{U}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{u}\frac{\partial \hat{U}}{\partial \hat{x}} + \hat{V}\frac{\partial \hat{u}}{\partial \hat{y}} + \hat{v}\frac{\partial \hat{U}}{\partial \hat{y}} &= -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{1}{R}\nabla^2 \hat{u} + \tilde{\epsilon}\hat{S}^x \\
\frac{\partial \hat{v}}{\partial \hat{t}} + \hat{U}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{u}\frac{\partial \hat{V}}{\partial \hat{x}} + \hat{V}\frac{\partial \hat{v}}{\partial \hat{y}} + \hat{v}\frac{\partial \hat{V}}{\partial \hat{y}} &= -\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{1}{R}\nabla^2 \hat{v} + \tilde{\epsilon}\hat{S}^y
\end{aligned}
$$

$$
\begin{aligned}
\hat{u}(\hat{x}, \hat{y} = 0, \hat{t}) &= \tilde{\epsilon}\hat{u}_{\text{wall}}(\hat{x}, \hat{t}) & \qquad \hat{u}(\hat{x}, \hat{y} \to \infty, \hat{t}) &\to \tilde{\epsilon}\hat{u}_{\infty}(\hat{x}, \hat{t}) \\
\hat{v}(\hat{x}, \hat{y} = 0, \hat{t}) &= \tilde{\epsilon}\hat{v}_{\text{wall}}(\hat{x}, \hat{t}) & \qquad \hat{v}(\hat{x}, \hat{y} \to \infty, \hat{t}) &\to \tilde{\epsilon}\hat{v}_{\infty}(\hat{x}, \hat{t}) \\
\hat{p}(\hat{x}, \hat{y} = 0, \hat{t}) &= \tilde{\epsilon}\hat{p}_{\text{wall}}(\hat{x}, \hat{t}) & \qquad \hat{p}(\hat{x}, \hat{y} \to \infty, \hat{t}) &\to \tilde{\epsilon}\hat{p}_{\infty}(\hat{x}, \hat{t})
\end{aligned}
$$

$$
R = \frac{\delta_r^* U_\infty^*}{\nu^*} = \sqrt{\frac{x_r^* U_\infty^*}{\nu^*}} = \sqrt{Re_{x_r^*}}
$$

⇒ **Given base flow** for a general boundary layer profile

⇒ **Disturbance** treated using **multiple–scale** approach

General receptivity formulation

$$x = \tilde{\epsilon}\hat{x}, \quad y = \hat{y}, \quad t = \hat{t},$$

$$U(x,y) = \hat{U}(\hat{x},\hat{y}), \quad V(x,y) = \frac{\hat{V}(\hat{x},\hat{y})}{\tilde{\epsilon}}, \quad S(x,y,t) = \hat{S}(\hat{x},\hat{y},\hat{t})$$

$$q(x,y,t) = (q_0(x,y) + \tilde{\epsilon} q_1(x,y) + \cdots)\, e^{\frac{i\theta(x)}{\tilde{\epsilon}} - i\omega t}, \qquad \frac{\partial \theta}{\partial x} = \alpha$$

0^{th} order $\boldsymbol{A}(\alpha,\omega,R)\,\mathrm{f}_0 \;=\; 0 \quad \Rightarrow \quad \boxed{\alpha, \tilde{\mathbf{f}}_0 \quad (\mathrm{f}_0 = c\tilde{\mathbf{f}}_0)}$

1^{st} order $\boldsymbol{A}(\alpha,\omega,R)\,\mathrm{f}_1 \;=\; -\boldsymbol{H}(\alpha,R)\frac{d\mathrm{f}_0}{dx} - \boldsymbol{C}(\alpha,R)\,\mathrm{f}_0 + \mathbf{y}(x,\omega)e^{-\frac{i\theta(x)}{\tilde{\epsilon}} + i\omega t}$

