ADJOINING DECLASSIFICATION AND ATTACK MODELS BY ABSTRACT INTERPRETATION

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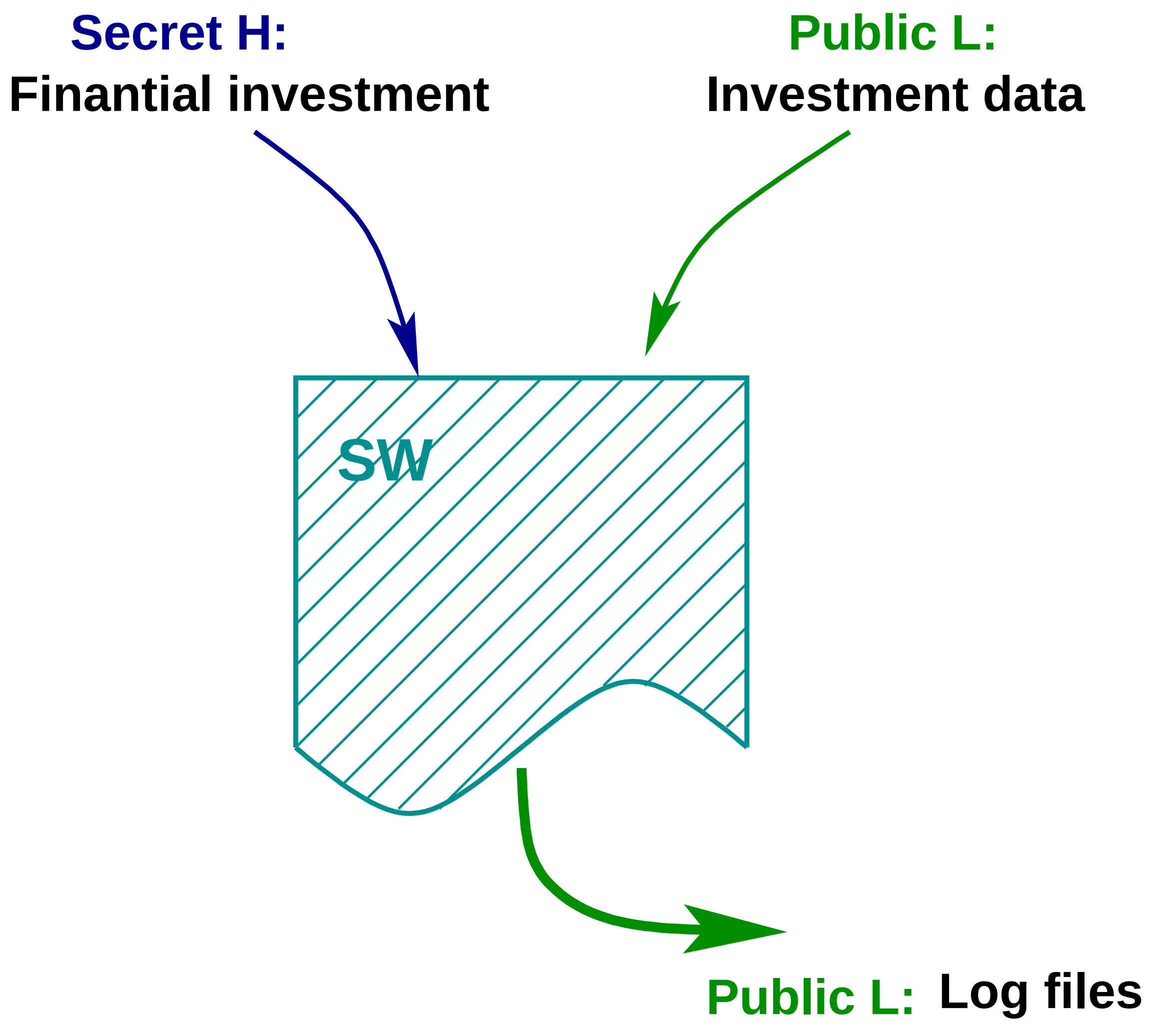
The Problem: Non-Interference

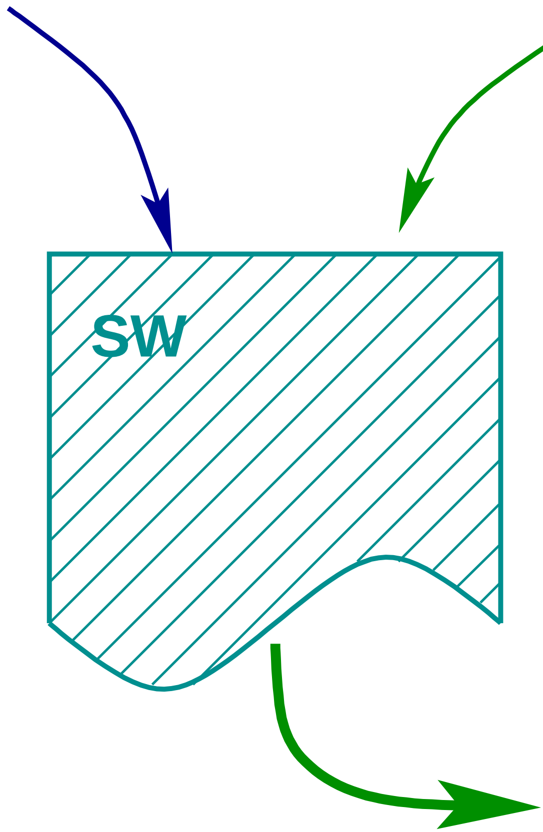


[Sabelfeld and Sands'01]

The Problem: Non-Interference

Secret H:
Finantial investment

Public L:
Investment data



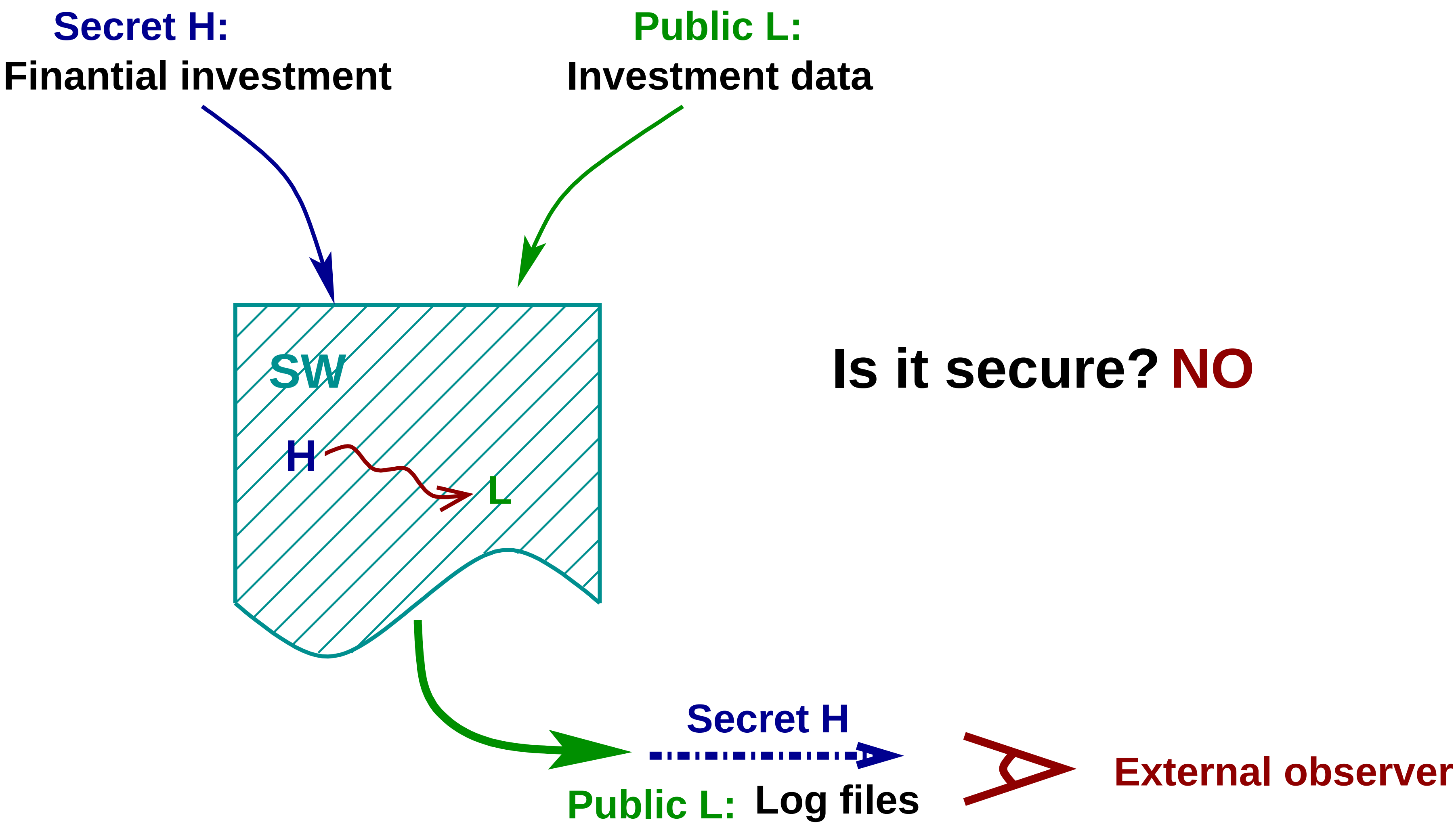
Is it secure?

Public L: Log files

External observer

[Sabelfeld and Sands'01]

The Problem: Non-Interference



Is it secure? NO

[Sabelfeld and Sands'01]

Background: Non-Interference

SECURITY PROPERTY: States which classes have not to interfere with other classes of objects.

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$$\Downarrow$$

Confi nement problem[Lampson'73]: *Preventing the results of computations leaking even partial information about the confidential inputs.*

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Confinement problem[Lampson'73]: *Preventing the results of computations leaking even partial information about the confidential inputs.*

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Non-interference policies require that any *change upon confidential data has not to be revealed through the observation of public data.*

- Many real systems are intended to leak some kind of information
- Even if a system satisfies non-interference, some kind of tests could reject it as insecure

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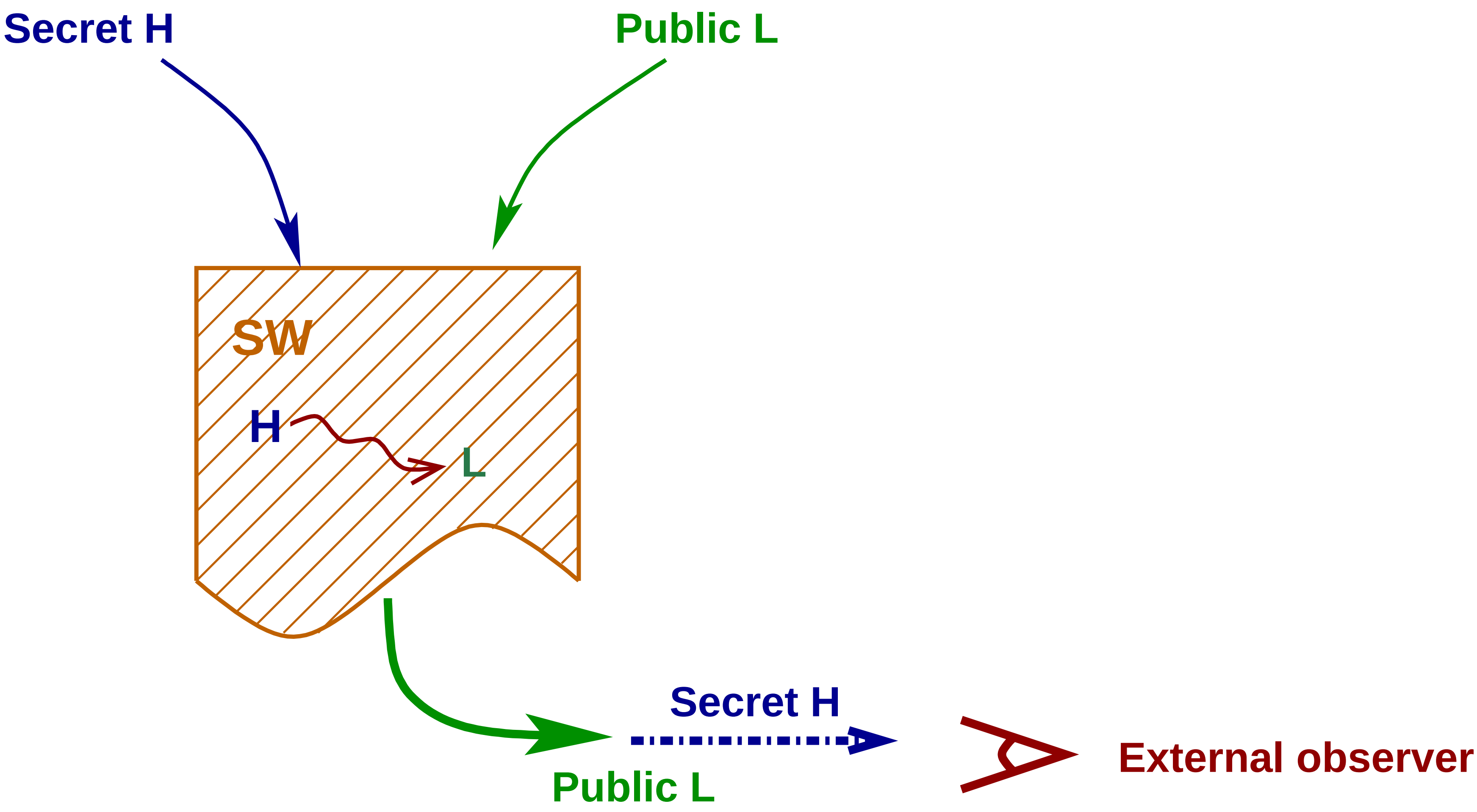
Non-interference policies require that any *change upon confidential data has not to be revealed through the observation of public data.*

- Characterizing released information: [Cohen'77], [Zdancewic & Myers'01], [Clark et al.'04], [Lowe'02];
- Constraining attackers: [Di Pierro et al.'02], [Laud'01].

Our idea: Abstracting Non-Interference



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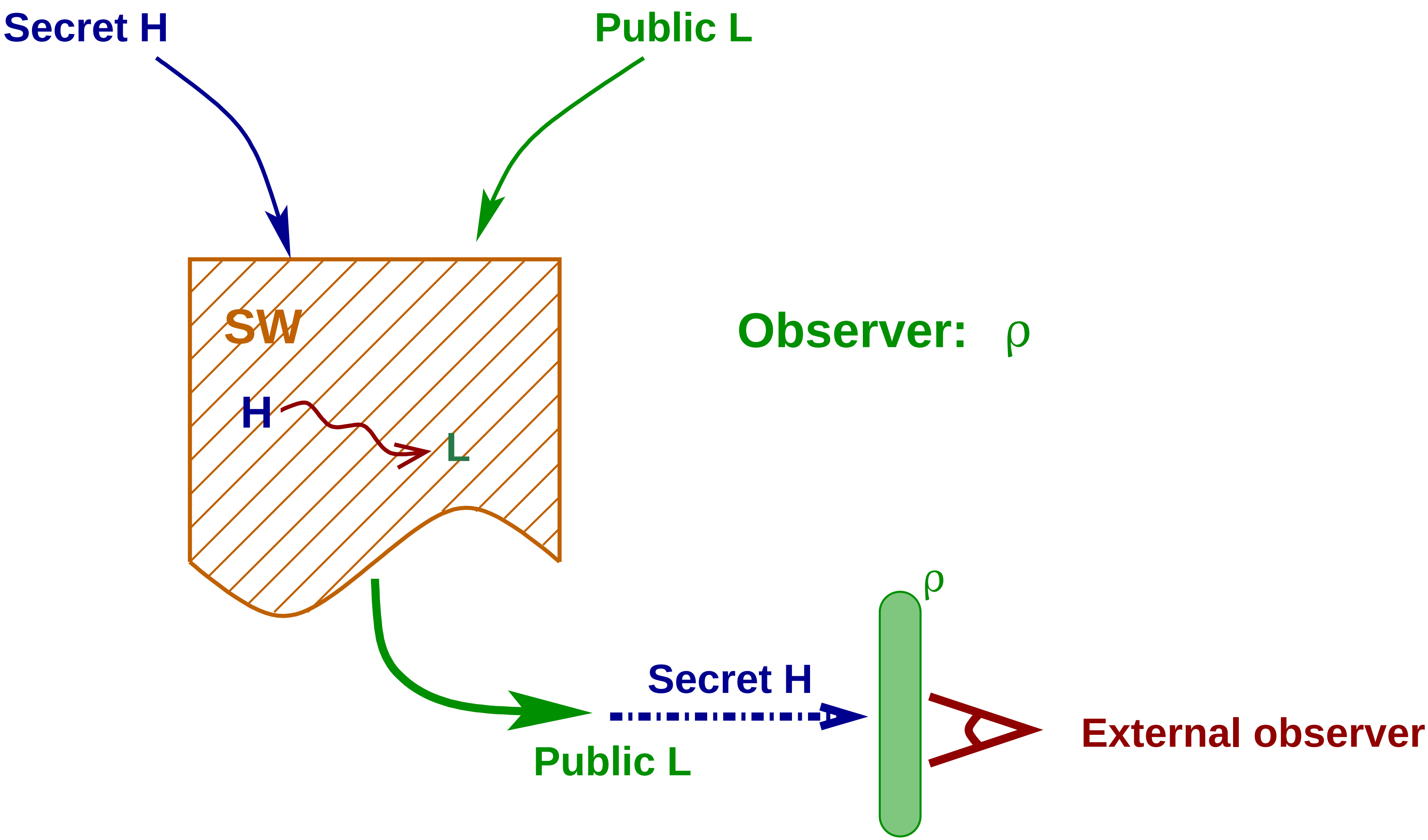

