Measuring Confinement via Weak Bisimulation (Extended Abstract)

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1 Introduction

It is well known that observability is closely related to vulnerability against confidentiality attacks: once a low level process is able to observe differences in the behaviour of a high level process it is able to deduce information about its secrets, and confidentiality is thus at least partially violated. However, if whatever the high level secrets are, the behaviours of the observed processes are all equivalent, then a potential attacker cannot reveal any secret information.

The recent literature has shown that perfect confinement is a requirement which is hardly met by the real-world systems. With this motivation, in previous work [1, 2], we began looking at approximative notions of confinement. This research is based on the study of quantitative versions of various process equivalences by re-casting them in terms of linear operators. It turned out that process equivalences such as tree isomorphism and bisimulation can be understood in terms of Probabilistic Abstract Interpretation [3]. In particular, two processes are equivalent if there exists a common probabilistic abstraction of both for an appropriate class of abstractions which depend on the notion of observation we are interested in. Within this setting we were then able to formulate approximate notions of these process equivalences. This leads naturally to the definition of corresponding approximate notions of confinement. Each notion has a quantity ε associated to it which lends itself to a statistical interpretation in terms of the effort needed to an attacker to break a system. This gives a measure of the confinement of the system.

The current work aims in extending this approach to accommodate another important notion of observability, namely probabilistic weak bisimulation. Weak bisimulation is of particular importance in the context of security as it is quite natural to assume that the actions of high level principals cannot be observed directly by the low level processes. The high level behaviour is thus modelled by τ actions, see e.g. [4, 5].

2 Background and Preliminaries

We present our approach in the framework of probabilistic transition systems. We adopt the so-called generative model of probability [6], and formally define a PTS by means of a variation of the general definition given in [7, Def 2].

Definition 1. *A* probabilistic transition system *(PTS) is a tuple* $(S, A, \longrightarrow, \pi_0)$*, where:*

- S *is a non-empty, finite set of* states*,*
- A *is a non-empty, finite set of* actions*,*
- $\longrightarrow \subseteq S \to Dist(A \times S)$ *is a* (generative) transition relation*, and*
- $\pi_0 \in Dist(S)$ *is an* initial distribution *on* S*.*

For $s \in S$, $\alpha \in A$ and $\pi \in Dist(S)$ we write as usual $s \xrightarrow{\alpha} \pi$ for $(s, \alpha, \pi) \in \longrightarrow$. By $s \xrightarrow{\alpha}_{\pi(t)} t$ we denote the transition to individual states t with probability $\pi(t)$.

We choose here to present our results for finite state systems as this allows us to concentrate on the description of the method by avoiding the technical details of a more involved (topological) treatment of infinite structures. Nevertheless, these results can be extended to the general case of infinite set of states along the lines sketched in [8], i.e. in an operator algebraic setting.

2.1 Operator Semantics of Probabilistic Transition Systems

A suitable mathematical framework for a quantitative study of process equivalences in concurrent systems is provided by linear algebra. Given a probabilistic transition relation $\longrightarrow \subseteq X \times [0,1] \times X$ we represent it by the matrix $\mathbf{M}(\longrightarrow)$:

$$(\mathbf{M}(\longrightarrow))_{xy} = \begin{cases} p \text{ if } x \xrightarrow{p_i} y \text{ and } p = \sum_i p_i \\ 0 \text{ otherwise} \end{cases}$$

We obtain this way a *stochastic matrix*, that is a positive matrix where the entries in each row sums up to one. The matrix $\mathbf{M}$ defines a linear map $\mathcal{V}(X) \to \mathcal{V}(X)$ on the vector space $\mathcal{V}(X)$ over a set X. This is defined as the space of all formal linear combinations of elements in X with coefficients in $\mathbb{R}$ which we can represent as infinite vectors with coefficients in $\mathbb{R}$ indexed by elements in X: $\mathcal{V}(X) = \{(c_x)_{x\in X} \mid c_x \in \mathbb{R}\}$.

Definition 2. *Given a probabilistic transition system $p = (S, A, \longrightarrow, \pi_0)$, we define its* matrix *or* operator representation $\mathbf{M} = (\mathbf{M}(p), \mathbf{M}(\pi_0))$ *as the direct sum of the operator representations of the transition relations for each $\alpha \in A$*

$$\mathbf{M}(p) = \bigoplus_{\alpha\in A} \mathbf{M}(\xrightarrow{\alpha}),$$

and $|A|$ copies of the vector $\boldsymbol{\pi}_0$ representing π_0: $\mathbf{M}(\pi_0) = \bigoplus_{\alpha\in A} \boldsymbol{\pi}_0$.