$\Downarrow$

Solvability condition: $\mathbf{y}^* \cdot$RHS$= 0$ $\Rightarrow$ equation for c

Standard multiple–scales but with $\mathbf{y}$

General receptivity formulation

$$\boxed{\frac{dc}{dx}+\frac{a_2}{a_1}c=\frac{\mathbf{y}^*\cdot\mathbf{y}}{a_1}e^{-\frac{i\theta}{\tilde{\epsilon}}+i\omega t}} \quad \Rightarrow \quad c(x_f)$$

$$\left| \begin{array}{lcl} a_1 & = & \mathbf{y}^*\cdot\left(\boldsymbol{H}\tilde{\mathbf{f}}_0\right) \\ a_2 & = & \mathbf{y}^*\cdot\left(\boldsymbol{H}\frac{d\tilde{\mathbf{f}}_0}{dx}+\boldsymbol{C}\tilde{\mathbf{f}}_0\right) \end{array}\right.$$

a_2 account for non–parallel corrections

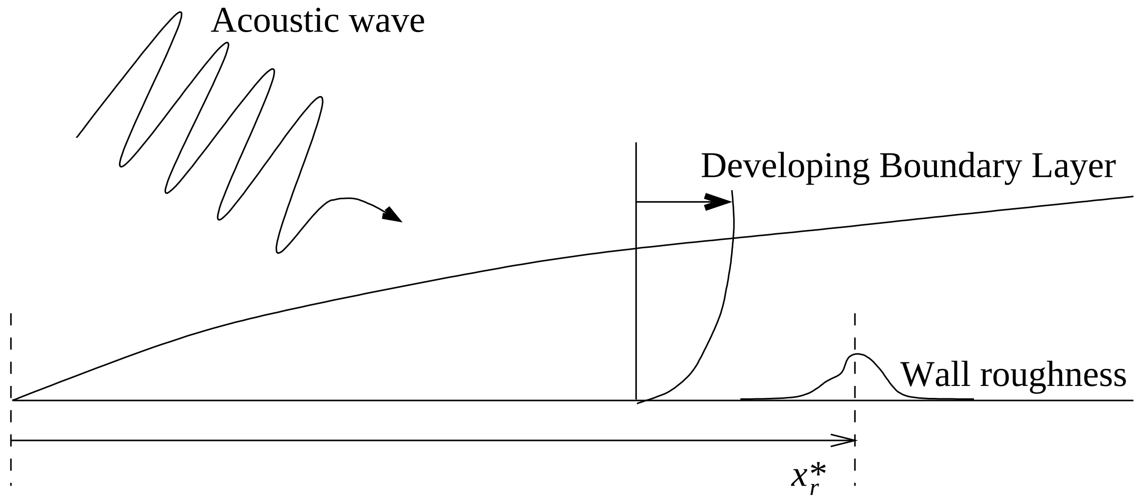
$$\mathbf{f}(x_f) \;=\; \left[\int_{x_0}^{x_f}\frac{\mathbf{y}^*(x)\cdot\mathbf{y}(x)\,e^{-\frac{i\theta(x)}{\tilde{\epsilon}}}}{a_1(x)}\,e^{\int_x^{x_f}-\frac{a_2(x')}{a_1(x')}\,dx'}\,dx\right]\tilde{\mathbf{f}}_0(x_f)e^{\frac{i\theta(x_f)}{\tilde{\epsilon}}}+\mathcal{O}(\tilde{\epsilon})$$

In a compact form:

$$\boxed{\mathbf{f}(x_f)=\tilde{\mathbf{f}}_0(x_f)\int_{x_0}^{x_f}\mathbf{r}(x)\cdot\mathbf{y}(x)\,dx+\mathcal{O}(\tilde{\epsilon})}$$

$\mathbf{r}(x)$ is the **adjoint** times a constant and represents the **sensitivity** of $\mathbf{f}(x_f)$ to $\mathbf{y}(x)$

Acoustic waves – wall roughness interaction



Regular perturbation series expansion:

$$\bar{\mathbf{v}}(x,y,t) = \mathbf{V}(x,y) + \epsilon \mathbf{v}_\epsilon(x,y)e^{-i\omega t} + \delta \mathbf{v}_\delta(x,y) + \epsilon\delta \mathbf{v}_{\epsilon\delta}(x,y)e^{-i\omega t} + \cdots$$