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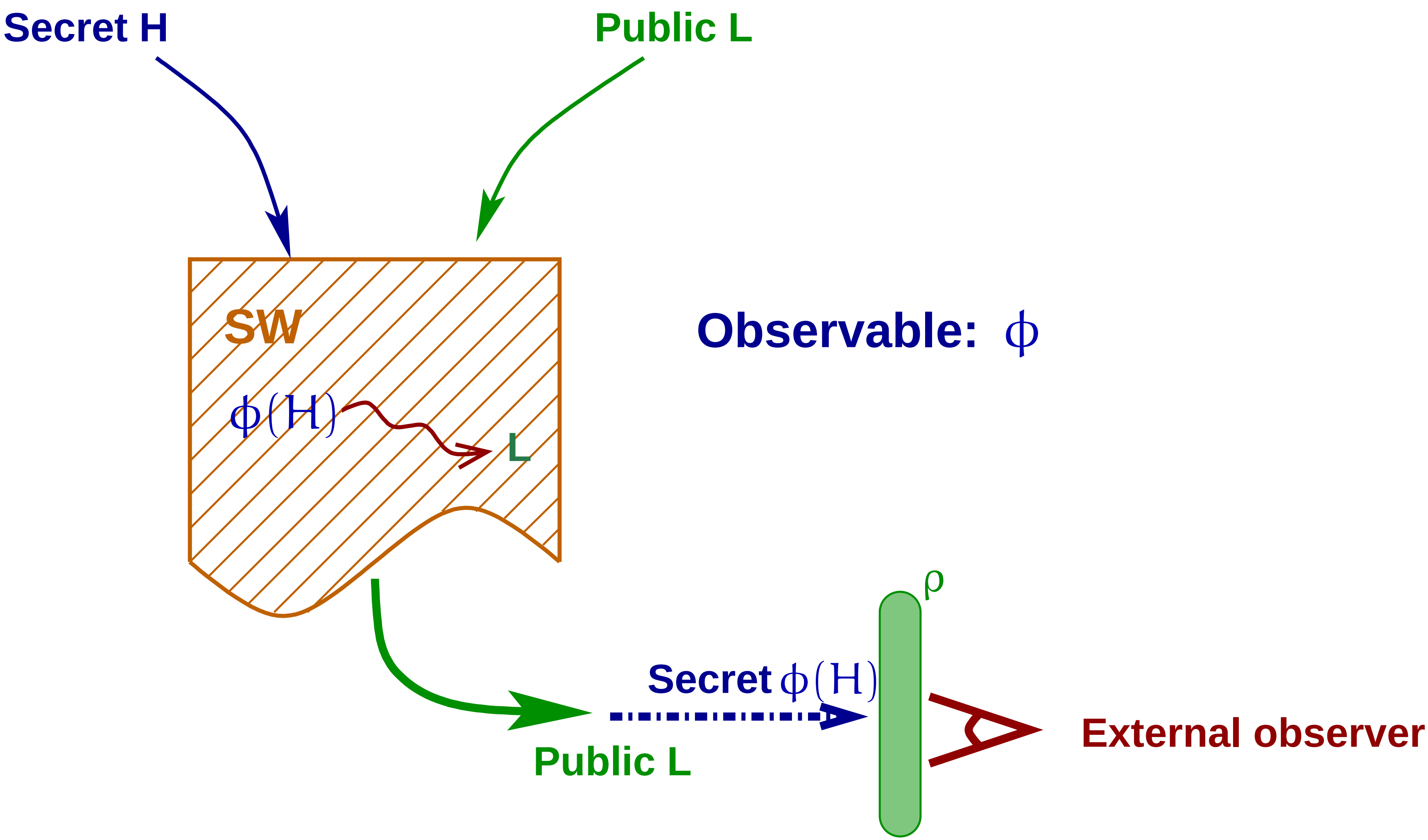
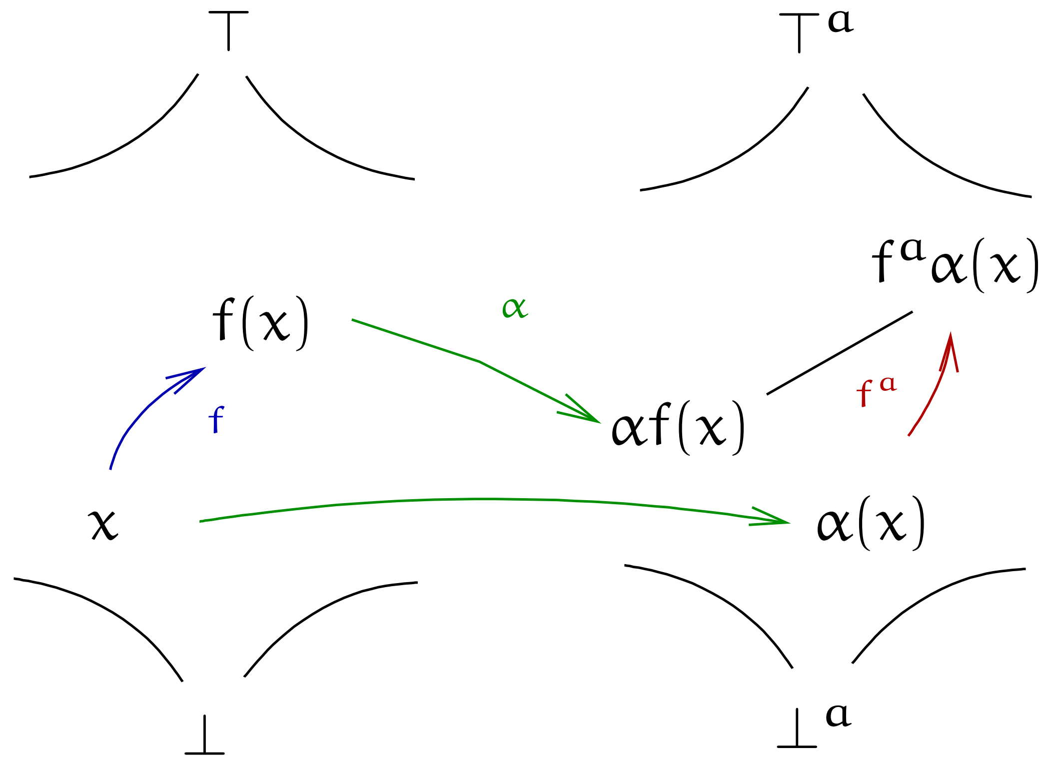


Abstract domain completeness

Let $< A, \alpha, \gamma, C >$ a Galois insertion. [Cousot & Cousot '77,'79]
$f : C \longrightarrow C$, $f^a = \alpha \circ f \circ \gamma : A \longrightarrow A$ (b.c.a. of f) and ρ=$\gamma \circ \alpha$

ρ correct for f



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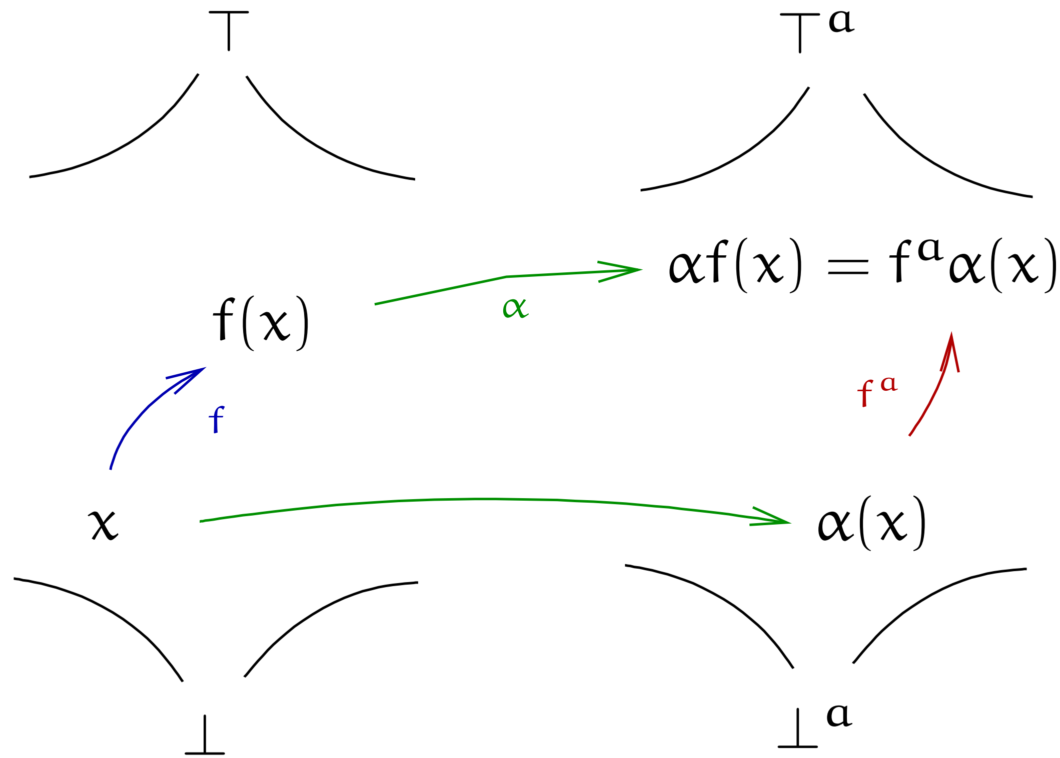
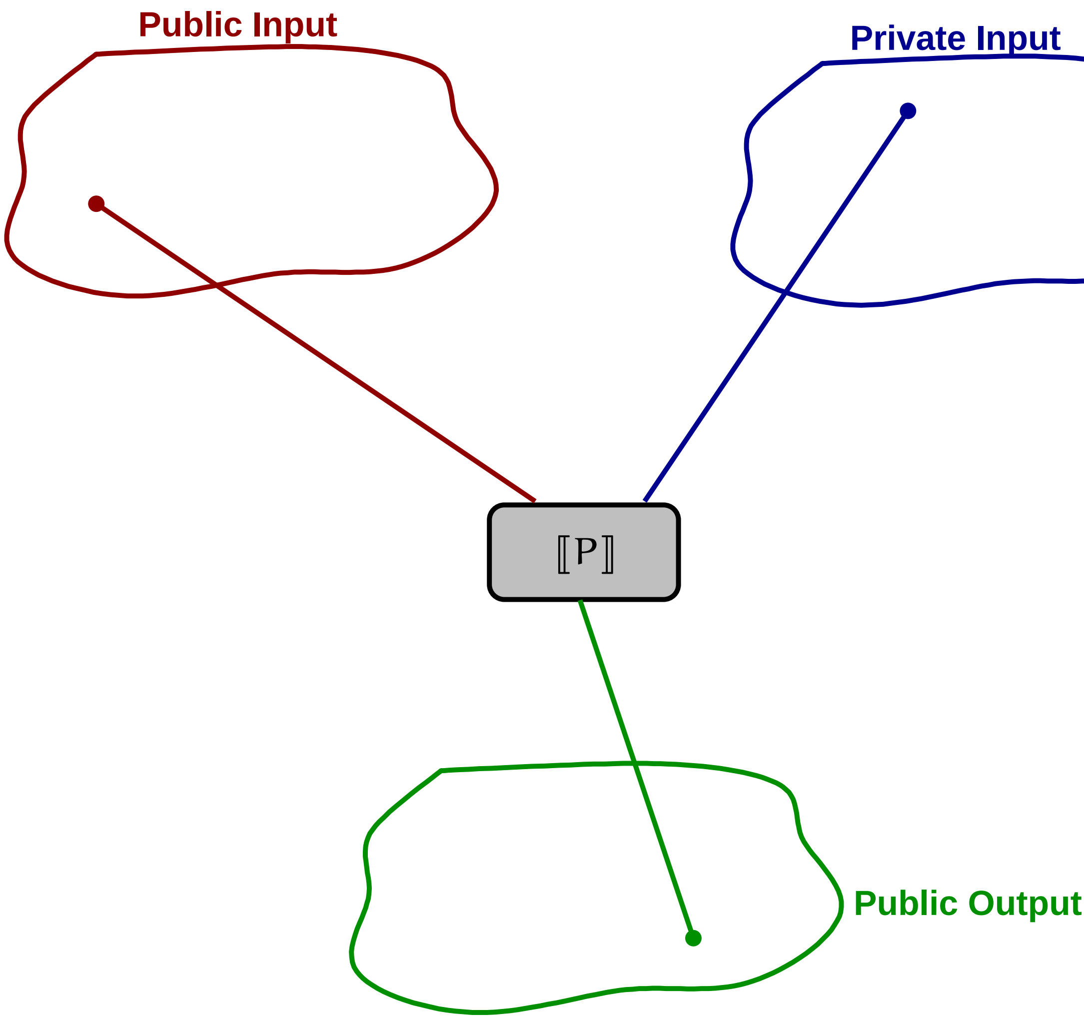
$|||$

$\rho f \rho = \rho f$



Standard non-interference



$$\forall l : \mathsf{L}\ , \forall h_1, h_2 : \mathsf{H}\ .\ [\![P]\!](h_1, l)^{\mathsf{L}} = [\![P]\!](h_2, l)^{\mathsf{L}}$$

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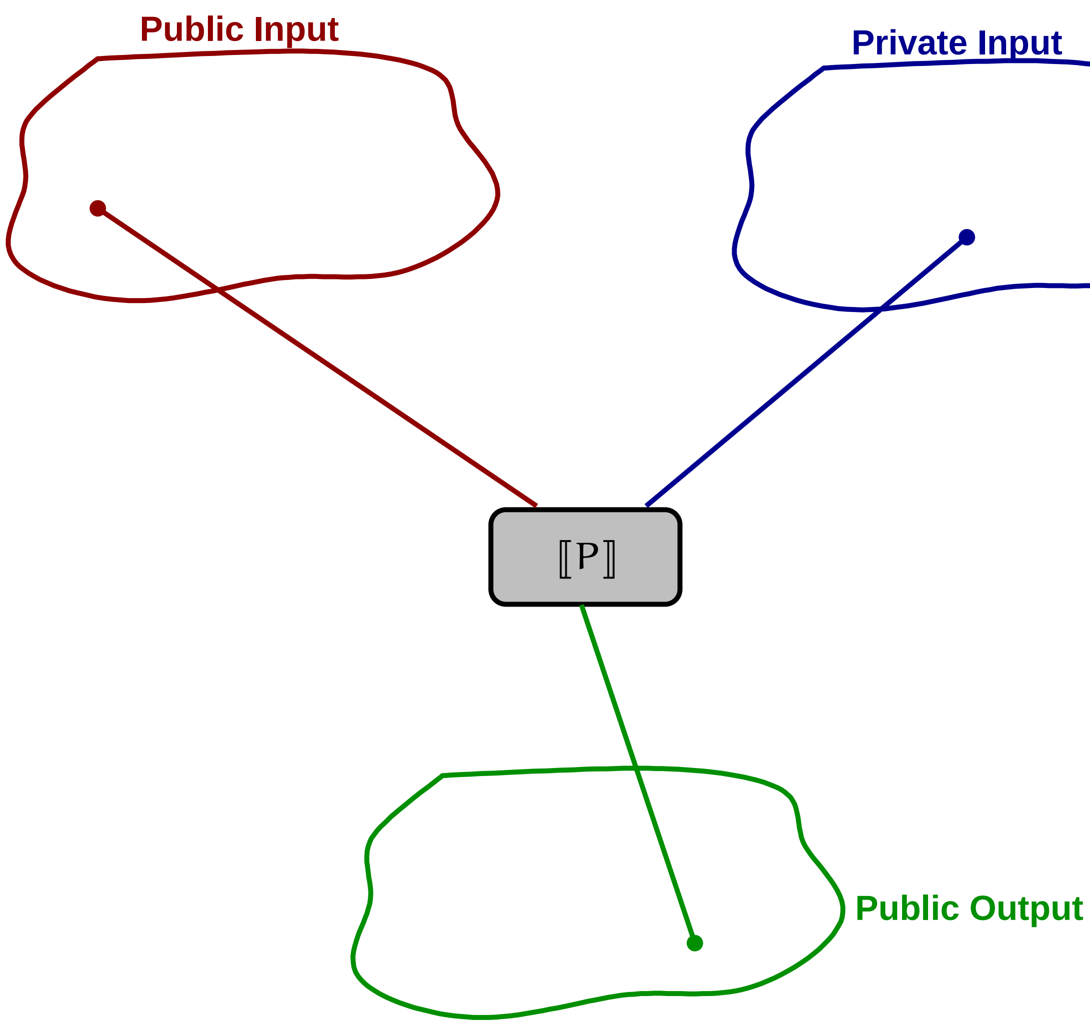

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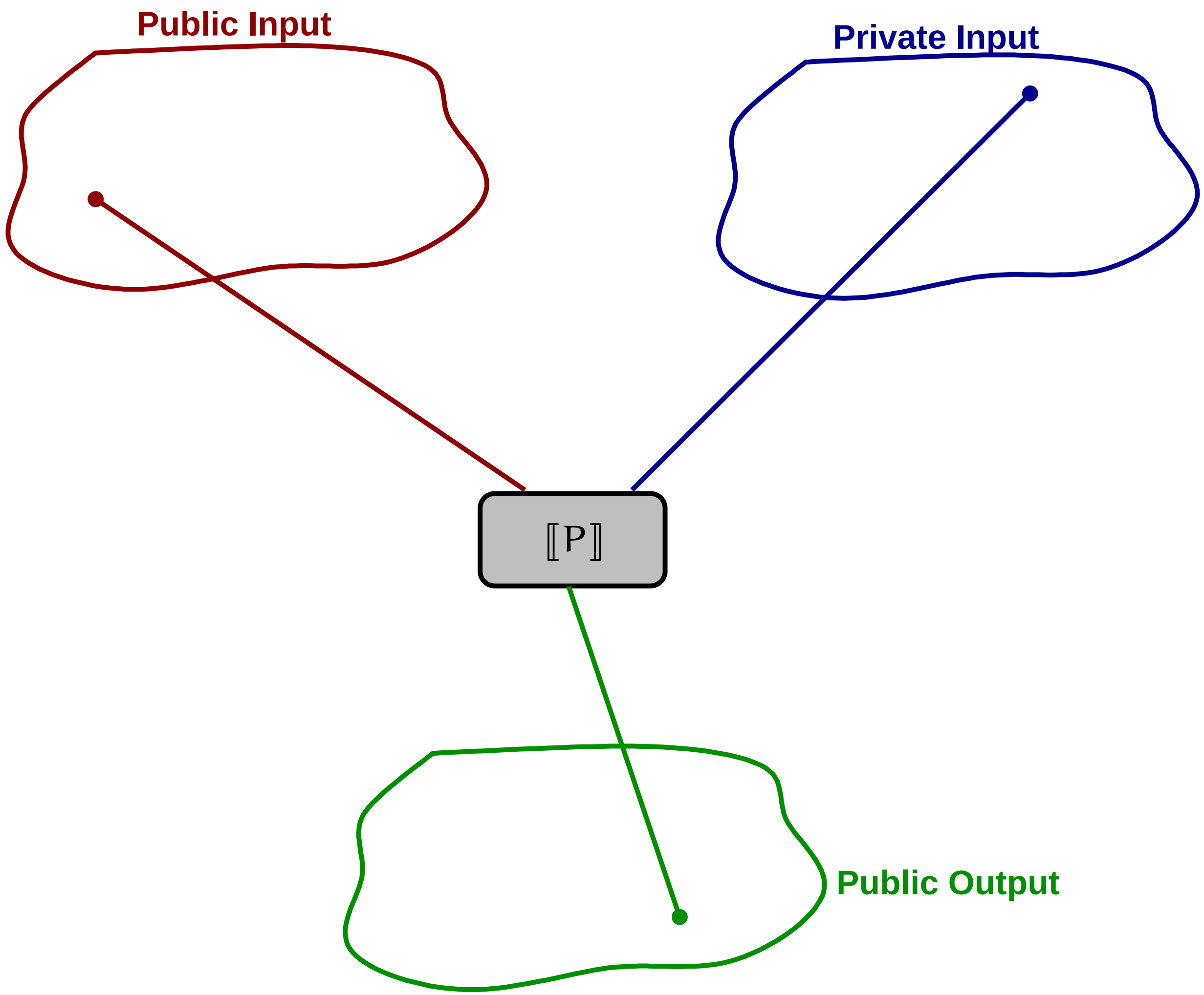

