Kőnig's Edge Coloring Theorem without augmenting paths

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We give a simple, self-contained proof of the following basic fact [1, 2] in matching theory:

Theorem *Every bipartite regular multigraph is factorizable.*

Proof: Assume G to be a counterexample with the smallest number of edges. Then G is r-regular for some integer $r \geq 1$. Let $e = uv$ be any edge of G. In G, remove nodes u and v, then add a set of edges F as to obtain a bipartite multigraph G' which is r-regular. Note that $|F| < r$ and G' has less edges than G. By assumption G' contains r disjoint 1-factors. Since $|F| < r$, at least one of these 1-factors, say M', is disjoint from F. Therefore $M = M' \cup \{e\}$ is a 1-factor of G. Moreover $G \setminus M$ has less edges than G and is factorizable. But then G is factorizable. $\square$

Note that the above proof does not use any alternating path argument.

References

[1] D. Kőnig, Ű ber Graphen und ihre Andwendung auf Determinantentheorie und Mengenlehre, Math. Ann. 77 (1916) 453–465.

[2] D. Kőnig, Graphok és alkalmazásuk a determinánsok és a halmazok elméletére, Math. Termész. Ért. 34 (1916) 104–119.