

# Glossary of Z notation

Based on appendix A in [The Way of Z](#).

## Names

a,b	identifiers
d,e	declarations (e.g., a: A; b, ...: B ...)
f,g	functions
m,n	numbers
p,q	predicates
s,t	sequences
x,y	expressions
A,B	sets
C,D	bags
Q,R	relations
S,T	schemas
X	schema text (e.g., d, d   p, or S)

## Logic

true	Logical true constant
false	Logical false constant
$\neg p$	Logical negation, <i>not</i>
$P \wedge q$	Logical conjunction, <i>and</i>
$P \vee q$	Logical disjunction, <i>or</i>
$P \Rightarrow q$	Logical implication
$P \Leftrightarrow q$	Logical equivalence
$\forall X \bullet q$	Universal quantification
$\exists X \bullet q$	Existential quantification
(let a == x; ... • p)	Local definition

## Relations

$A \leftrightarrow B$	Binary relation ( $\mathbb{P} (A \times B)$ )
$A \mapsto b$	Maplet ((a,b))
Dom R	Domain of a relation
ran R	Range of a relation
$Q \ ; \ R$	Forward relational composition
$Q \ \circ \ R$	Backward relational composition ( $R \ ; \ Q$ )
$A \triangleleft R$	Domain restriction
$A \triangleleft R$	Domain anti-restriction
$A \triangleright R$	Range restriction
$A \triangleright R$	Range anti-restriction
$R \ \downarrow \ A$	Relational image
$R \ \sim$	Inverse of relation
$R \ +$	Transitive closure
$Q \ \oplus \ R$	Relational overriding

## Definitions

a == x	Abbreviation definition
a ::= b   ...	Free type definition
[a]	Introduction of a given set (or [a,...])
a _	Prefix operator
_ a	Postfix operator
_ a _	Infix operator

## Sets and expressions

$x = y$	Equality
$x \neq y$	Inequality
$x \in A$	Set membership
$x \notin A$	Non-membership
$\emptyset$	Empty set
$A \subseteq B$	Set inclusion
$A \subset B$	Strict set inclusion
{ x, y, ... }	Set display
{ X • x }	Set comprehension
( $\lambda X \bullet x$ )	Lambda expression
(let a == x; ... • y)	Local definition
if p then x else y	Conditional expression
(x,y, ...)	Tuple
(x,y)	Pair
$A \times B \times \dots$	Cartesian product
$\mathbb{P}A$	Power set
$A \cap B$	Set intersection
$A \cup B$	Set union
$A \setminus B$	Set difference
first x	First element of an ordered pair
second x	Second element of an ordered pair
# A	Number of elements in a set

## Functions

$A \leftrightarrow B$	Partial functions
$A \rightarrow B$	Total functions
$A \rightsquigarrow B$	Partial injections
$A \mapsto B$	Total injections
$A \twoheadrightarrow B$	Bijections
f x	Function application (or f(x))

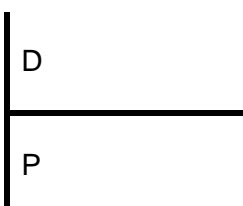
## Numbers

$\mathbb{Z}$	Set of integers
$\mathbb{N}$	Set of natural numbers { 0, 1, 2, ... }
$\mathbb{N}_1$	Set of strictly positive numbers { 1, 2, ... }
$m+n$	Addition
$m-n$	Subtraction
$m*n$	Multiplication
$m \text{ div } n$	Division
$m \text{ mod } n$	Remainder (modulus)
$m \leq n$	Less than or equal
$m < n$	Less than
$m \geq n$	Greater than or equal
$m > n$	Greater than
$m .. n$	Number range
$\min A$	Minimum of a set of numbers
$\max A$	Maximum of a set of numbers

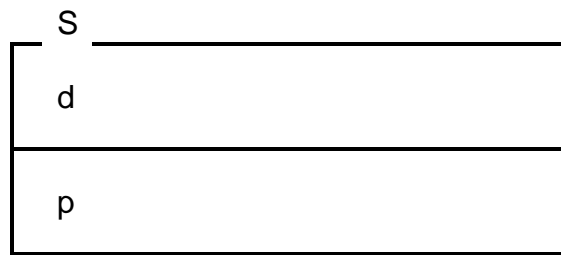
## Sequences

$\text{seq } A$	Set of finite sequences
$\text{seq}_1 A$	Set of non-empty finite sequences
$\text{iseq } A$	Set of finite injective sequences
$\langle \rangle$	Empty sequence
$\langle x, y, \dots \rangle$	Sequence display
$s \hat{\ } t$	Sequence concatenation
$\text{head } s$	First element of a sequence
$\text{tail } s$	All but the head element of a sequence
$\text{Last } s$	Last element of a sequence
$\text{front } s$	All but the last element of a sequence
$s \text{ in } t$	Sequence segment relation

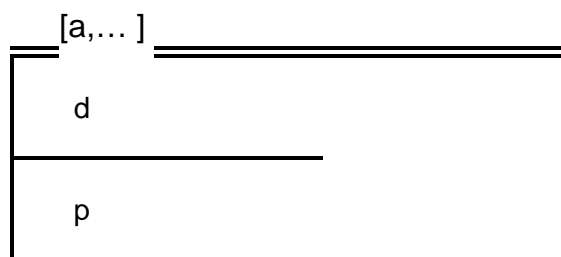
## Axiomatic definition



## Schema



## Generic Definition



## Schema Calculus

$S \triangleq [ X ]$	Horizontal schema
$[ T; \dots   \dots ]$	Schema inclusion
$z.a$	Component selection (given $z:S$ )
$\theta S$	Binding
$\neg S$	Schema negation
$S \wedge T$	Schema conjunction
$S \vee T$	Schema disjunction
$S \ddagger T$	Schema composition
$S \gg T$	Schema piping

## Conventions

$A?$	Input to an operation
$A!$	Output from an operation
$A$	State component before an operation
$A'$	State component after an operation
$S$	State schema before an operation
$S'$	State schema after an operation
$\Delta S$	Change of state
$\Xi S$	No change of state