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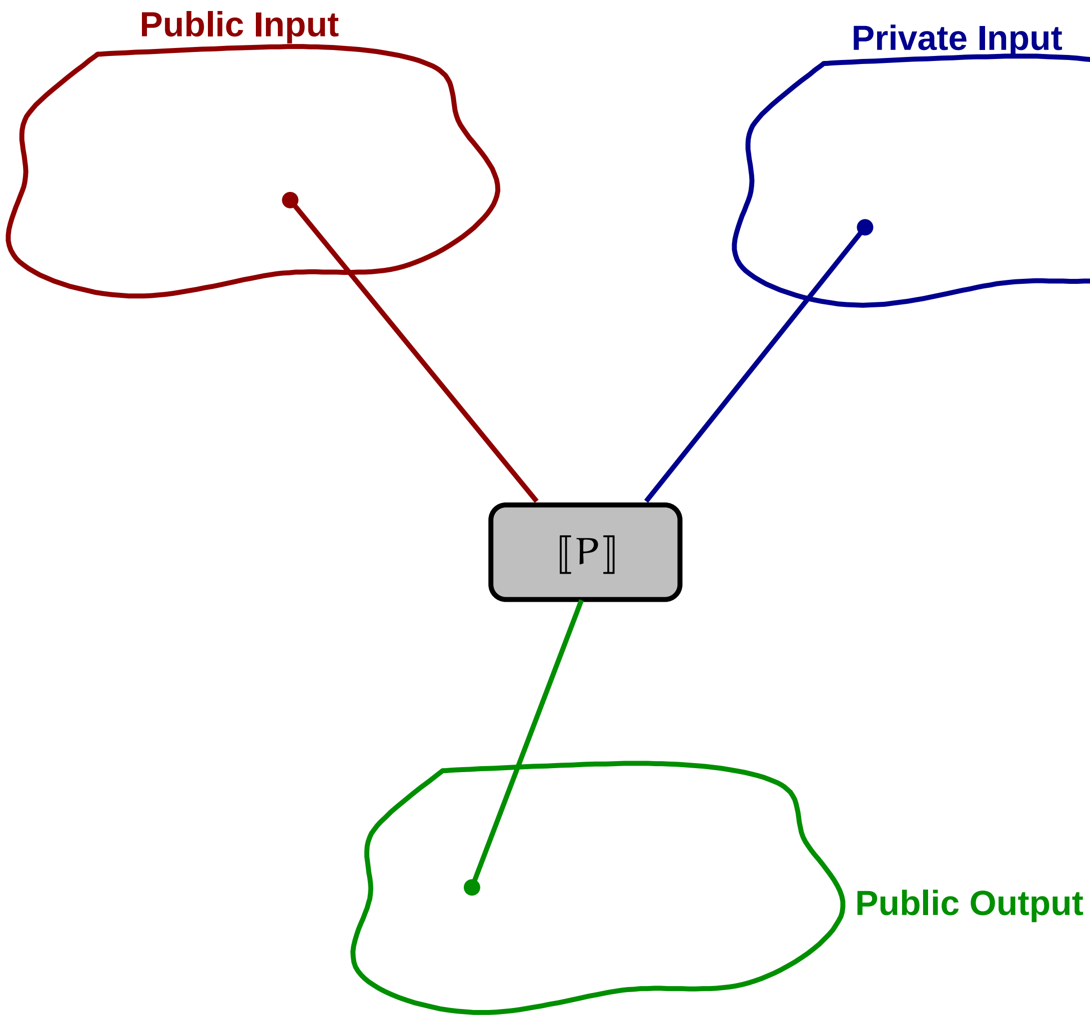

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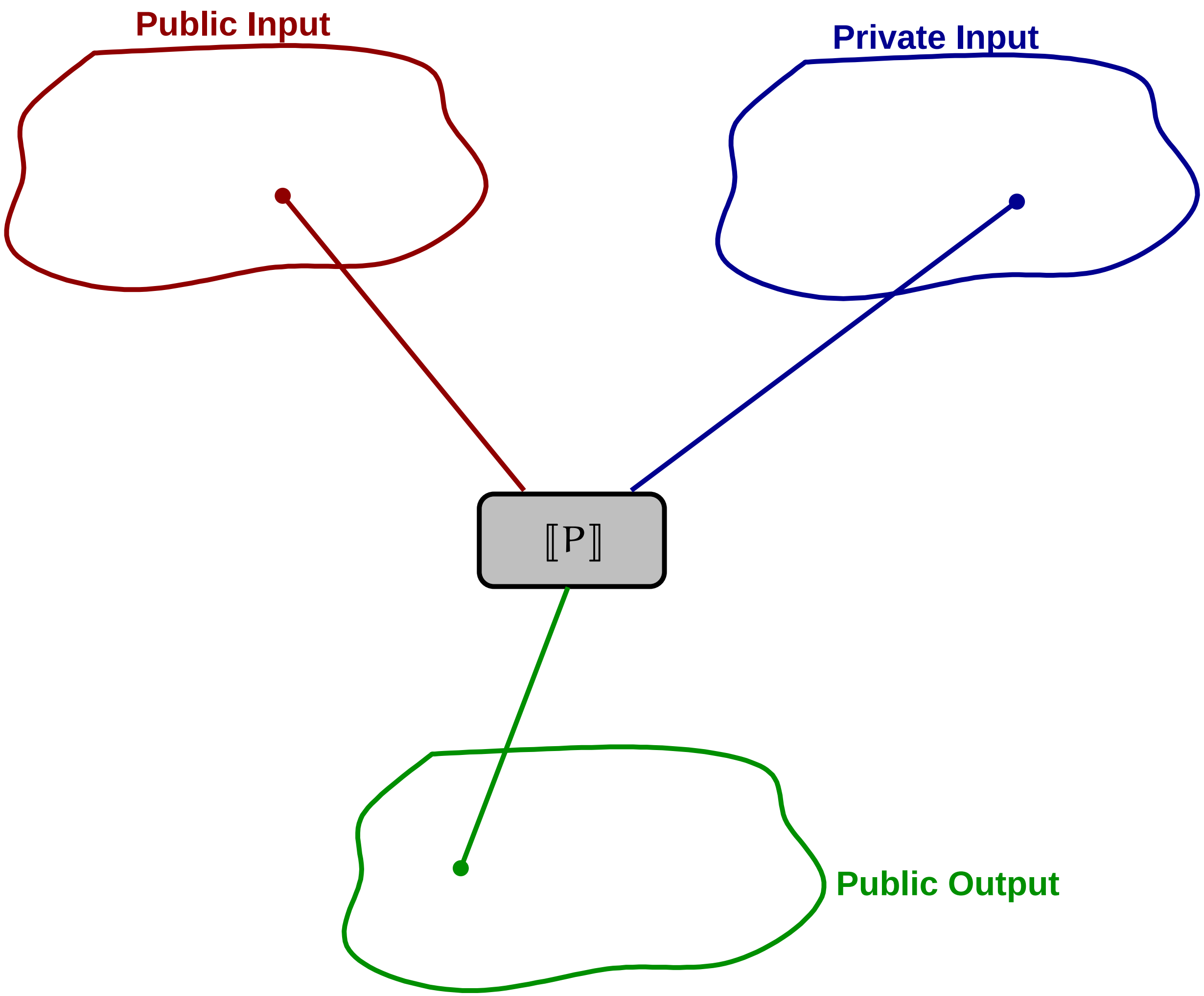

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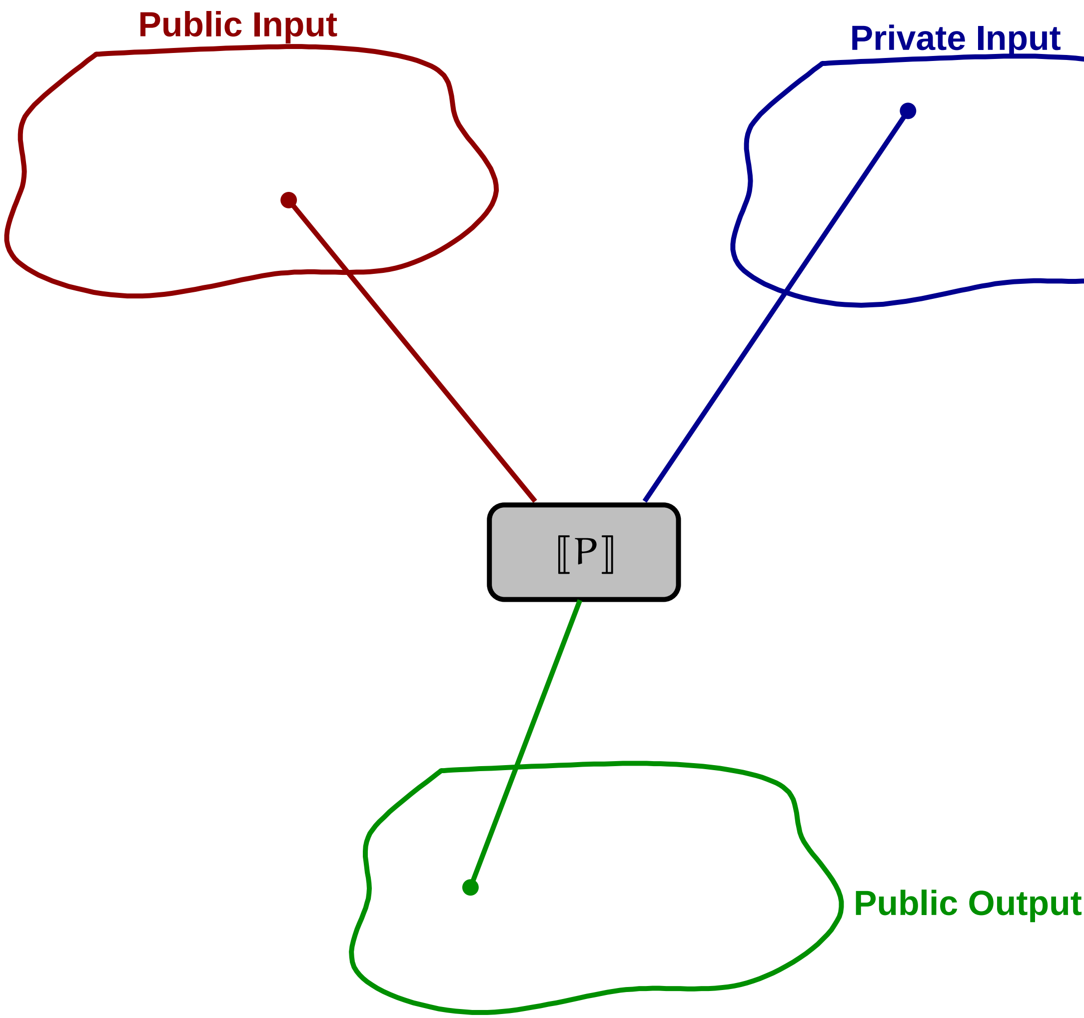

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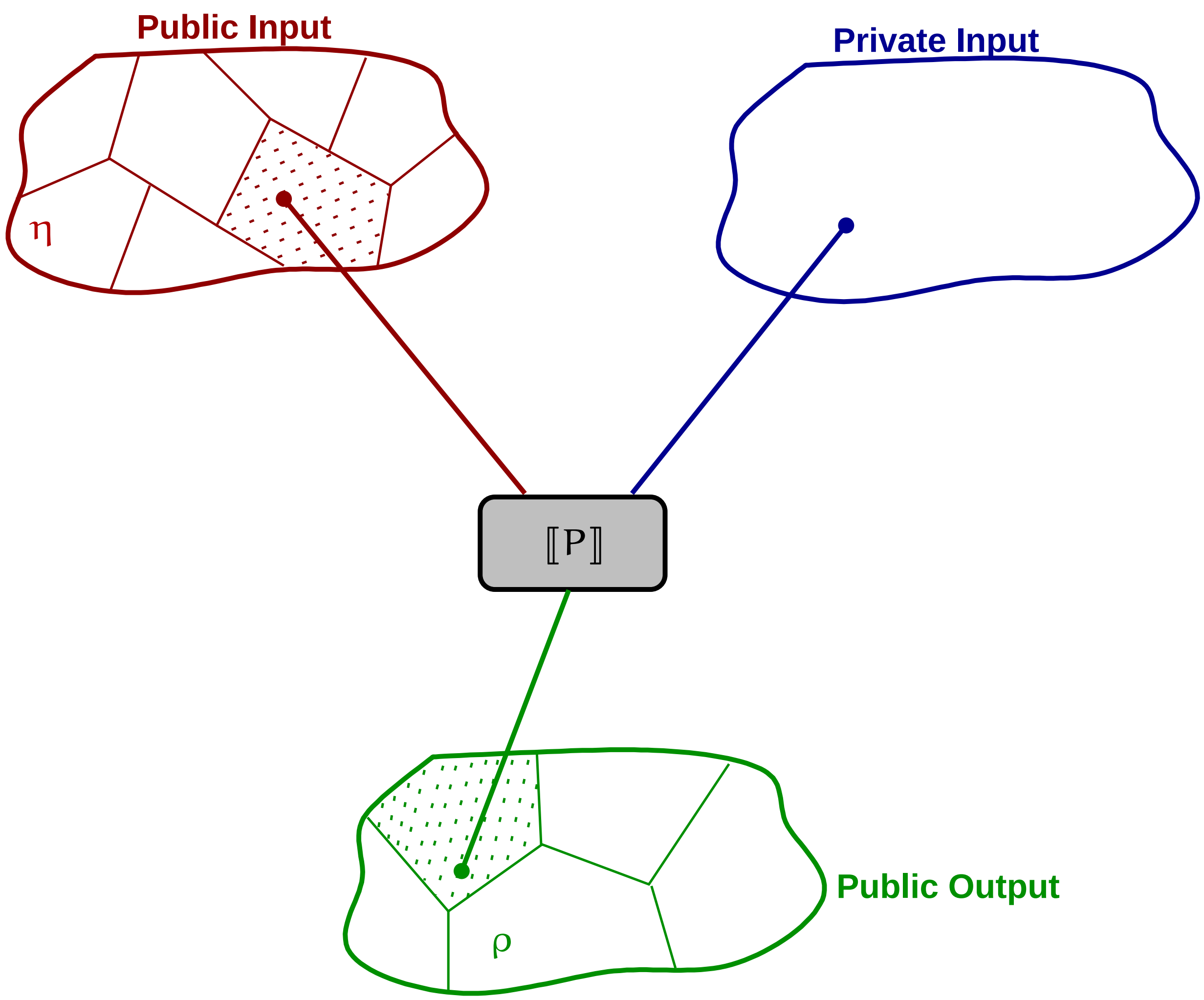
Abstracting non-interference I: Narrow ANI

[POPL'04]



$$\rho, \eta \in \mathit{Abs}(\wp(\mathbb{V}^{L})) \colon [\eta]P(\rho) \colon \eta(l_1) = \eta(l_2) \Rightarrow \rho(⟦P⟧(h_1, l_1)^{L}) = \rho(⟦P⟧(h_2, l_2)^{L})$$

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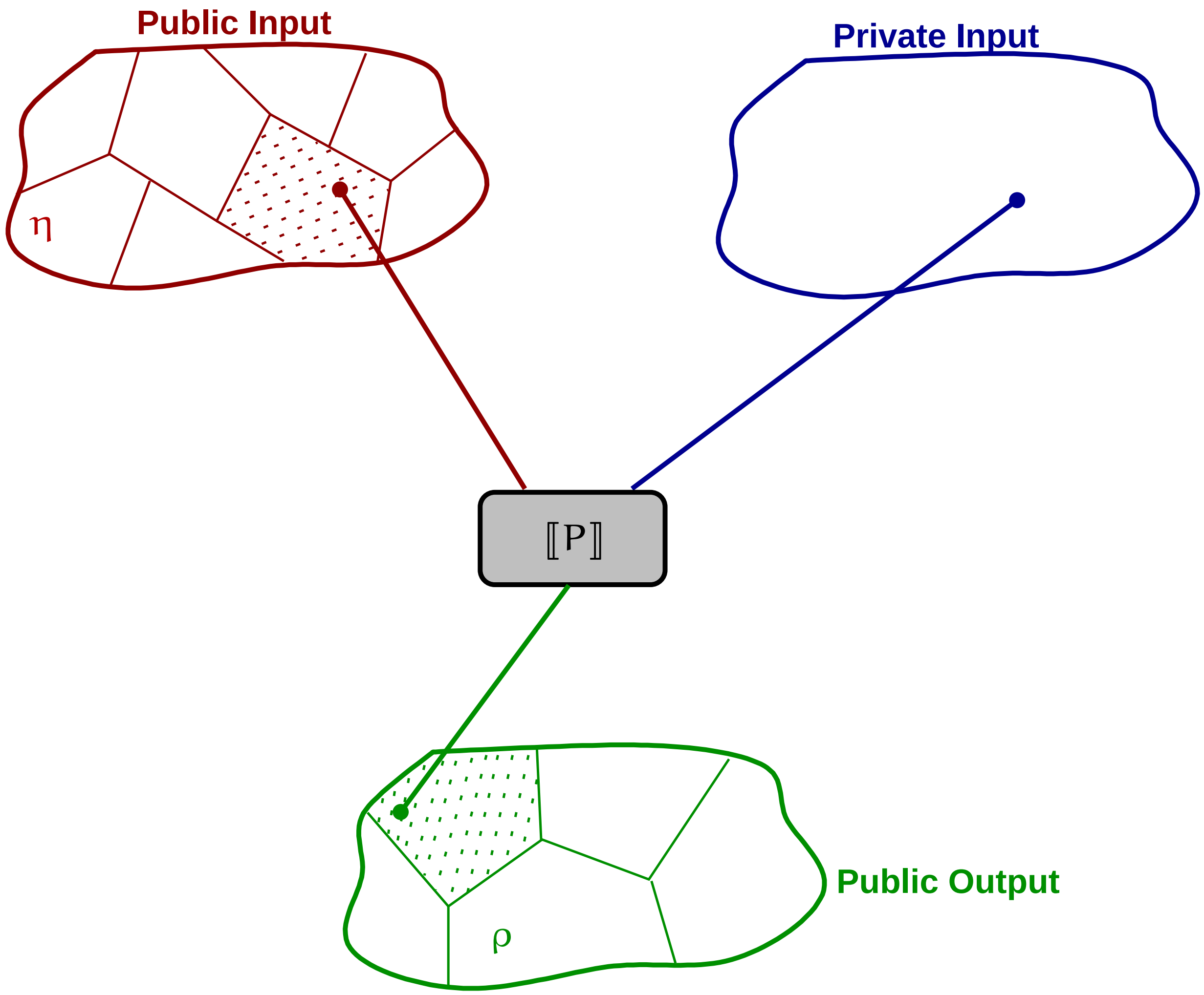

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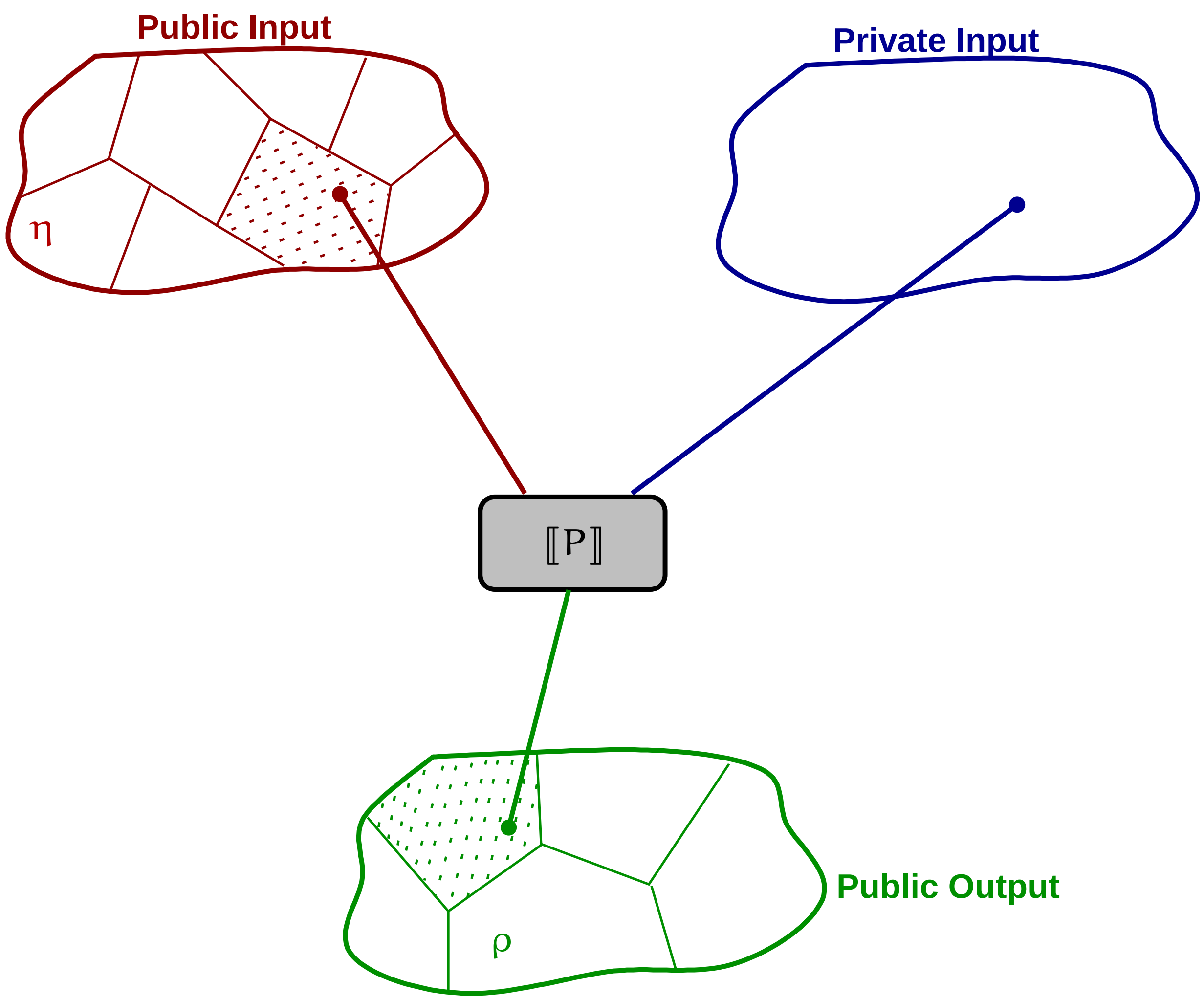