Base flow: Blasius solution

Disturbance: three problem at three orders ϵ, δ, $\epsilon\delta$

Acoustic waves – wall roughness interaction

Boundary conditions:

$$\begin{array}{rclccl} u & \to & 1+\epsilon e^{-i\omega t} & \text{as} & y & \to \infty \\ v & \to & 0 & \text{as} & y & \to \infty \\ u=v & = & 0 & \text{at} & y & = \delta h(x) \end{array}$$

Linearization of boundary conditions if $\delta \ll \delta_{\mathsf{ST}}$ and $\delta \ll \lambda_h$:

$$\bar{\mathbf{v}}(x,y,t) = \bar{\mathbf{v}}(x,0,t) + \delta h(x) \frac{\partial \bar{\mathbf{v}}(x,y,t)}{\partial y}\bigg|_{y=0} + \delta^2 h^2(x) \frac{1}{2} \frac{\partial^2 \bar{\mathbf{v}}(x,y,t)}{\partial y^2}\bigg|_{y=0} + \cdots$$

$$\begin{array}{lrcl} \text{Order } \epsilon & \mathbf{v}_\epsilon(x,0) & = & 0 \\ \text{Order } \delta & \mathbf{v}_\delta(x,0) & = & -h(x) \left.\dfrac{\partial \mathbf{V}(x,y)}{\partial y}\right|_{y=0} \\ \text{Order } \epsilon\delta & \mathbf{v}_{\epsilon\delta}(x,0) & = & -h(x) \left.\dfrac{\partial \mathbf{v}_\epsilon(x,y)}{\partial y}\right|_{y=0} \end{array}$$

Order ϵ

Multiple–scales:

0th order

$$\frac{\partial v_{0\epsilon}}{\partial y} = 0 \qquad u_{0\epsilon} \to 1 \quad \text{as} \quad y \to \infty$$

$$-i\omega u_{0\epsilon} - \frac{1}{R}\frac{\partial^2 u_{0\epsilon}}{\partial y^2} = 0 \qquad v_{0\epsilon} \to 0 \quad \text{as} \quad y \to \infty$$

$$\frac{\partial p_{0\epsilon}}{\partial y} = 0 \qquad u_{0\epsilon} = 0 \quad \text{at} \quad y = 0$$

1st order

$$\frac{\partial v_{1\epsilon}}{\partial y} = 0 \qquad u_{1\epsilon} \to 0 \quad \text{as} \quad y \to \infty$$

$$-i\omega u_{1\epsilon} - \frac{1}{R}\frac{\partial^2 u_{1\epsilon}}{\partial y^2} = -u_{0\epsilon}\frac{\partial U}{\partial x} - V\frac{\partial u_{0\epsilon}}{\partial y} \qquad v_{1\epsilon} \to 0 \quad \text{as} \quad y \to \infty$$

$$\frac{\partial p_{1\epsilon}}{\partial y} = -u_{0\epsilon}\frac{\partial V}{\partial x} \qquad u_{1\epsilon} = 0 \quad \text{at} \quad y = 0$$

For $R = 582$ and $F = 49.34 \cdot 10^{-6}$ $(F = \omega^* \nu^* / U_\infty^{*2})$ the correction $u_{1\epsilon} \simeq 0.01 u_{0\epsilon}$

$$\boxed{\epsilon \mathbf{v}_\epsilon(x,y)e^{-i\omega t} = \epsilon(1 - e^{-\sqrt{-i\omega R}\,y}, 0)e^{-i\omega t}}$$

Order δ

$$
\begin{aligned}
\frac{\partial u_{0\delta}}{\partial x} + \frac{\partial v_{0\delta}}{\partial y} &= 0 \\
\frac{\partial u_{0\delta}}{\partial x}U - \frac{1}{R}\left(\frac{\partial^2 u_{0\delta}}{\partial x^2} + \frac{\partial^2 u_{0\delta}}{\partial y^2}\right) + \frac{\partial U}{\partial y}v_{0\delta} + \frac{\partial p_{0\delta}}{\partial x} &= 0 \\
\frac{\partial v_{0\delta}}{\partial x}U - \frac{1}{R}\left(\frac{\partial^2 v_{0\delta}}{\partial x^2} + \frac{\partial^2 v_{0\delta}}{\partial y^2}\right) + \frac{\partial p_{0\delta}}{\partial y} &= 0
\end{aligned}
$$

$$
u_{0\delta}(x,0) = -h(x)\left.\frac{\partial U}{\partial y}\right|_{y=0}
$$

$$
v_{0\delta}(x,0) = -h(x)\left.\frac{\partial V}{\partial y}\right|_{y=0}
$$

$$
\boxed{\boldsymbol{L}(x, \frac{\partial}{\partial x})\bar{\mathbf{f}}_\delta(x) = \mathbf{y}_\delta(x)h(x)}
$$