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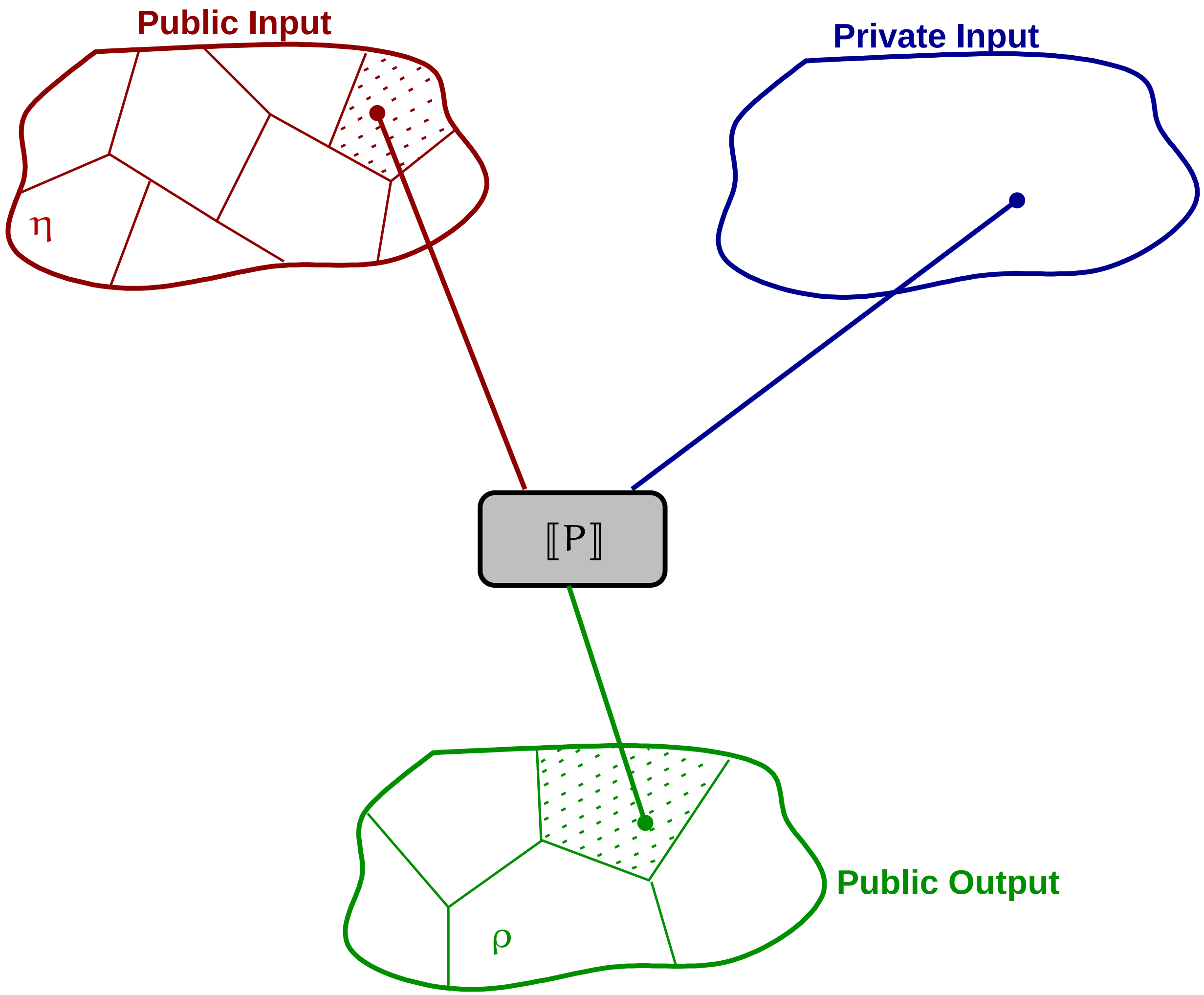

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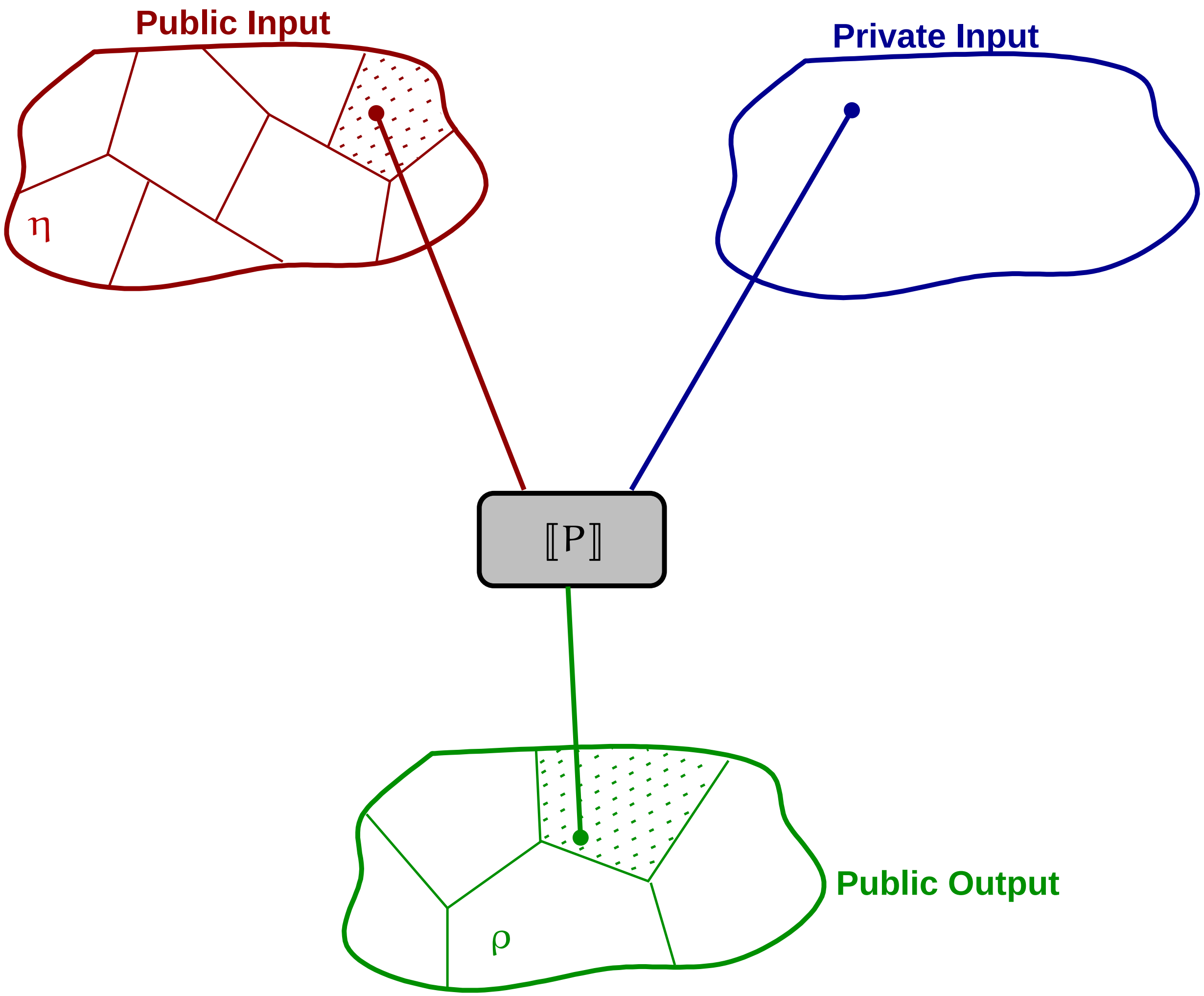

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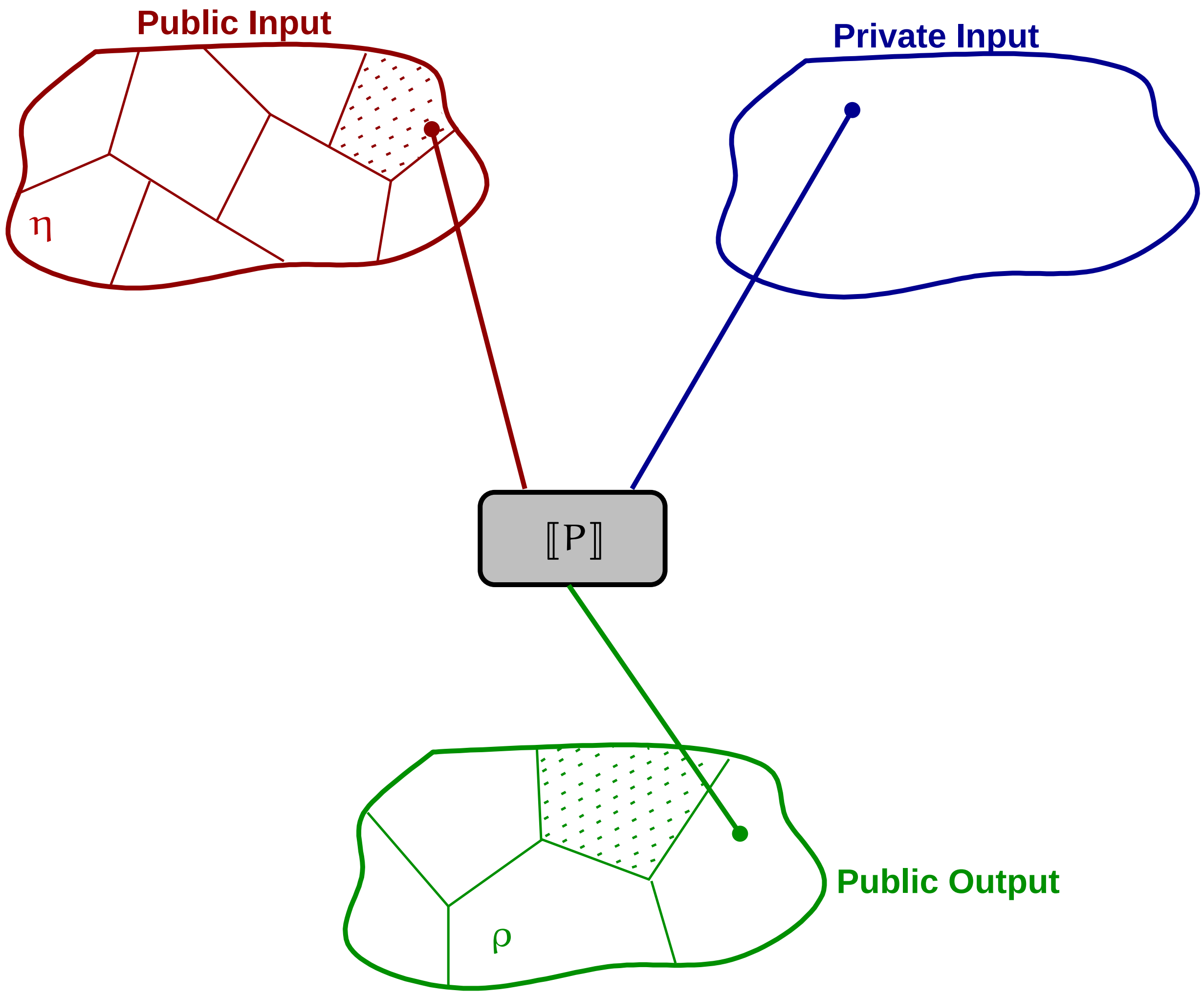

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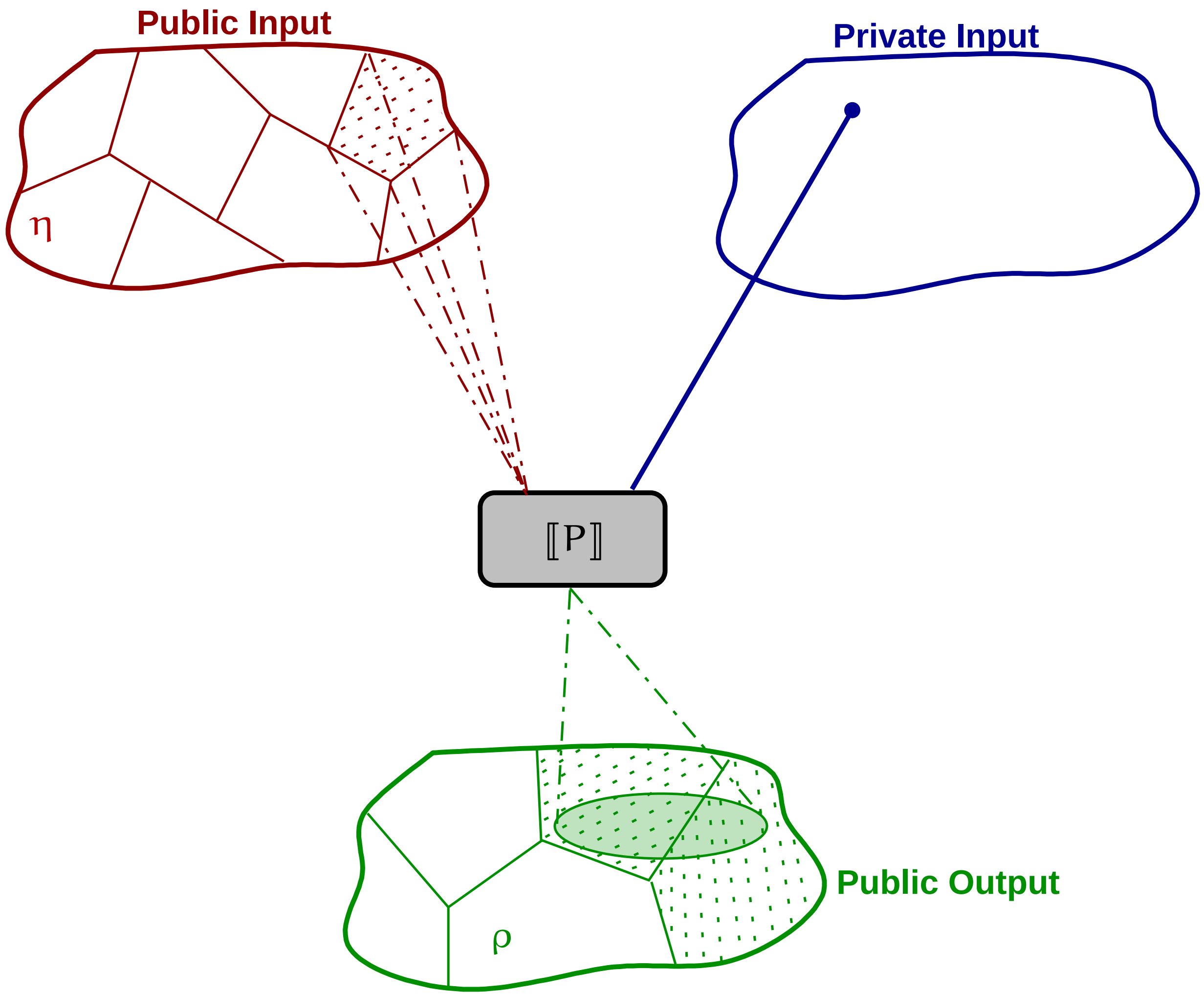

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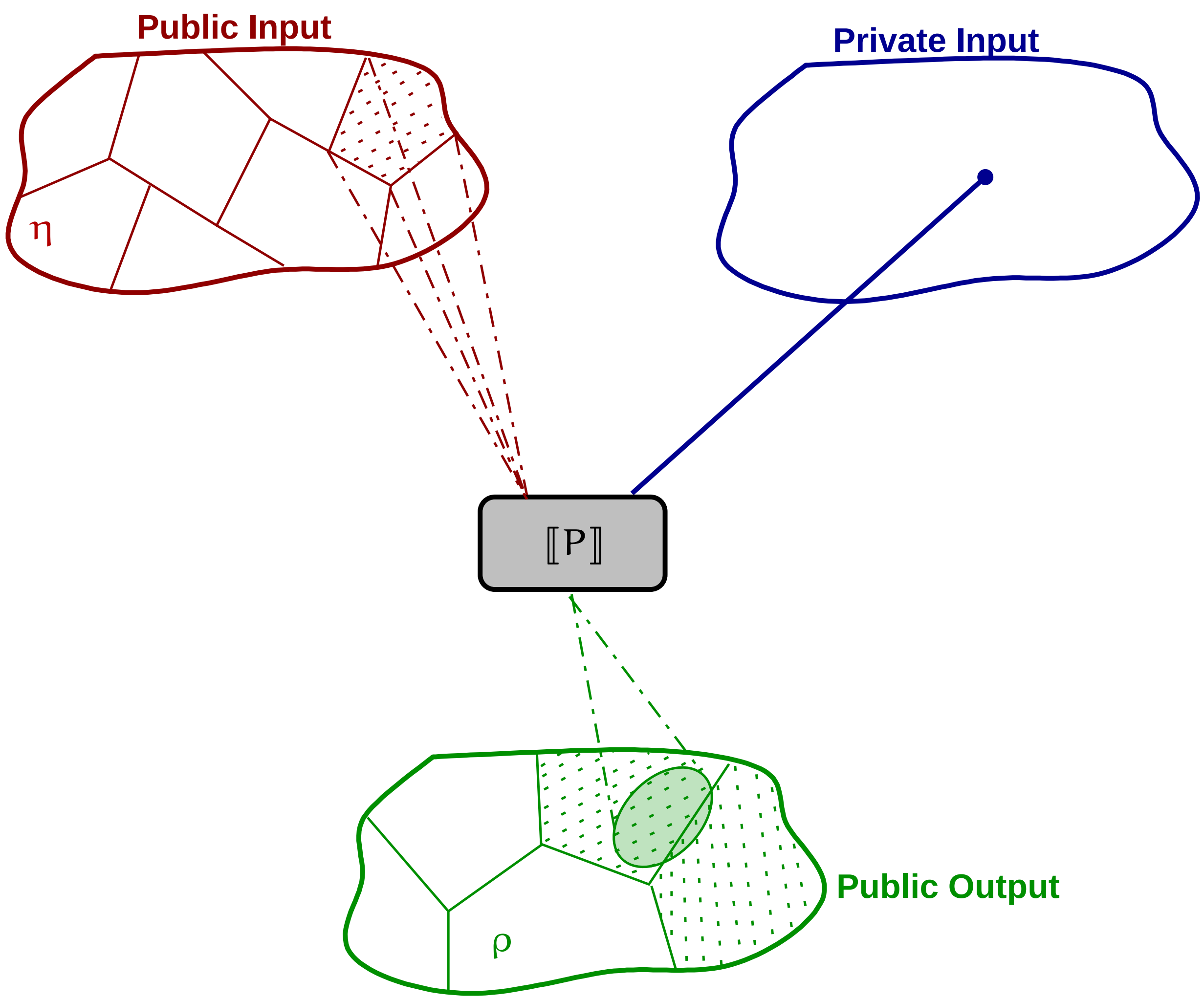

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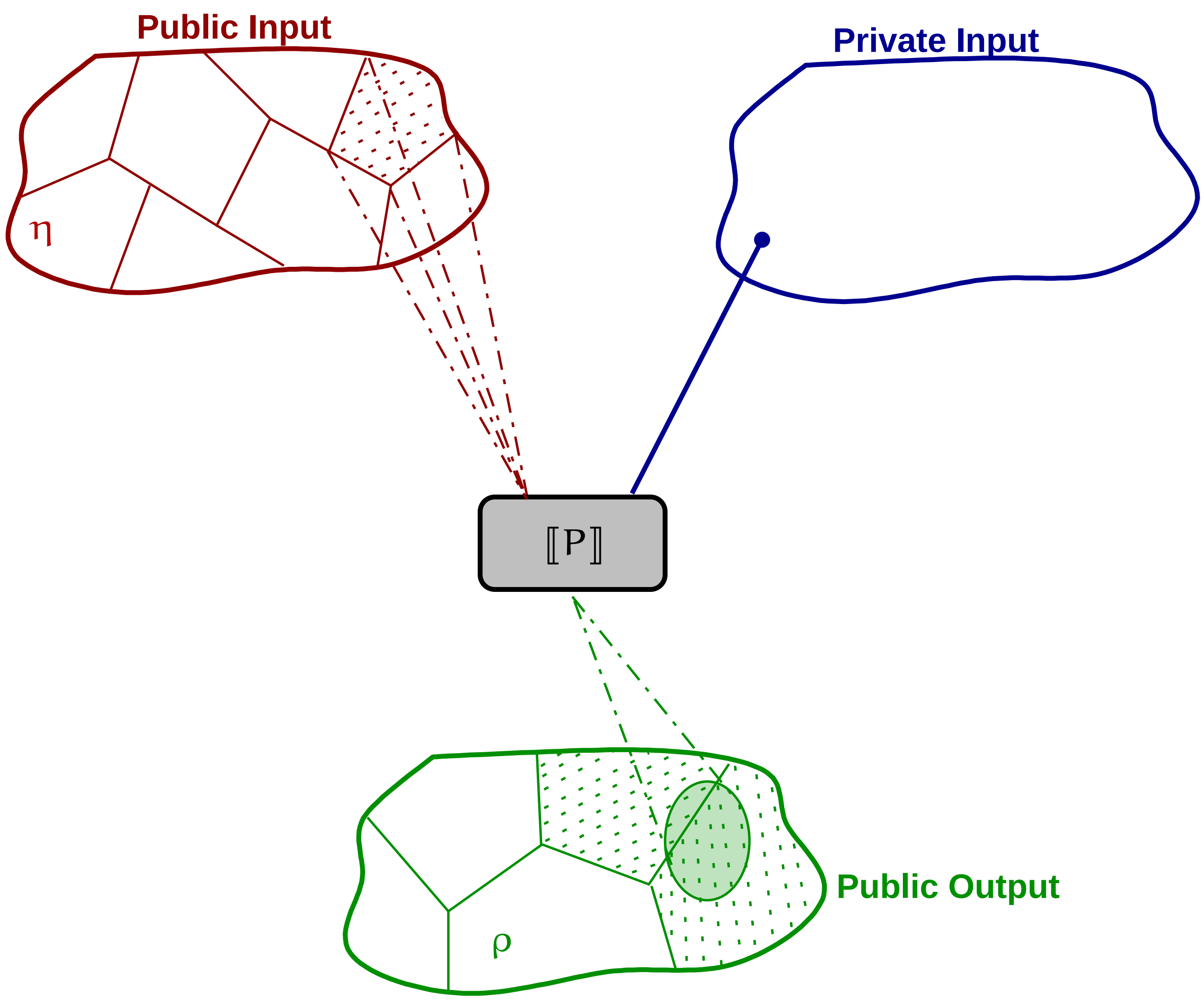

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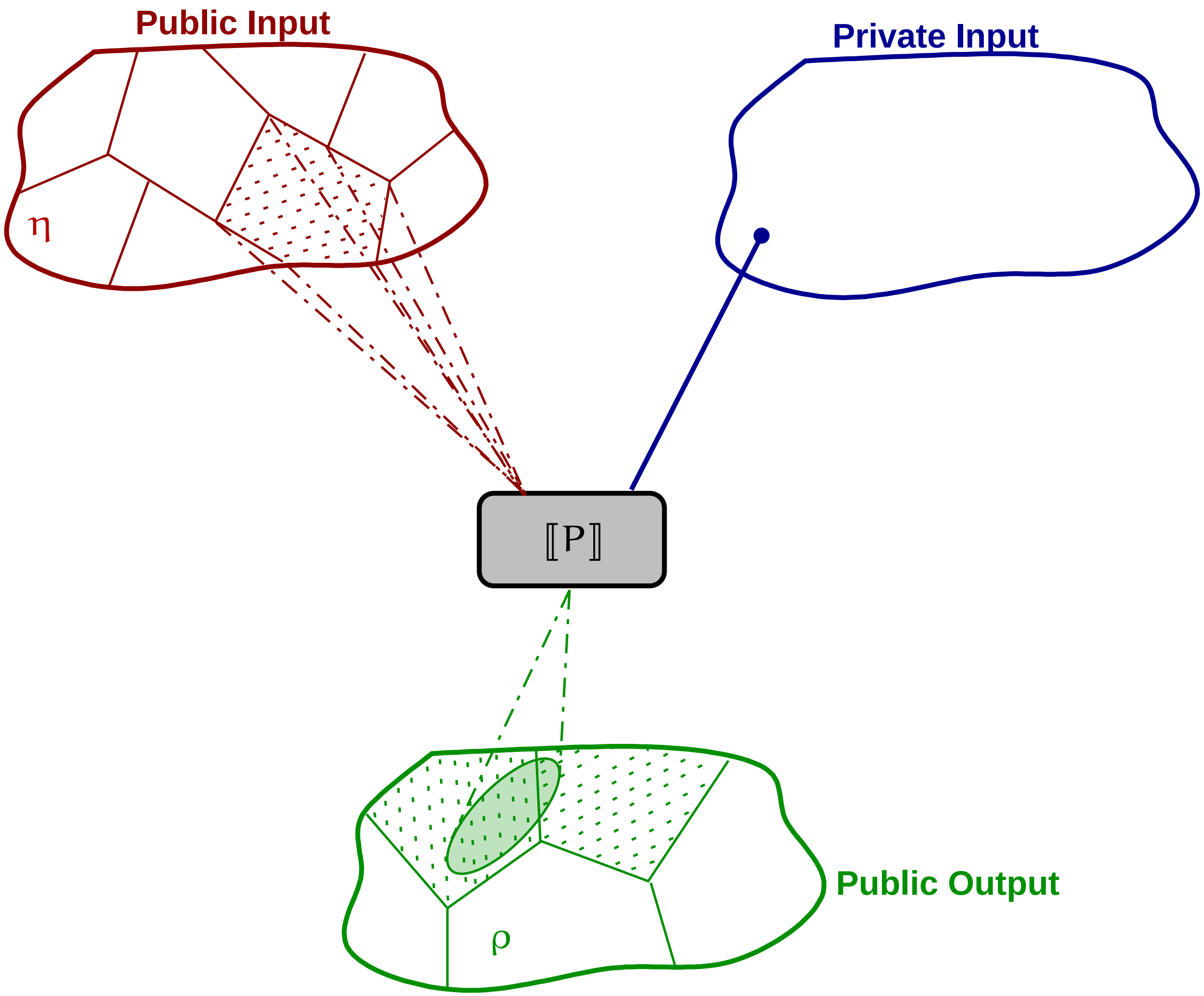

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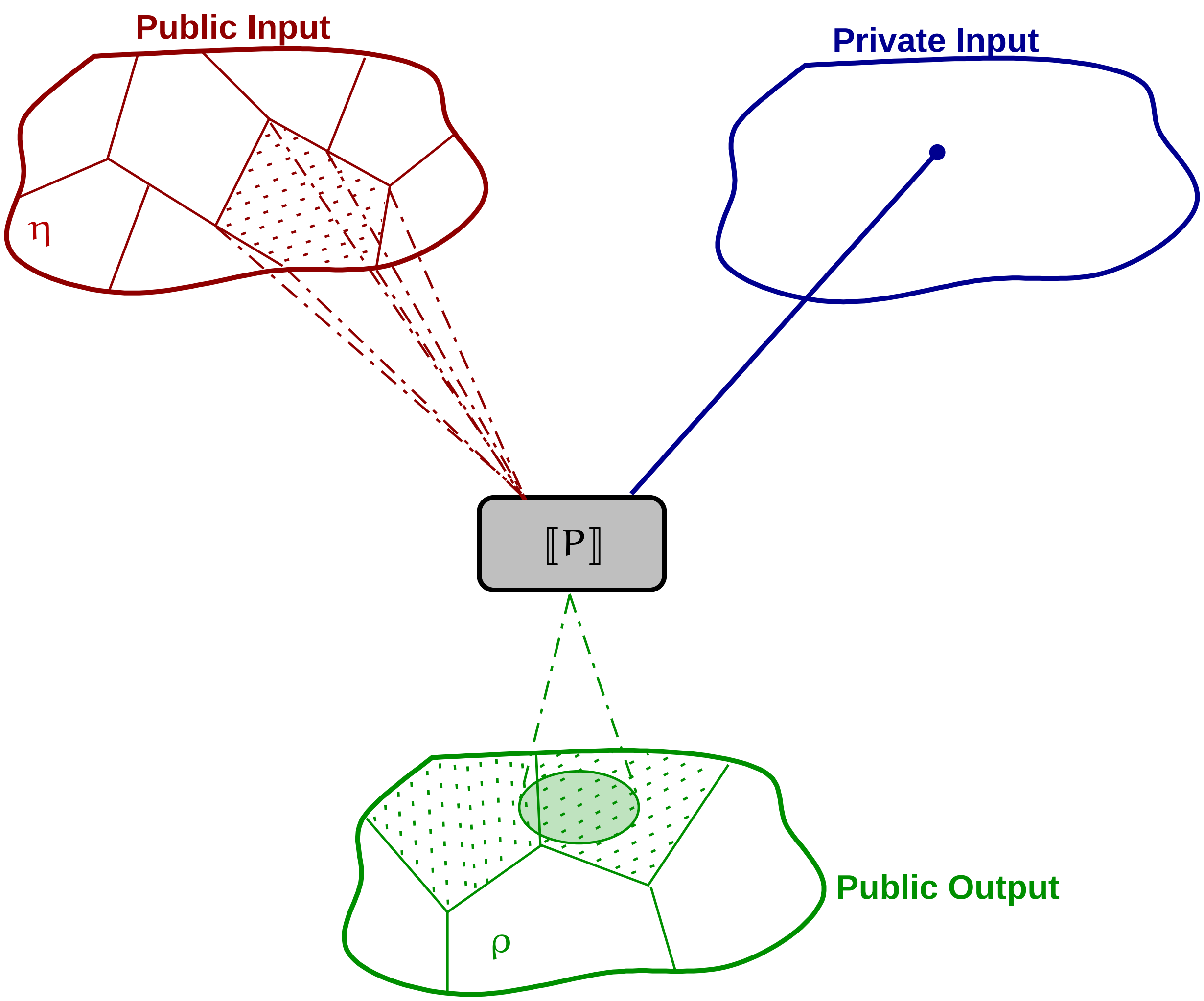

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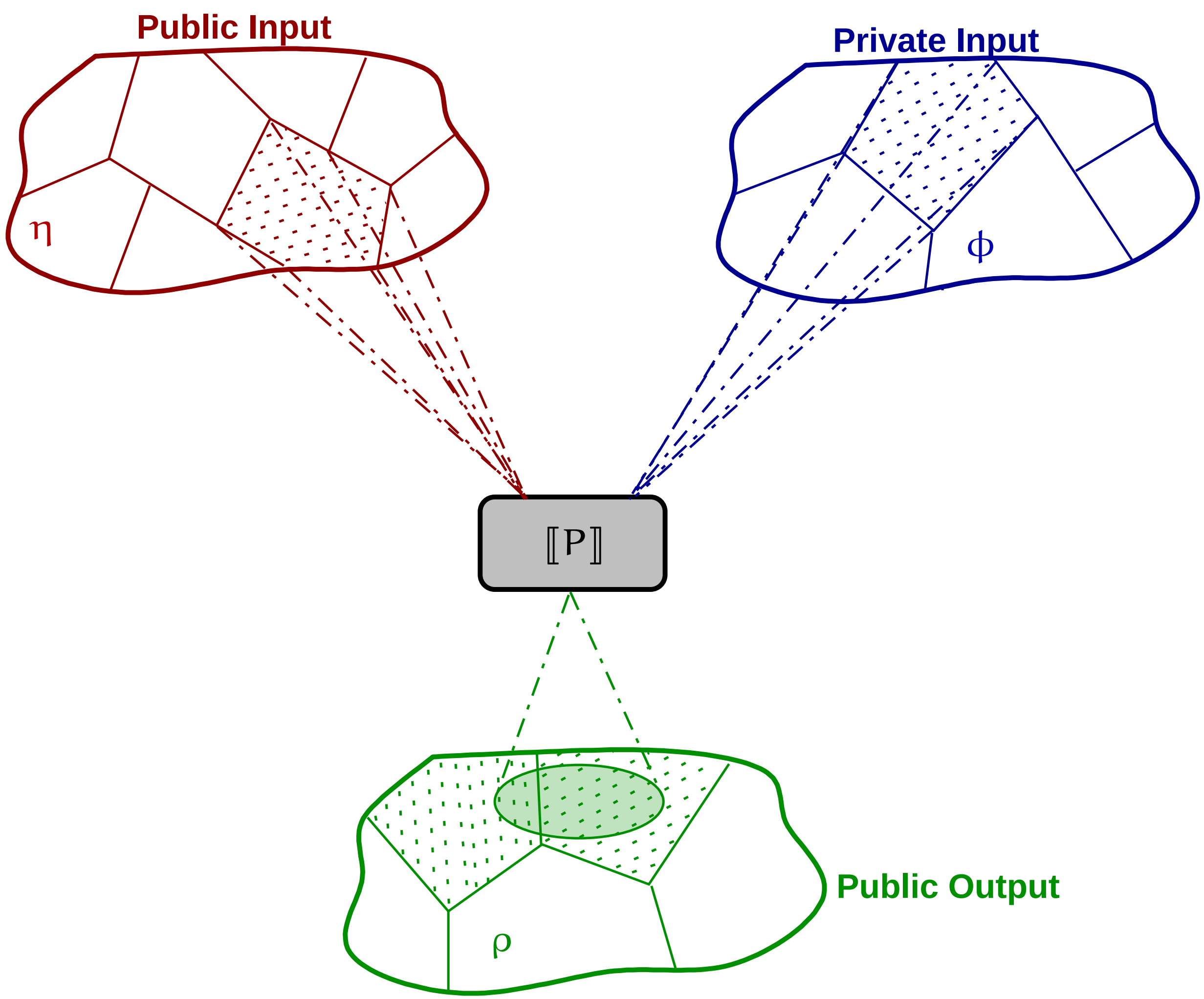

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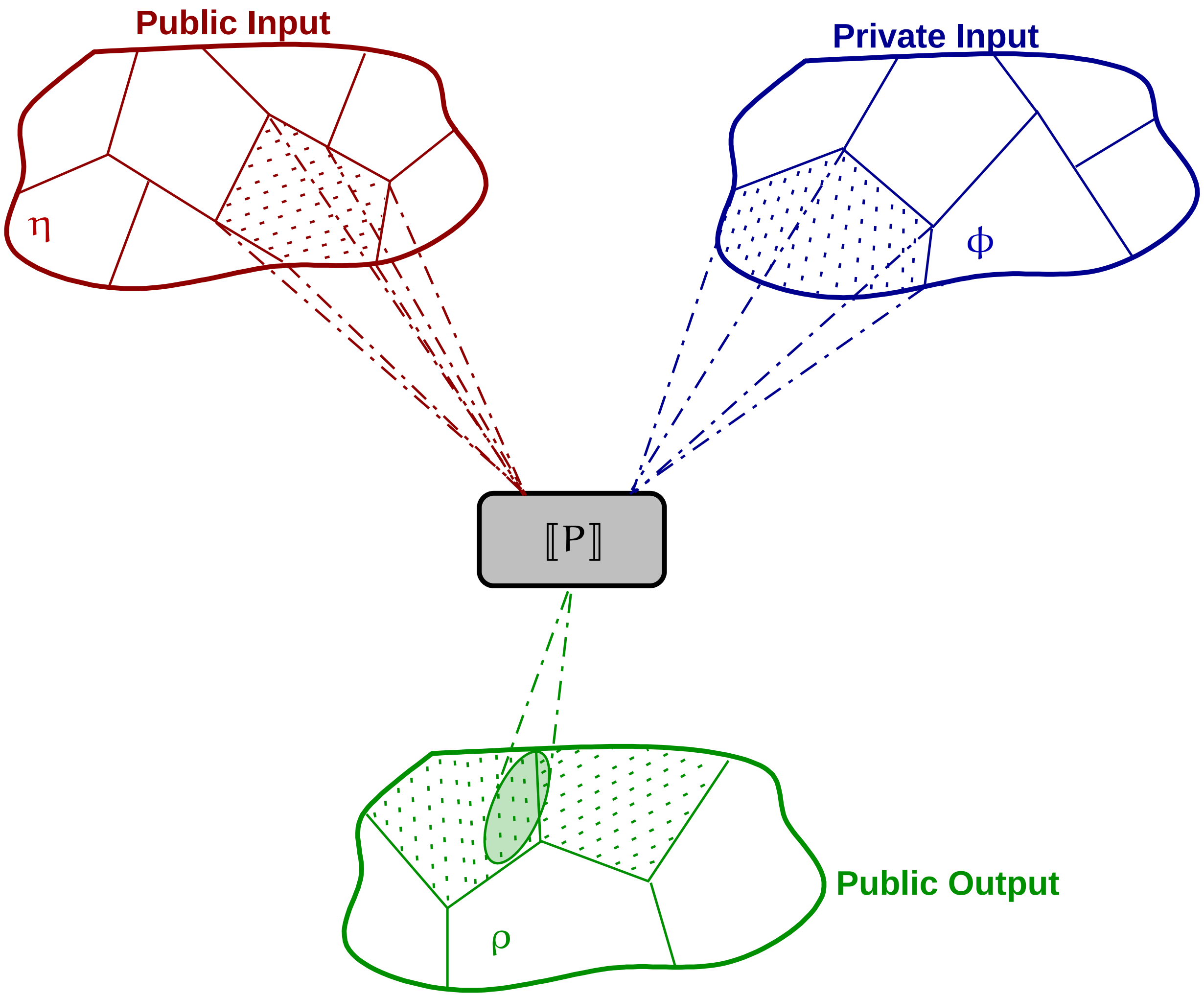