$\bar{\mathbf{f}}_\delta(x)\, e^{-i\int \alpha\, dx}$ appears at order $\epsilon\delta$.

Adjoint definition:

$$
\mathbf{g}\, e^{i\int \alpha\, dx} \cdot \boldsymbol{L}(\mathbf{f}) = \mathbf{f} \cdot \hat{\boldsymbol{L}}(\mathbf{g}\, e^{i\int \alpha\, dx}) = \mathbf{f} \cdot \hat{\boldsymbol{L}}(x,\alpha)\mathbf{g}\, e^{i\int \alpha\, dx}
$$

$\bar{\mathbf{f}}_\delta = \mathbf{f}_\delta\, e^{i\int \alpha\, dx}$ and $\frac{\partial}{\partial x} = i\alpha$ introduced:

$$
\boxed{\boldsymbol{A}\,(\alpha, R)\,\mathbf{f}_\delta(x) = \mathbf{y}_\delta(x)h(x)e^{-i\int \alpha\, dx}}
$$

Order $\epsilon\delta$

Application of the non–homogeneous multiple–scales approach:

$\mathcal{O}(0)$: Eigenvalue problem

$$\boldsymbol{A}(\alpha,\omega,R)\,\mathbf{f}_{0\epsilon\delta}(x) = 0 \quad \Rightarrow \quad \boxed{\alpha, \tilde{\mathbf{f}}_{0\epsilon\delta}} \qquad (\mathbf{f}_{0\epsilon\delta} = c\tilde{\mathbf{f}}_{0\epsilon\delta})$$

$\mathcal{O}(\tilde{\epsilon})$: Singular problem

$$\boldsymbol{A}(\alpha,\omega,R)\,\mathbf{f}_{1\epsilon\delta}(x) = -\boldsymbol{H}(\alpha,R)\frac{d\mathbf{f}_{0\epsilon\delta}}{dx} - \boldsymbol{C}(\alpha,R)\,\mathbf{f}_{0\epsilon\delta} + \tilde{\mathbf{y}}_{\epsilon\delta}(x,\omega)e^{-i\int_{x_0}^{x}\alpha\,dx'}$$

$\Downarrow$

Solvability condition $\Rightarrow$ c

$$\boxed{\mathbf{f}_{\epsilon\delta}(x_f)e^{-i\omega t} = \tilde{\mathbf{f}}_{0\epsilon\delta}(x_f)\left(\int_{x_0}^{x_f}\mathbf{r}(x)\cdot\tilde{\mathbf{y}}_{\epsilon\delta}(x)\,dx\right)e^{-i\omega t} + \mathcal{O}(\tilde{\epsilon})}$$

Application of multiple–scales approach

What is $\tilde{\mathbf{y}}_{\epsilon\delta}(x)$?

1. from the equations:

$$S^x = \left(-i\alpha u_\epsilon \hat{u}_\delta - \hat{v}_\delta \frac{\partial u_\epsilon}{\partial y}\right) h(x)$$

$$S^y = \left(-i\alpha u_\epsilon \hat{v}_\delta\right) h(x)$$

2. from boundary conditions:

$$u_{\epsilon\delta\text{wall}} = \left(-\left.\frac{\partial u_\epsilon}{\partial y}\right|_{y=0}\right) h(x)$$

$$\tilde{\mathbf{y}}_{\epsilon\delta}(x) = \hat{\mathbf{y}}_{\epsilon\delta}(x) h(x)$$

$$\Downarrow$$

$$\int_{x_0}^{x_f} \mathbf{r}(x) \cdot \tilde{\mathbf{y}}_{\epsilon\delta}(x)\, dx = \int_{x_0}^{x_f} \mathbf{r}(x) \cdot \hat{\mathbf{y}}_{\epsilon\delta}(x) h(x)\, dx = \int_{x_0}^{x_f} r_h(x) h(x)\, dx$$

Application of multiple–scales approach

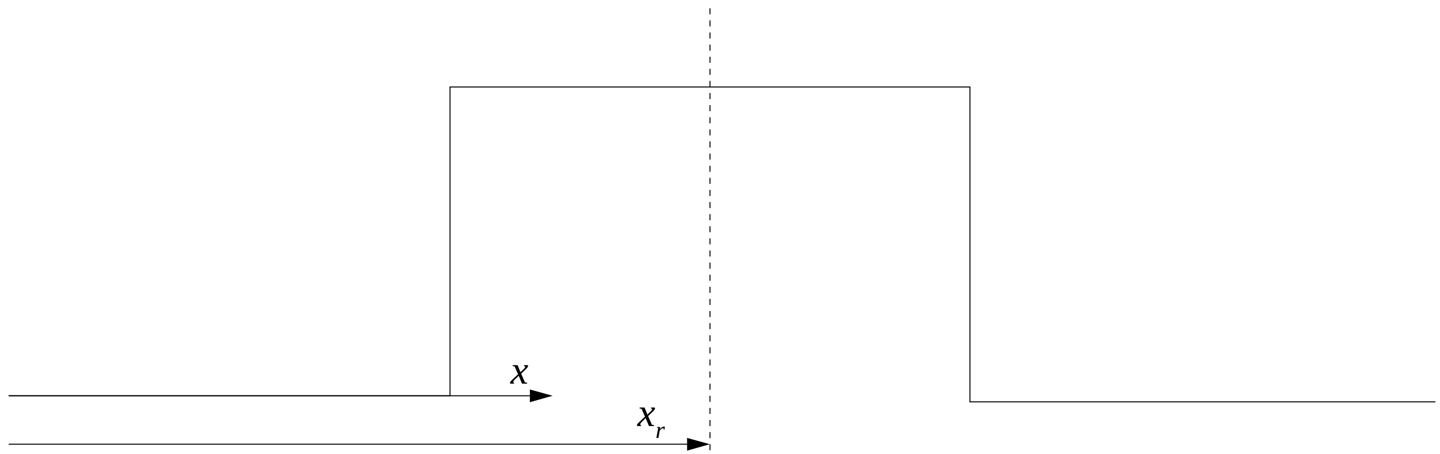
$$\mathbf{f}_{\epsilon\delta}(x_f)e^{-i\omega t} = \tilde{\mathbf{f}}_{0\epsilon\delta}(x_f)\left(\int_{x_0}^{x_f} r_h(x)h(x)\,dx\right)e^{-i\omega t} + \mathcal{O}(\tilde{\epsilon})$$

$$\max\left|\tilde{u}_{0\epsilon\delta}(x_f,y)\right| = 1 \Rightarrow \quad A(x_f) = \left|\epsilon\delta\int_{x_0}^{x_f} r_h(x)h(x)\,dx\right|$$

$A(x_f)$ depends on the normalization of $\tilde{\mathbf{f}}_{0\epsilon\delta}$

$$\begin{aligned} r_h(x) &= \frac{\mathbf{y}^*\cdot\widehat{\mathbf{y}}_{\epsilon\delta}}{\mathbf{y}^*\cdot\left(\boldsymbol{H}\tilde{\mathbf{f}}_0\right)}\,\mathrm{EXP}\left[-\int_x^{x_f}\left(\frac{\mathbf{y}^*\cdot\left(\boldsymbol{H}\frac{d\tilde{\mathbf{f}}_0}{dx}+\boldsymbol{C}\tilde{\mathbf{f}}_0\right)}{\mathbf{y}^*\cdot\left(\boldsymbol{H}\tilde{\mathbf{f}}_0\right)} - \frac{i\alpha}{\tilde{\epsilon}}\right)dx'\right] \\ &= \widehat{r}_h(x)\,e^{-\int_x^{x_f} a(x')\,dx'} \end{aligned}$$