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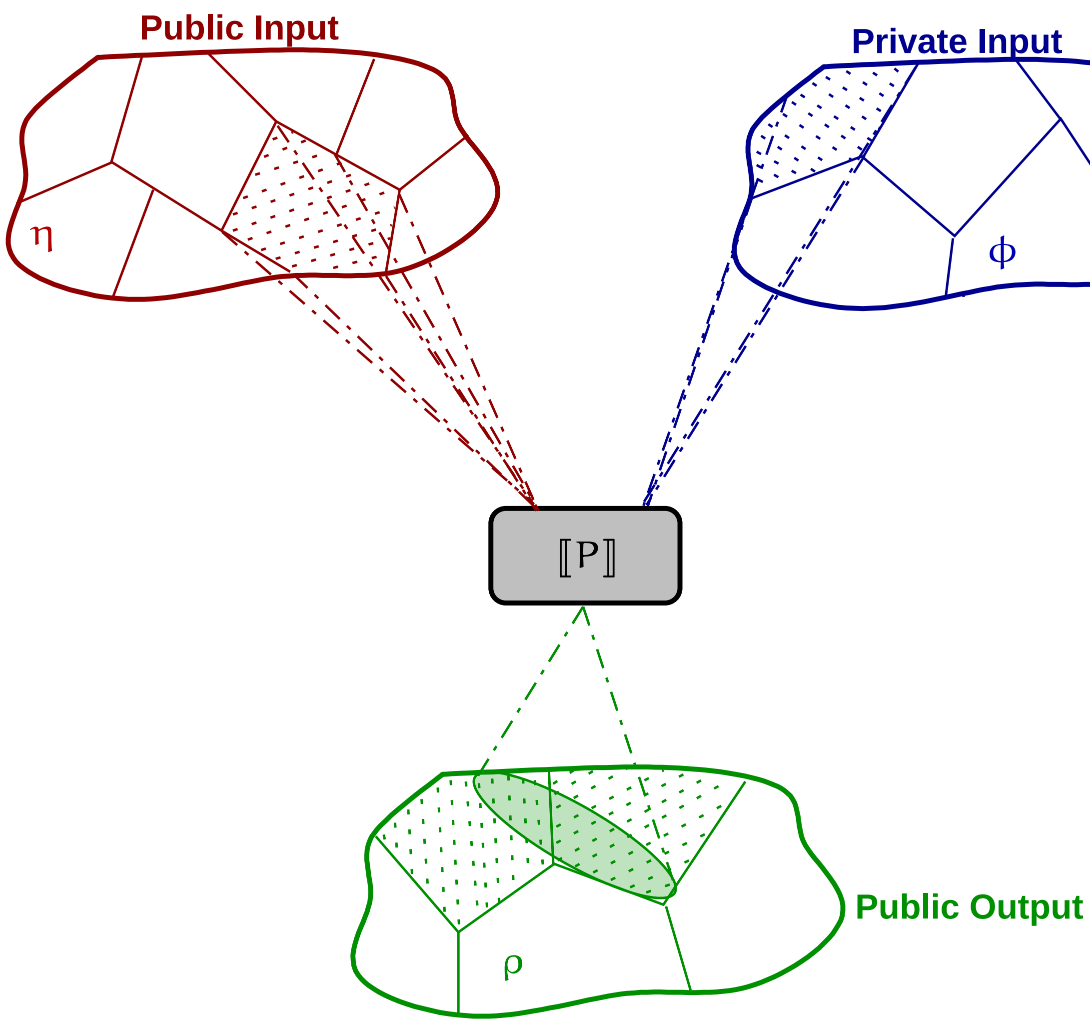

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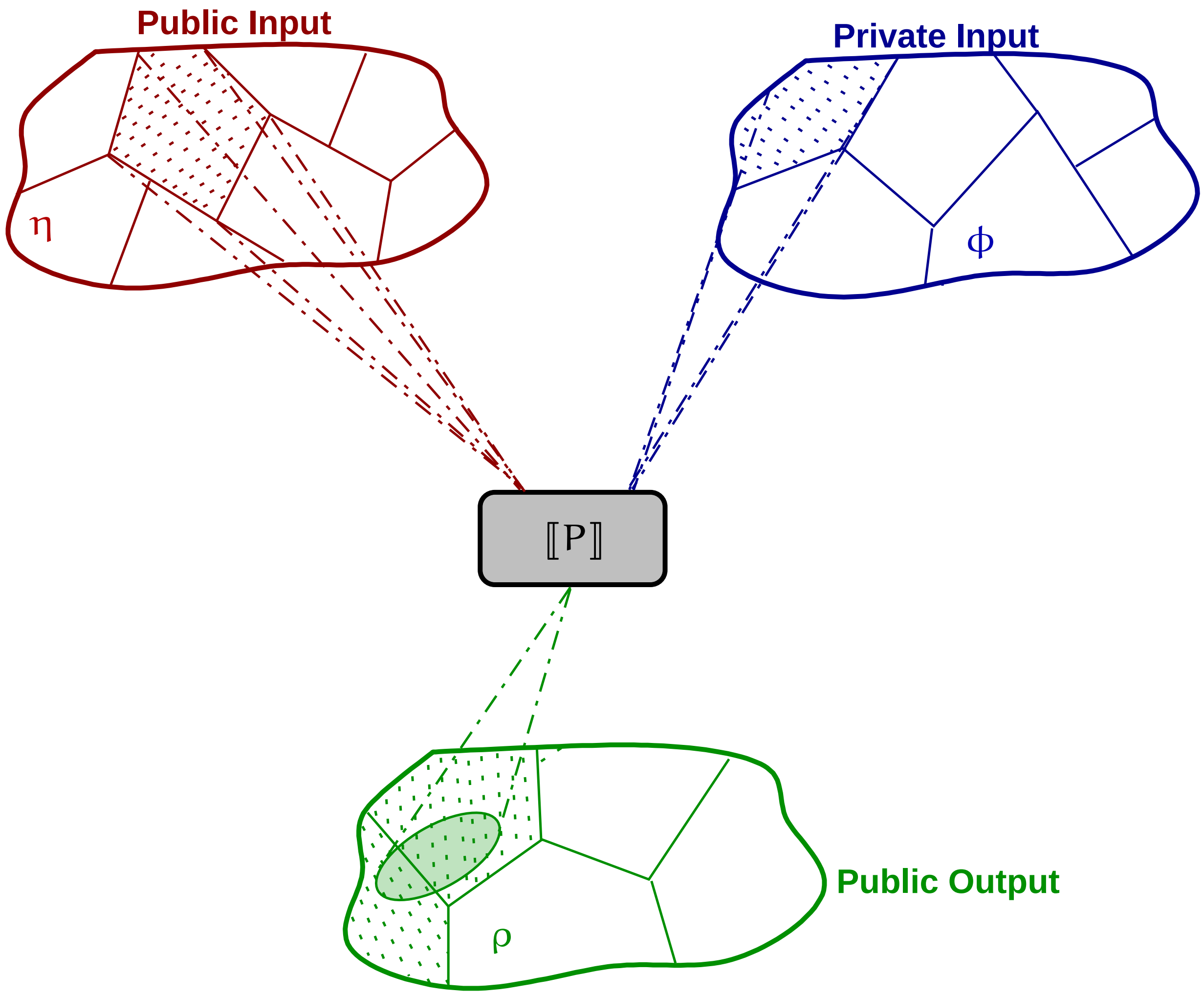

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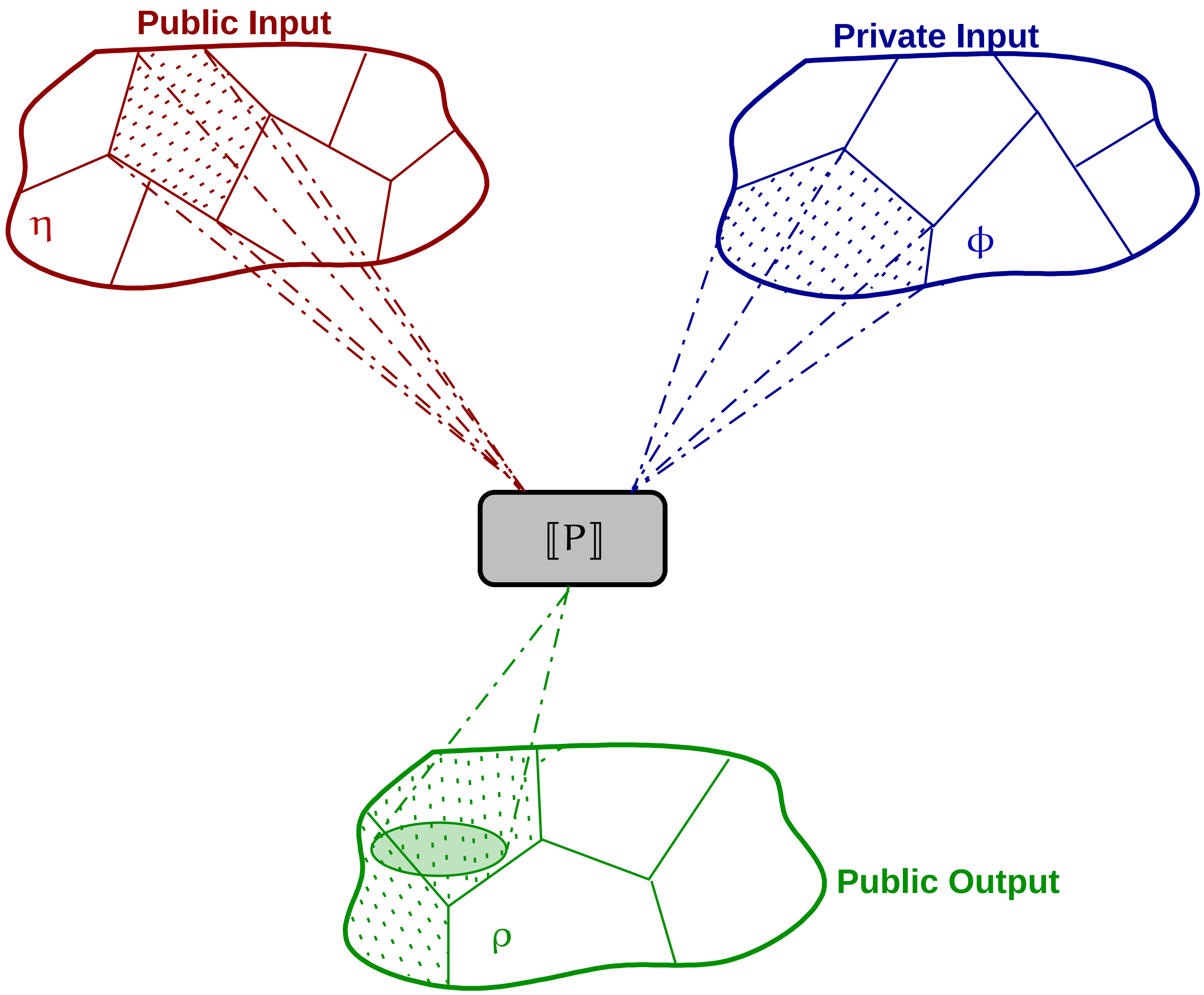

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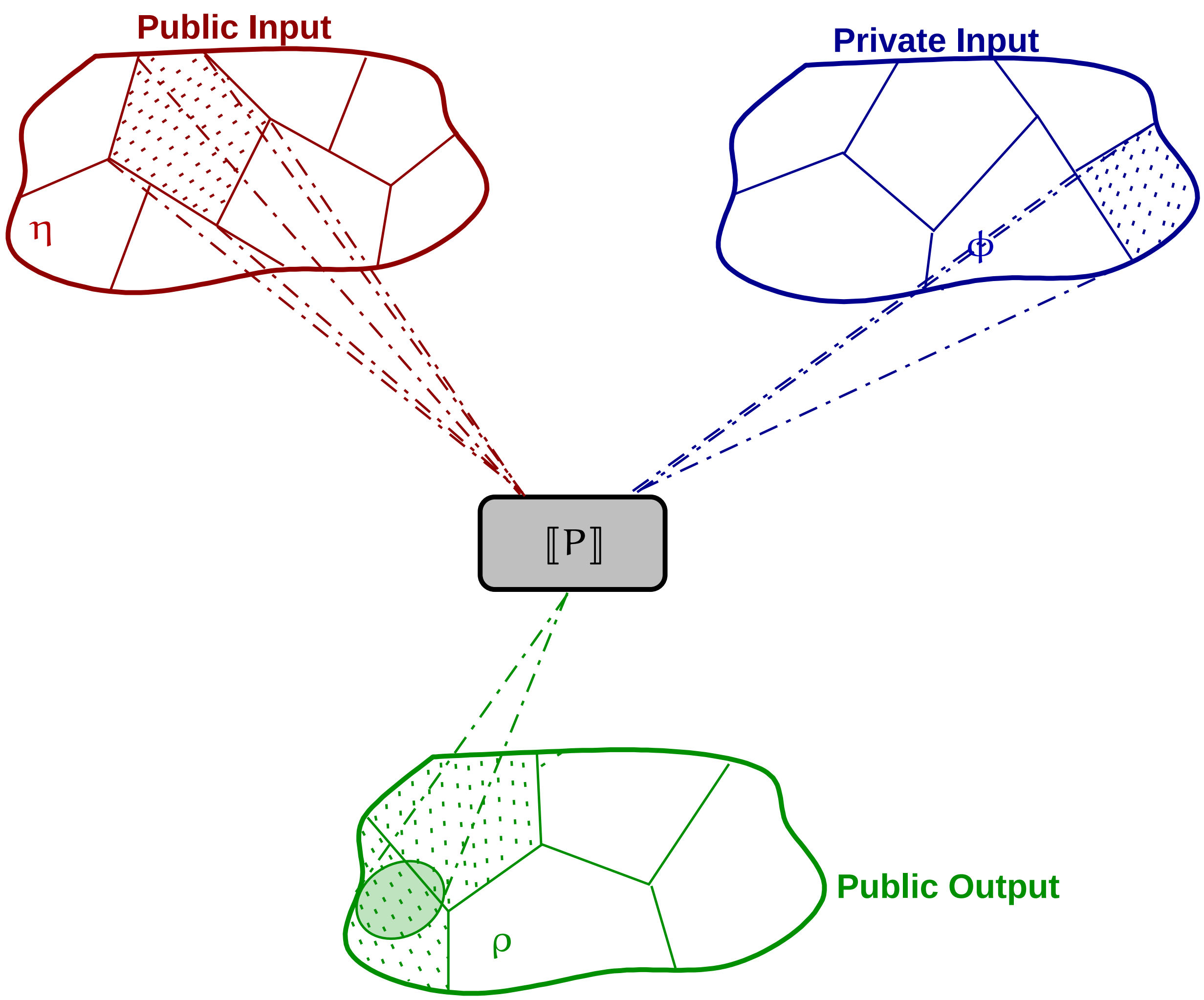

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Examples

EXAMPLE I:

$$\textbf{while } h \textbf{ do } (l := l + 2;\ h := h - 1).$$

$$\boxed{\text{Standard Non-Interference} \equiv [id]P(id)}$$

$$h = 0,\ l = 1 \rightsquigarrow l = 1$$
$$h = 1,\ l = 1 \rightsquigarrow l = 3$$
$$h = n,\ l = 1 \rightsquigarrow l = 1 + 2n$$

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$$\Downarrow$$

$$\boxed{[\mathit{id}]\mathrm{P}(\mathit{Par})}$$

$$h = 0,\ l = 1 \leadsto \mathit{Par}(l) = \text{odd}$$
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Examples

EXAMPLE II:

$$P = \ l := 2 * l * h^2.$$

$$\boxed{[\mathit{Par}]P(\mathit{Sign})}$$

$$h = 1,\ l = 4\ (\mathit{Par}(4) = \text{even}) \rightsquigarrow \mathit{Sign}(l) = +$$
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$$\boxed{[Par]P(Sign)}$$

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$$\Downarrow$$

$$\boxed{(Par)P(Sign)}$$

$$h = -3,\ Par(l) = \text{even} \rightsquigarrow Sign(l) = \text{I don't know}$$
$$h = 1,\ Par(l) = \text{even} \rightsquigarrow Sign(l) = \text{I don't know}$$

Examples

EXAMPLE III:

$$P = \quad l := l * h^2.$$

$$\boxed{(\mathit{id})P(\mathit{Par})}$$

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$$h = n,\ l = 1 \ \rightsquigarrow\ \mathit{Par}(l) = \mathit{Par}(n)$$

Examples

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$$\Downarrow$$

$$\boxed{(id)P(Sign \leadsto\!\!\!\!\parallel Par)}$$

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Deriving output attackers

Abstract interpretation provides advanced methods for designing abstractions (refi nement, simplifi cation, compression ...)

Designing abstractions = designing attackers

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$$\Downarrow$$

- Characterize the most concrete ρ such that $(\eta)P(\phi \rightsquigarrow\!\!\!\!\rightarrow\!\!]\!\!]\, \rho)$

  [The most powerful *public observer*]

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Transforming abstractions = transforming attackers

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Transforming abstractions = transforming attackers

$$\Downarrow$$

- Characterize the most concrete δ such that $(\delta)P(\phi \rightsquigarrow\rrbracket \delta)$

  [The most powerful *canonical* public observer]

$\Rightarrow$ This would provide a certificate for security.

Abstract declassification

Consider a program P and its finite computations.

A passive attacker may be able to learn some information by observing the system but, by assumption, that information leakage is allowed by the security policy.

[Zdancewic and Myers 2001]

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- We want to characterize the most abstract *private observable* property such that $(\eta)P(\phi \Rightarrow \rho)$

  [The maximal amount of information disclosed]

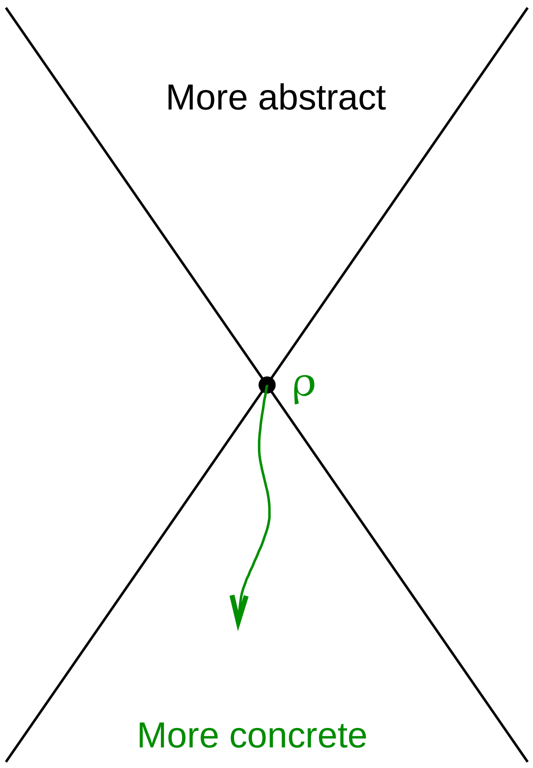
$\Rightarrow$ This would provide a certificate for disclosed secrets.