Comparisons

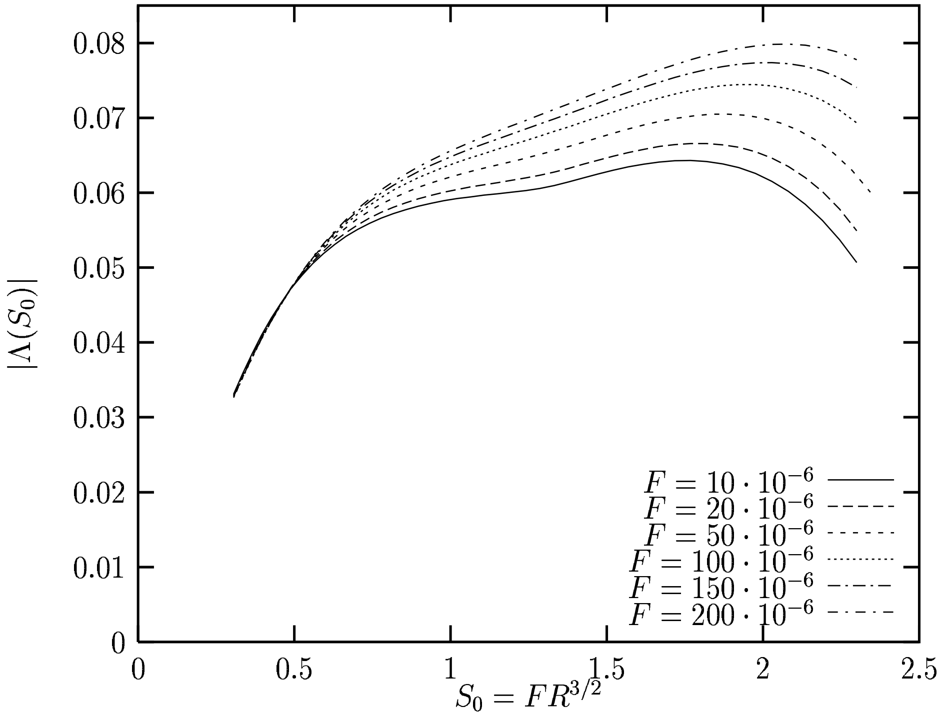


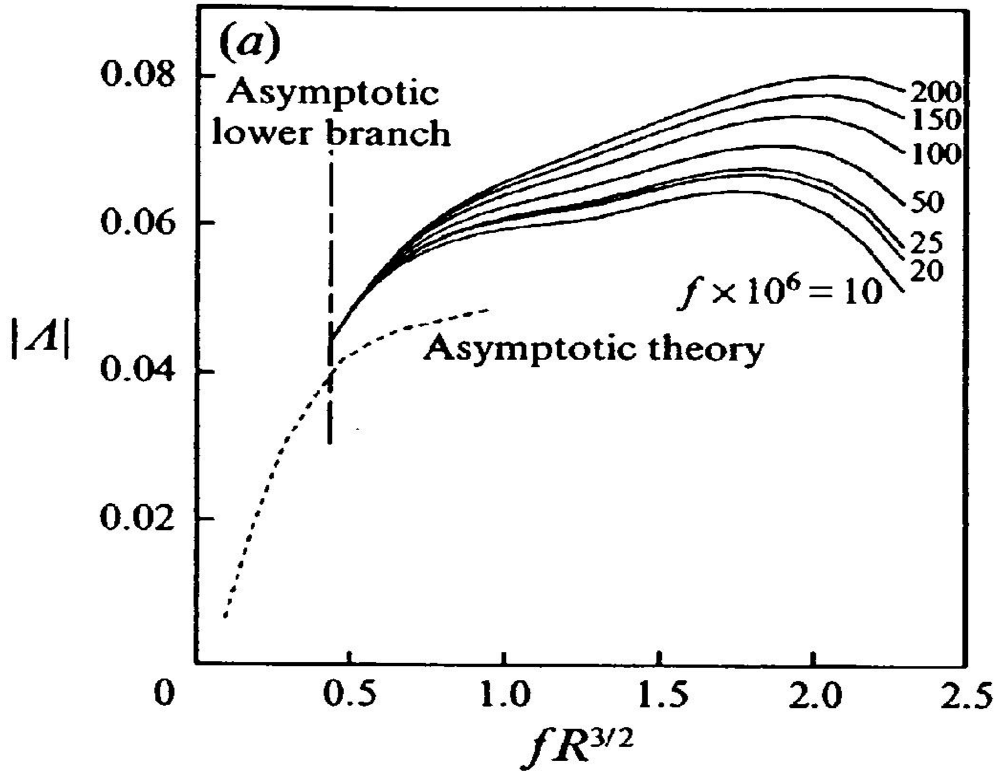
$$A(x_f) = \left| \epsilon\delta \int_{x_0}^{x_f} h(x)\widehat{r}_h(x) e^{-\int_x^{x_f} a(x')\,dx'}\,dx \right|$$

$$A(x_f) = \left| \epsilon\delta e^{-\int_{x_r}^{x_f} a(x')\,dx'} \int_{-\infty}^{\infty} h(x)\widehat{r}_h(x) e^{-\int_x^{x_r} a(x')\,dx'}\,dx \right|$$

Crouch: $A(x_r) = \left| \epsilon\delta \widehat{r}_h(x_r) \int_{-\infty}^{\infty} h(x) e^{-i\bar{\alpha}x}\,dx \right| = |\epsilon\delta\widehat{r}_h(x_r)H(\bar{\alpha})|$

Comparisons





$$\hat{r}_h(x_r) = \Lambda(S_0) = -\frac{1}{R}\left(\frac{dU}{dy}\frac{d\tilde{U}}{dy} + \frac{du}{dy}\frac{d\tilde{u}_{\alpha\omega}}{dy}\right)_{y=0}$$

$$S_0 = FR^{3/2},\ F = \frac{\omega^* \nu^*}{U_\infty^{*2}} = \frac{\omega}{R}$$

Comparisons

Saric *et al.*: $R = 582$, $F = 49.34 \cdot 10^{-6}$, $R_f = \sqrt{Re_{x_f^*}} = 1121$

$$\left| \int_{-\infty}^{\infty} h(x) \widehat{r}_h(x) e^{-\int_x^{x_r} a(x')\, dx'}\, dx \right| = \frac{A(x_r)}{\epsilon\delta} \quad = 1.22976 \text{ non–parallel}$$

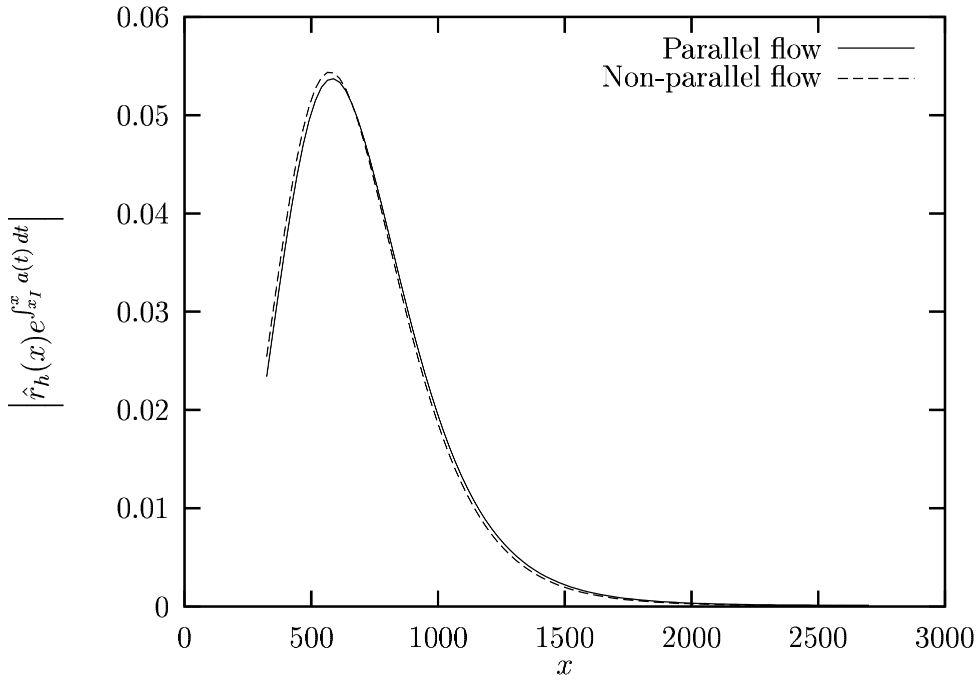
$$|\epsilon\delta \widehat{r}_h(x_r) H(\alpha)| = \frac{A(x_r)}{\epsilon\delta} \quad = 1.22283 \text{ Crouch}$$