Observer vs Observable

Consider $\models (\eta)P(\phi \rightsquigarrow\!\!\!\!\rightarrow\!\rrbracket \rho)$: *In order to keep non-interference...*

Observer vs Observable

Consider $\models (\eta)P(\phi \rightsquigarrow\!\!]\, \rho)$: *In order to keep non-interference...*



$uco(\wp(\mathbb{V}^{L}))$

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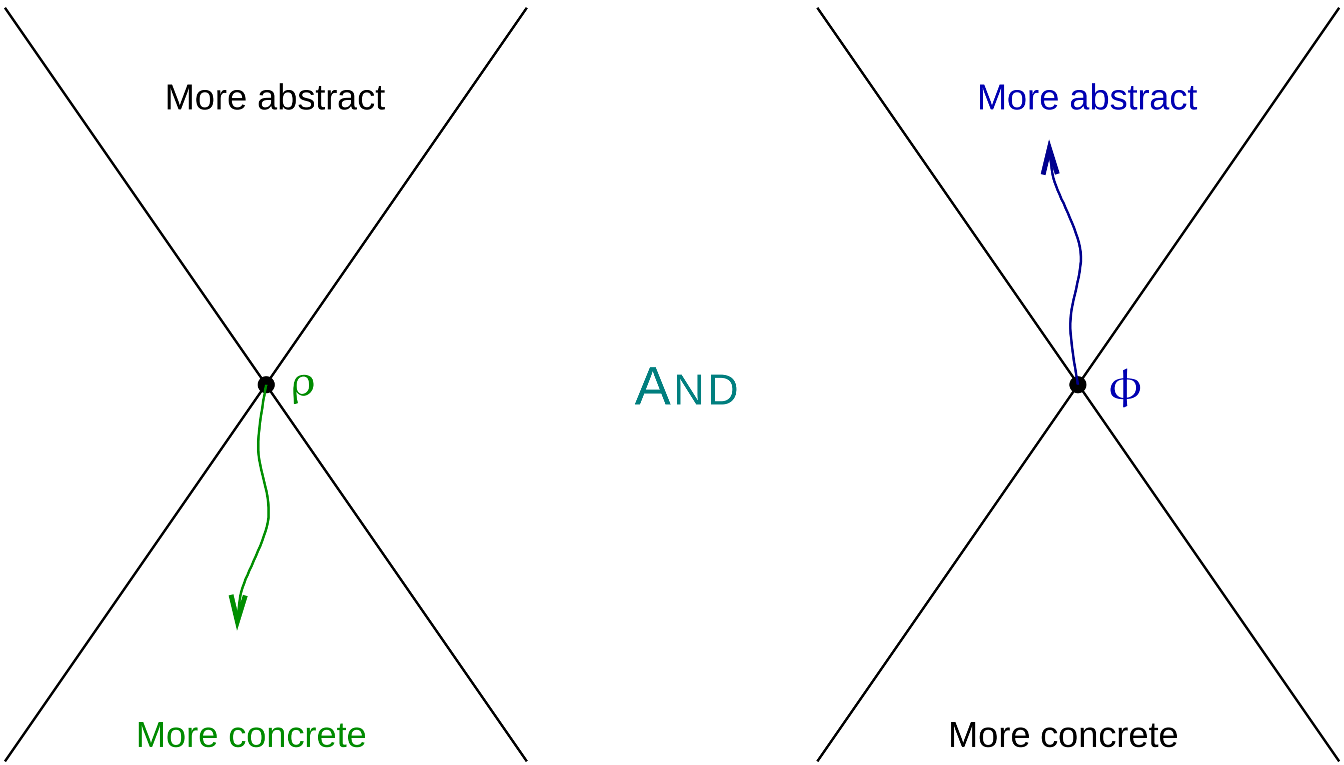

$uco(\wp(\mathbb{V}^{\mathsf{L}}))$ $uco(\wp(\mathbb{V}^{\mathsf{H}}))$

ANI: A completeness problem

Recall that [Joshi & Leino'00]

$$P \text{ is } \textit{secure} \qquad \text{iff} \qquad HH\ ;\ P;\ HH \ \doteq\ P\ ;\ HH$$

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$$\mathrm{P} \text{ is } \textit{secure} \qquad \text{iff} \qquad \mathsf{HH}\ ;\ \mathrm{P};\ \mathsf{HH}\ \doteq\ \mathrm{P}\ ;\ \mathsf{HH}$$

$$\text{Let } \mathrm{X} = \langle \mathrm{X}^{\mathsf{H}}, \mathrm{X}^{\mathsf{L}} \rangle \Rightarrow \mathcal{H}(\mathrm{X}) \stackrel{\text{def}}{=} \langle \top^{\mathsf{H}}, \mathrm{X}^{\mathsf{L}} \rangle \in \textit{uco}(\wp(\mathbb{V}))$$

$$\begin{array}{ccc} \mathsf{HH}\ ;\ \mathrm{P};\ \mathsf{HH} & \doteq & \mathrm{P}\ ;\ \mathsf{HH} \\ & \Downarrow & \\ \mathcal{H} \circ [\![\mathrm{P}]\!] \circ \mathcal{H} & = & \mathcal{H} \circ [\![\mathrm{P}]\!] \end{array}$$

ANI: A completeness problem

Recall that [Joshi & Leino'00]

$$\mathrm{P} \text{ is } \textit{secure} \qquad \text{iff} \qquad \mathrm{H\,H} \ ; \mathrm{P}; \ \mathrm{H\,H} \ \doteq \ \mathrm{P} \ ; \ \mathrm{H\,H}$$

$$\text{Let } X = \langle X^{\mathrm{H}}, X^{\mathrm{L}} \rangle \Rightarrow \mathcal{H}(X) \stackrel{\text{def}}{=} \langle \top^{\mathrm{H}}, X^{\mathrm{L}} \rangle \in \textit{uco}(\wp(\mathbb{V}))$$

$$\mathrm{H\,H} \ ; \ \mathrm{P}; \ \mathrm{H\,H} \quad \doteq \quad \mathrm{P} \ ; \ \mathrm{H\,H}$$

$$\Downarrow$$

$$\mathcal{H} \circ [\![\mathrm{P}]\!] \circ \mathcal{H} \quad = \quad \mathcal{H} \circ [\![\mathrm{P}]\!]$$

$$\boxed{\Rightarrow \text{A COMPLETENESS PROBLEM}}$$

ANI: A completeness problem

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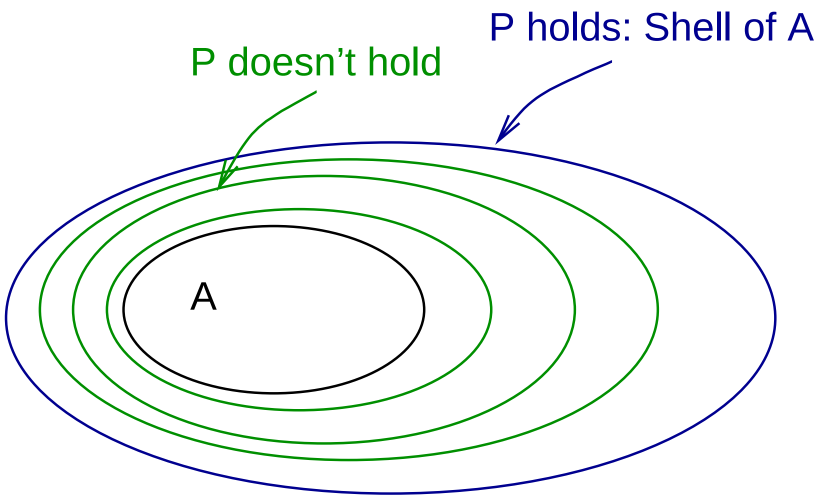
$$\boxed{\text{COMPLETENESS = NON-INTERFERENCE}}$$

$$\Downarrow$$

- Transform $\mathcal{H}$ vs *Core*;
- Transform $\mathcal{H}$ vs *Shell*. [Giacobazzi et al.'00]

Completeness shells and cores

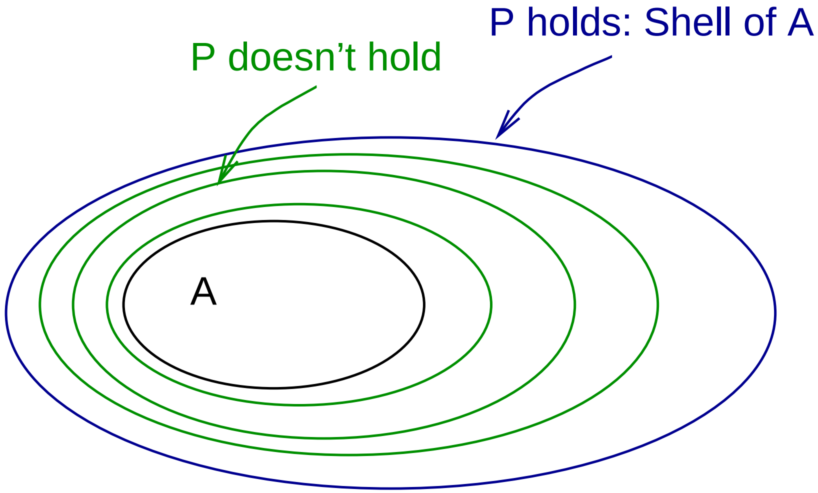
[Giacobazzi et al.'00]



$$R_f \stackrel{\text{def}}{=} \lambda \rho . \mathcal{M}\left(\bigcup_{y \in \rho} max(f^{-1}(\downarrow y)) \right)$$

Completeness shells and cores

[Giacobazzi et al.'00]

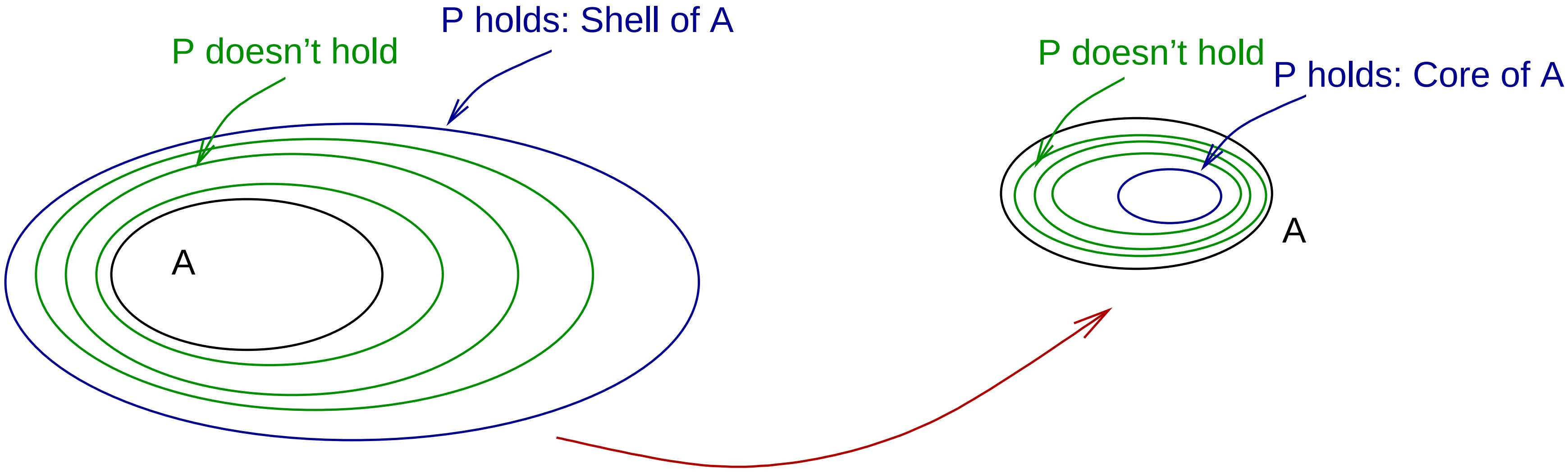


$$R_f \stackrel{\text{def}}{=} \lambda\rho.\mathcal{M}\Big(\bigcup_{y\in\rho} \mathit{max}(f^{-1}(\downarrow y))\Big)$$

- *Absolute shell* of ρ: $\mathcal{R}_f(\rho) = \mathit{gfp}^{\sqsubseteq}_{\rho}\lambda\varphi.\rho \sqcap R_f^{\mathcal{B}}(\varphi)$;
- *Relative shell* of η *relative to* ρ: $\mathcal{R}_f^{\rho}(\eta) = \eta \sqcap R_f(\rho)$.

Completeness shells and cores

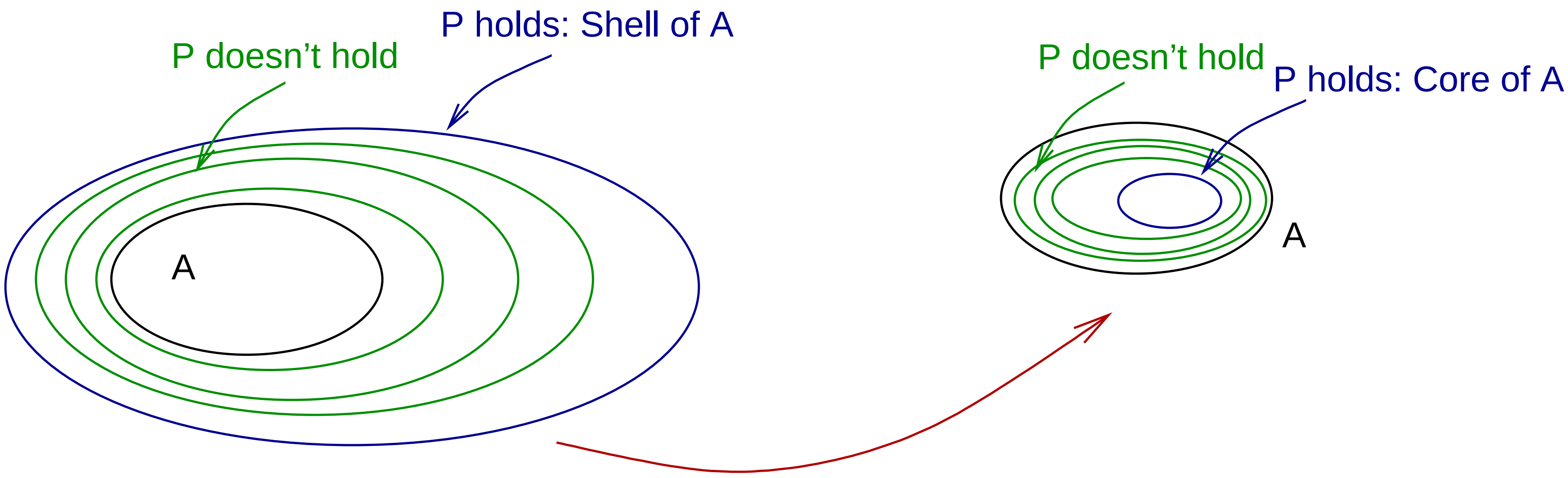
[Giacobazzi et al.'00]



$$C_f \stackrel{\text{def}}{=} \lambda\rho.\left\{ y \in C \;\middle|\; \mathit{max}(f^{-1}(\downarrow y)) \subseteq \rho \right\}$$

Completeness shells and cores

[Giacobazzi et al.'00]



$$C_f \stackrel{\text{def}}{=} \lambda\rho.\left\{\, y \in C \;\middle|\; \mathit{max}(f^{-1}(\downarrow y)) \subseteq \rho \,\right\}$$

- *Absolute core* of ρ: $\mathcal{C}_f(\rho) = \mathit{lfp}^{\sqsubseteq}_{\rho}\lambda\varphi.\rho \sqcup C_f^{\mathcal{B}}(\varphi)$;
- *Relative core* of ρ *relative to* η: $\mathcal{C}_f^{\eta}(\rho) = \rho \sqcup C_f(\eta)$.

ANI as completeness

$$\text{Let } \rho \in uco(\wp(\mathbb{V}^{L})) \Rightarrow \mathcal{H}_{\rho}(X) \stackrel{\text{def}}{=} \langle \top^{H}, \rho(X^{L}) \rangle \in uco(\wp(\mathbb{V}))$$

- *Narrow abstract non-interference:* $\mathcal{H}_{\rho} \circ [\![P]\!] \circ \mathcal{H}_{\eta} = \mathcal{H}_{\rho} \circ [\![P]\!]$;
- *Abstract non-interference:* $\mathcal{H}_{\rho} \circ [\![P]\!]^{\eta,\phi} \circ \mathcal{H}_{\eta} = \mathcal{H}_{\rho} \circ [\![P]\!]^{\eta,\phi}$

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Let $\rho \in uco(\wp(\mathbb{V}^{\mathsf{L}}))\Rightarrow \mathcal{H}_{\rho}(X) \stackrel{\text{def}}{=} \langle \top^{\mathsf{H}}, \rho(X^{\mathsf{L}})\rangle \in uco(\wp(\mathbb{V}))$

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$$\Downarrow$$

- PUBLIC OBSERVER AS COMPLETENESS CORE: $\mathcal{C}^{\mathcal{H}_{\eta}}_{[\![P]\!]^{\eta,\phi}}(\mathcal{H}) = (\eta)[\![P]\!](\phi \rightsquigarrow\!\!\!\!\!\rrbracket \mathit{id})$

ANI as completeness

Let $\rho \in uco(\wp(\mathbb{V}^{L})) \Rightarrow \mathcal{H}_\rho(X) \stackrel{\text{def}}{=} \langle \top^{H}, \rho(X^{L}) \rangle \in uco(\wp(\mathbb{V}))$

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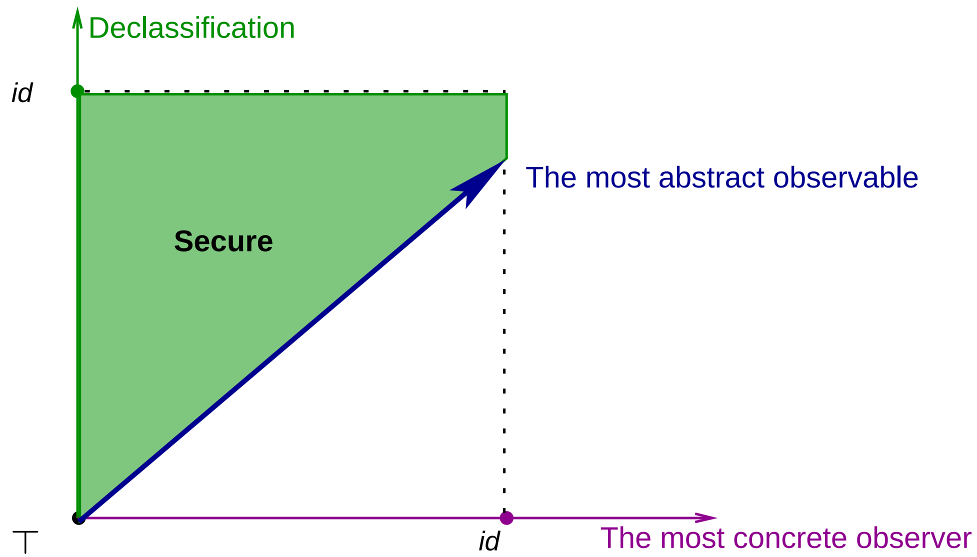
- PUBLIC OBSERVER AS COMPLETENESS CORE: $\mathcal{C}^{\mathcal{H}_\eta}_{[\![P]\!]^{\eta,\phi}}(\mathcal{H}) \;=\; (\eta)[\![P]\!](\phi \rightsquigarrow\!\!\!]\!\!] \, id)$
- PRIVATE OBSERVABLE AS COMPLETENESS SHELL: $(\eta)P(\mathcal{R}^{\mathcal{H}_\rho}_{[\![P]\!]^{\eta},id}(\mathcal{H}_\eta) \Rightarrow \rho)$

ANI as completeness

- PUBLIC OBSERVER AS COMPLETENESS CORE: $\mathcal{C}^{\mathcal{H}_\eta}_{[\![P]\!]^\eta,\phi}(\mathcal{H}) = (\eta)[\![P]\!](\phi \rightsquigarrow\!\!\| id)$
- PRIVATE OBSERVABLE AS COMPLETENESS SHELL: $(\eta)P(\mathcal{R}^{\mathcal{H}_\rho}_{[\![P]\!]^\eta,id}(\mathcal{H}_\eta) \Rightarrow \rho)$
- ADJOINING ATTACKERS AND DECLASSIFICATION

$$id \sqsubseteq (\eta)[\![P]\!](id \rightsquigarrow\!\!\| id) \Leftrightarrow \mathcal{P}(\sqcap_{L\in\eta}\mathcal{M}(\Pi_P(\eta, id)_{|L})) \sqsubseteq \top$$



A discussion

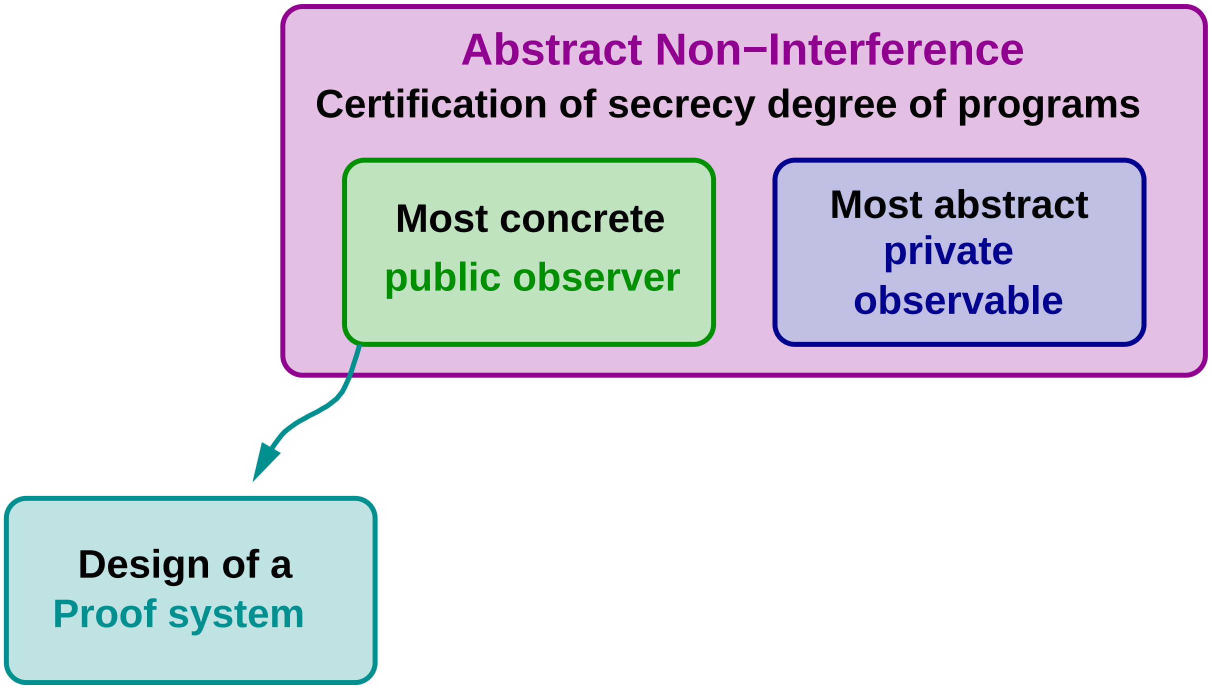
Abstract Non−Interference

Certification of secrecy degree of programs

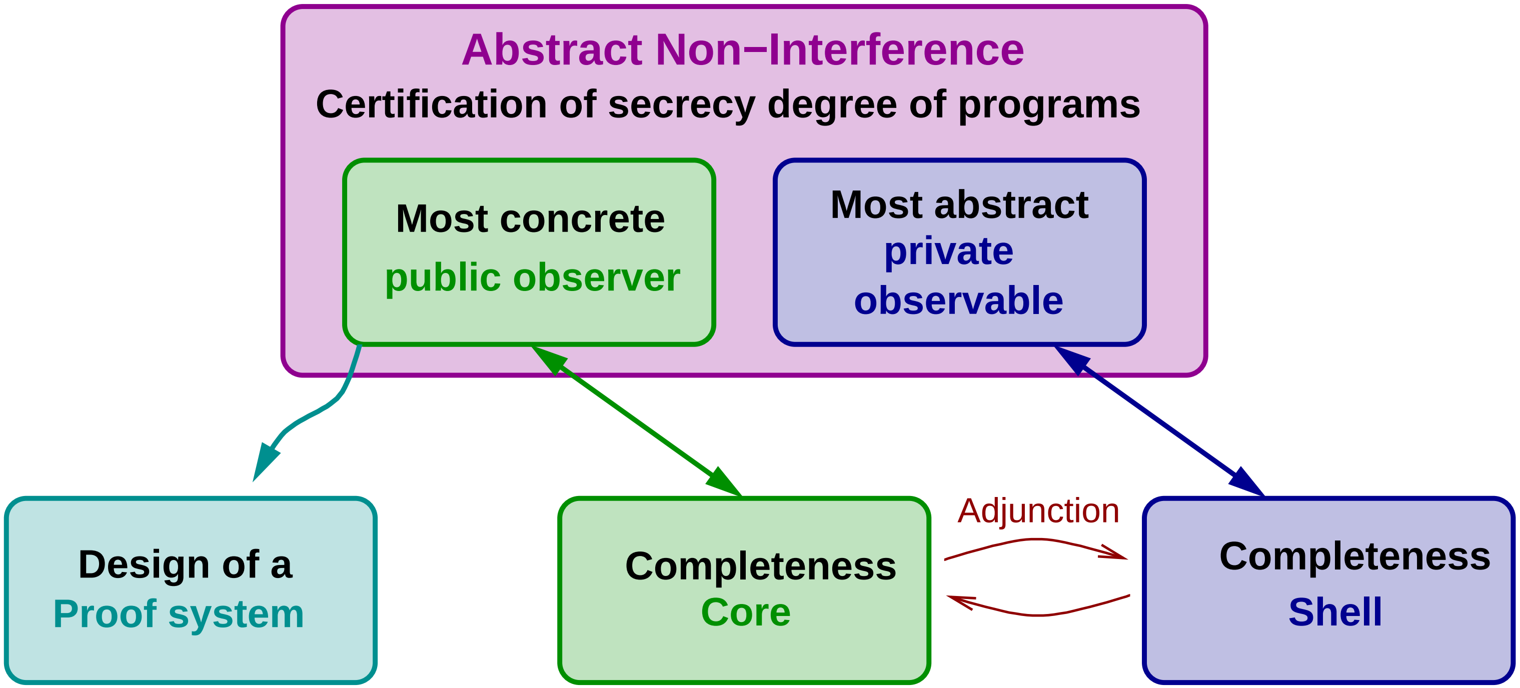
Most concrete
public observer

Most abstract
private observable

A discussion



A discussion



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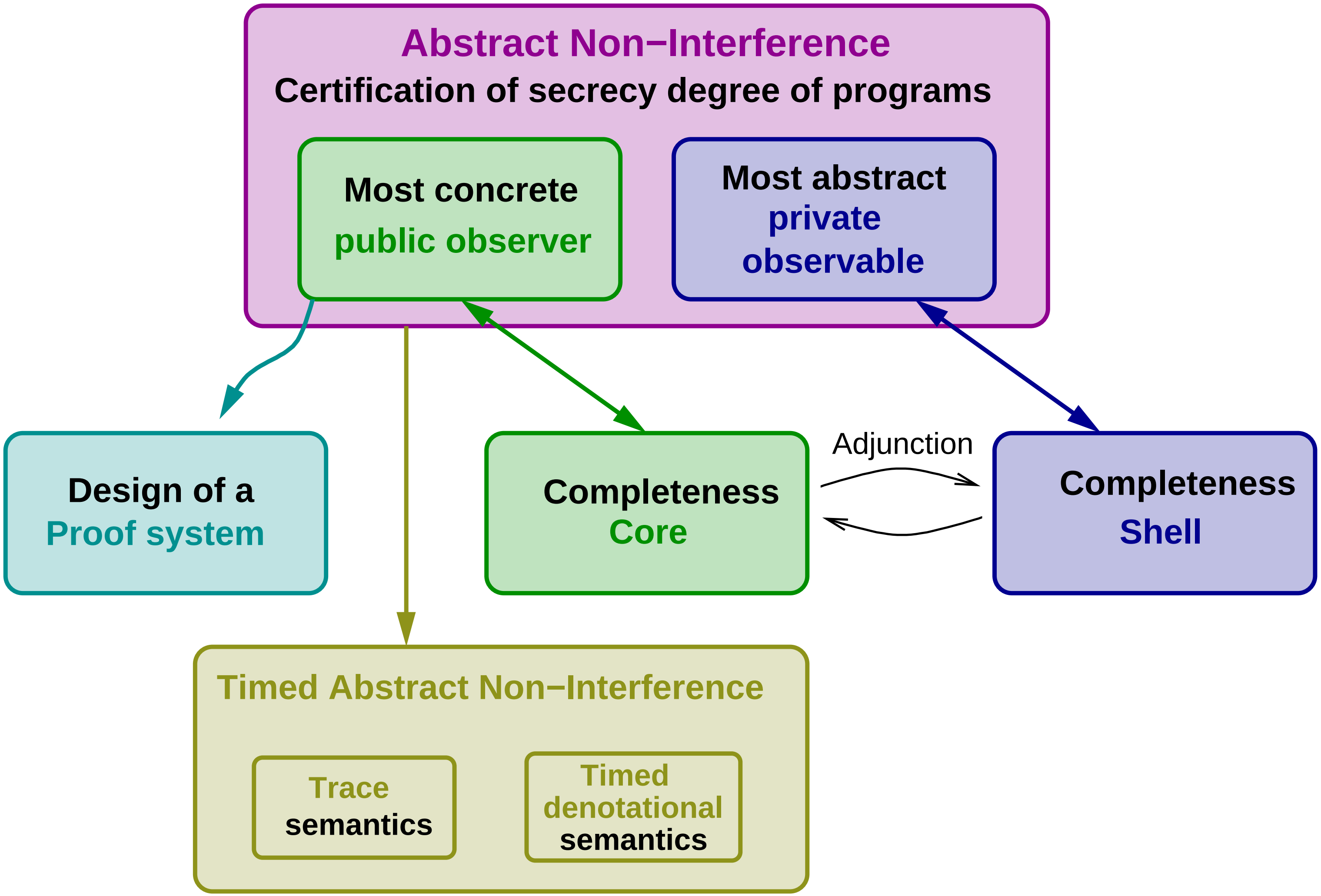

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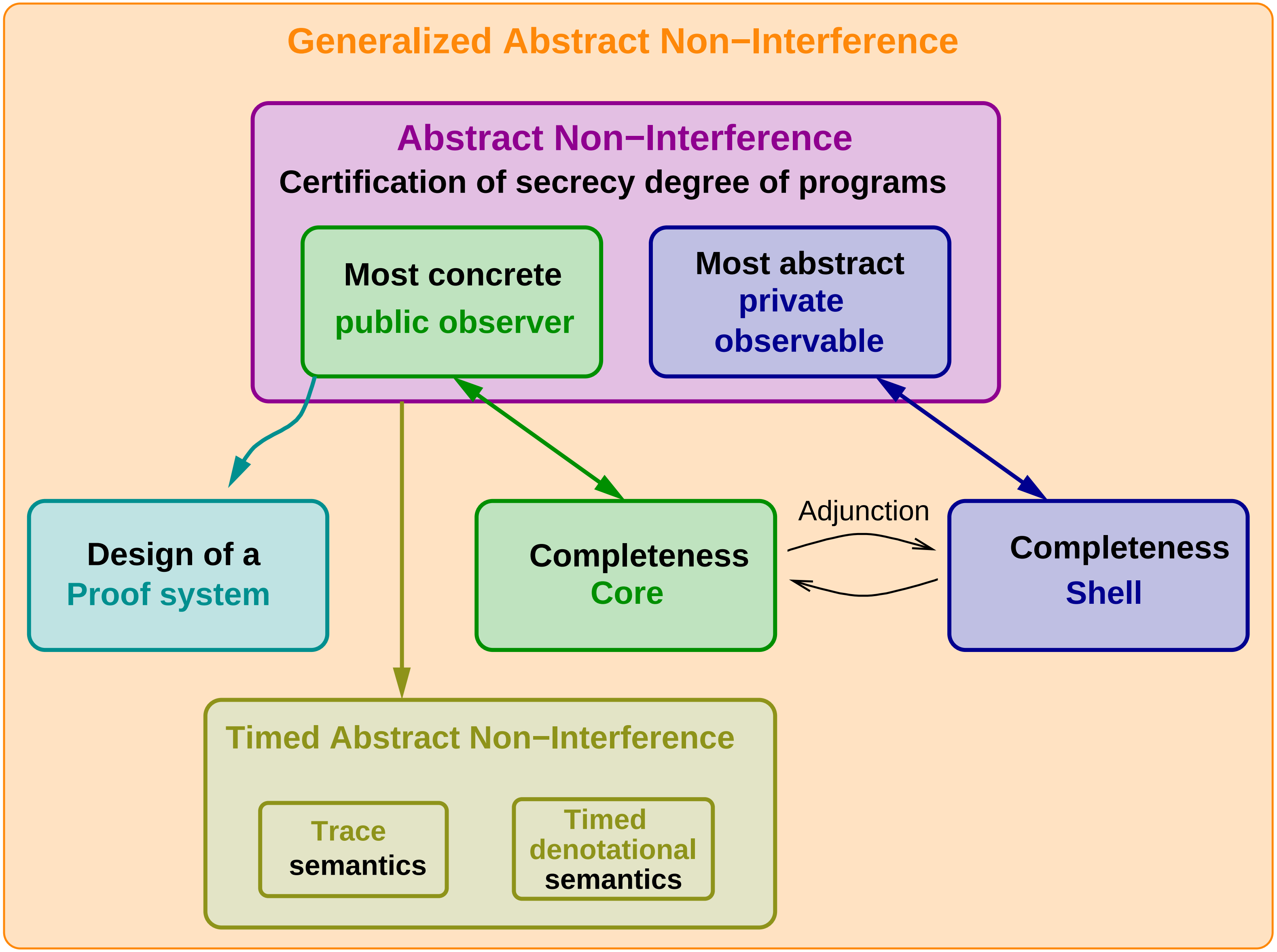
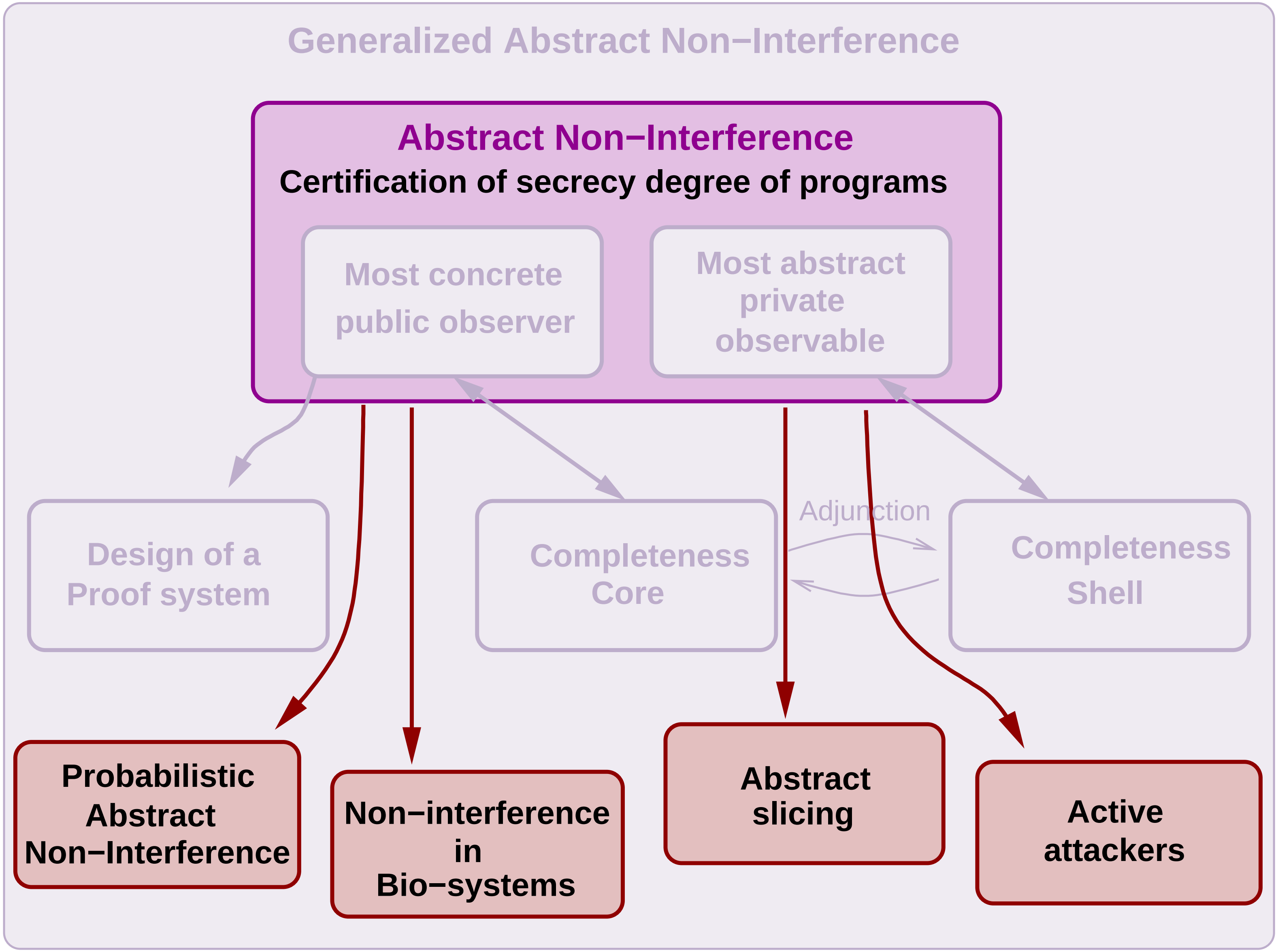
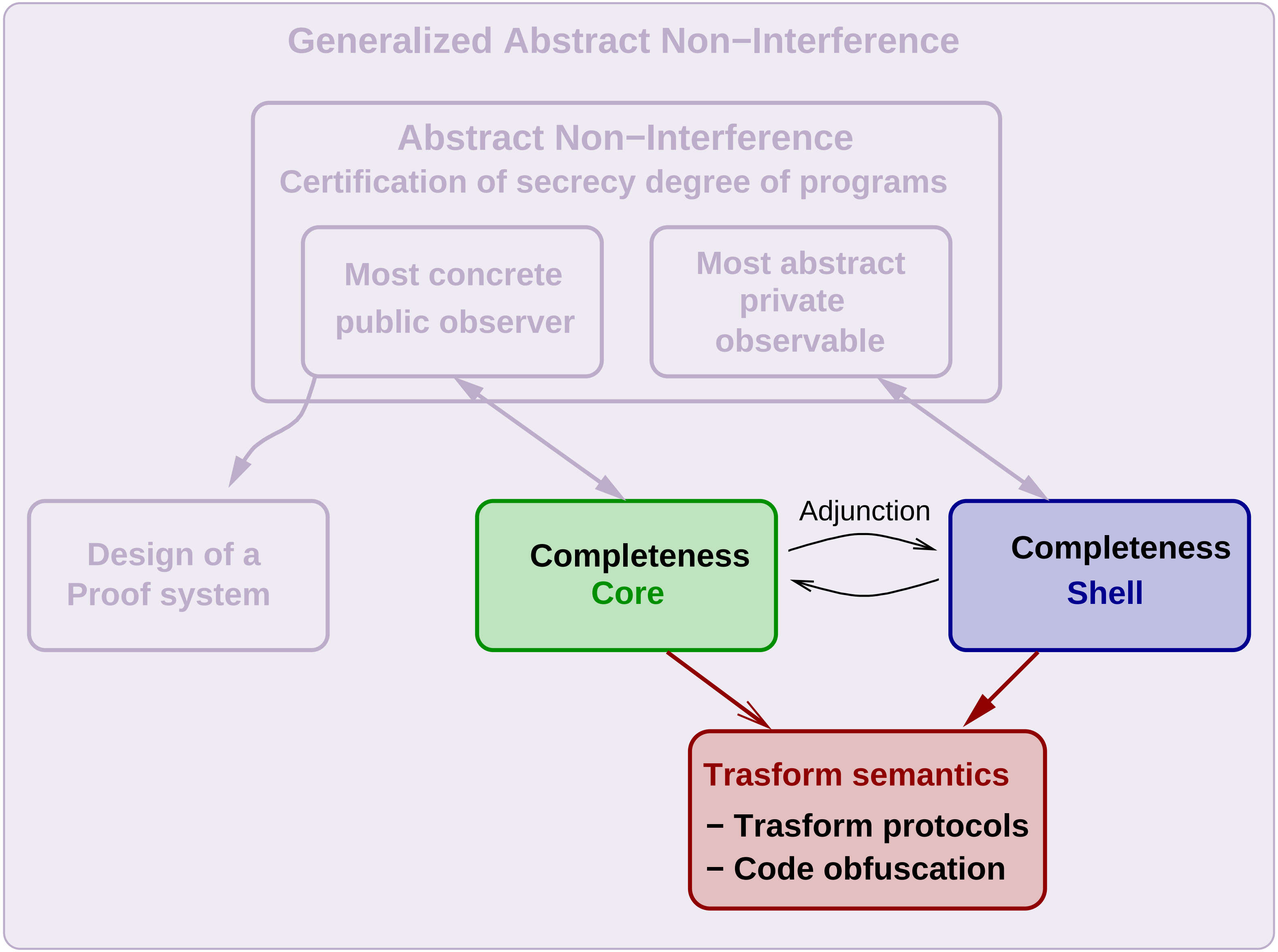


A discussion: Future works



A discussion: Future works



A discussion: Future works