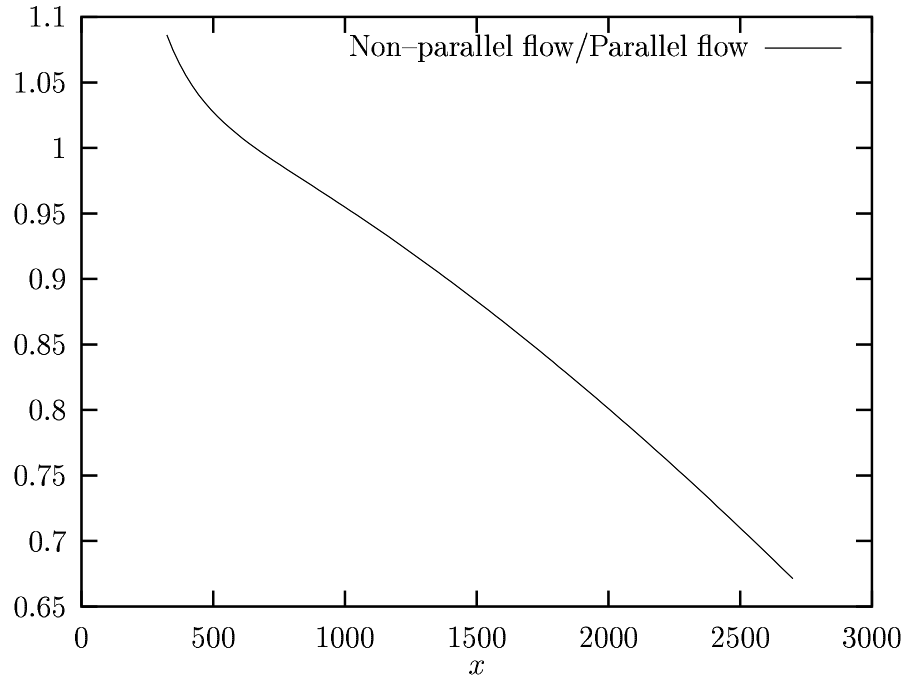
$$\frac{A(x_r)}{\epsilon\delta} \left| e^{-\int_{x_r}^{x_f} a(x')\, dx'} \right| = \frac{A(x_f)}{\epsilon\delta} \quad = 126.139 \text{ non–parallel}$$

$$\frac{A(x_r)}{\epsilon\delta} \left| e^{i \int_{x_r}^{x_f} \alpha(x')\, dx'} \right| = \frac{A(x_f)}{\epsilon\delta} \quad = 96.063 \text{ Crouch}$$

At R_f: **difference** between parallel and non–parallel due to the amplification

Non–parallel corrections





$$A(x_f) = \left| \epsilon \delta e^{-\int_{x_I}^{x_f} a(x')\, dx'} \int_{-\infty}^{\infty} h(x) \widehat{r}_h(x) e^{-\int_{x}^{x_I} a(x')\, dx'}\, dx \right|$$

Conclusions

- √ Corrections for **non–parallel effects** without additional costs implied
- √ General validity for **real geometries**
- √ Suitable for more complex boundary layer profiles and **wing design**
- √ **Fast tool**: possible integration in an industrial code for the transition prediction

Further developments

- ♢ Integration in an **industrial code** for the transition prediction (wing design)
- ♢ Application to different and more **complex boundary layers**: Falkner–Skan, stagnation point or given base flow from real problem
- ♢ Extension to **3D**