For the sake of simplicity we will denote by $\mathbf{PM}$ the multiplication of a *direct sum* $\bigoplus_\alpha \mathbf{P}$ of the same matrix $\mathbf{P}$ with the matrix $\mathbf{M} = \bigoplus_\alpha \mathbf{M}_\alpha$. By the properties of the direct sum this is the same as $\bigoplus_\alpha (\mathbf{PM}_\alpha)$.

2.2 Abstractions of Probabilistic Transition Systems

In [1, 8] we have shown how some special classes of linear operators can be used to define abstractions of the PTS semantics into various process equivalences as well as to define approximate versions of the process equivalences. This approach is based on a technique developed in the framework of *Probabilistic Abstract Interpretation* [3, 9] which allows us to define an abstract operator (semantics) in terms of the concrete one, the given abstraction $\mathbf{A}$ and its Moore-Penrose pseudo inverse $\mathbf{A}^\dagger$, see e.g. [10]. More precisely, given a linear operator Φ expressing the probabilistic semantics of a concrete system, and a linear abstraction function $\mathbf{A} : \mathcal{V} \mapsto \mathcal{W}$ from the concrete domain into an abstract domain $\mathcal{W}$, we compute the Moore-Penrose pseudo-inverse $\mathbf{G} = \mathbf{A}^\dagger$ of $\mathbf{A}$. The abstract semantics can then be defined as the linear operator on the abstract domain $\mathcal{W}$ as $\Psi = \mathbf{A} \circ \Phi \circ \mathbf{G}$.

A simple class of abstraction operators is represented by *classification matrices*. These are (stochastic) matrices corresponding to a particular type of abstraction which stems from an equivalence relation on a set X. Intuitively, a classification operator is a $0/1$-matrix whose rows represent the elements of the set X, while the columns are associated to the equivalence classes in the quotient set induced by the relation. An entry of such a matrix is 1 iff the row-element belongs to the column-class. In the finite case, it can be shown that there is a one-to-one correspondence between these operators and equivalence relations. The classification matrix $\mathbf{K}$ associated to a probabilistic bisimulation equivalence on a PTS $p = (S, A, \rightarrow, \pi_0)$ can then be used to define a probabilistic abstract interpretation of p. Since $\mathbf{K}$ is a classification matrix, its Moore-Penrose pseudo-inverse $\mathbf{K}^\dagger$ is just the transpose of $\mathbf{K}$ normalised such that every row sum is 1. By using classification operators we can characterise probabilistic bisimulation as shown in the following proposition [1].

Proposition 1. *Given the operator representations $\mathbf{M}(p)$ and $\mathbf{M}(q)$ of two probabilistic transition systems $p = (S, A, \longrightarrow, s_0)$ and $q = (S', A, \longrightarrow', s'_0)$, then p and q are probabilistic bisimilar iff there exist classification matrices $\mathbf{K}_p$ and $\mathbf{K}_q$ of dimension $(|S| \times n)$ and $(|S'| \times n)$ respectively, for some n such that*

$$\mathbf{K}_p^\dagger \cdot \mathbf{M}(p) \cdot \mathbf{K}_p = \mathbf{K}_q^\dagger \cdot \mathbf{M}(q) \cdot \mathbf{K}_q.$$

3 Weak Bisimulation

Several authors have argued that bisimulation is for many purposes a much too strong requirement and suggested a number of weaker semantics (see [11] for a detailed account). Weak bisimulation was introduced in [12] as a bisimulation which abstracts from internal computation by considering transitions of the form $\Rightarrow\xrightarrow{\alpha}\Rightarrow$, where $\Rightarrow$ is the transitive, reflexive closure of $\xrightarrow{\tau}$, and τ is the internal action and represents any invisible computation.

We define weak bisimulation following the treatment for fully probabilistic processes in [13]. An important step for this treatment is the definition of the probability of reaching a state or a certain class of states by sequences of actions (or *traces*) of the form $\xrightarrow{\tau}^* \xrightarrow{\alpha} \xrightarrow{\tau}^*$. This probability is defined for strings in a generic language $\Lambda \subset A^*$ recursively as follows:

$$\begin{aligned} \mathcal{P}(s, \Lambda, C) &= \quad 1 \quad \text{if } s \in C \text{ and } \varepsilon \in \Lambda \\ \mathcal{P}(s, \Lambda, C) &= \sum_{(a,t)\in A\times S} P(s,a,t) \cdot \mathcal{P}(t, \Lambda/a, C) \quad \text{otherwise} \end{aligned}$$

where Λ/a denotes the set of all strings λ such that $a\lambda \in \Lambda$, and ε denotes the empty string. By considering the language $\Lambda = \tau^* a \tau^* \cup \varepsilon$, the notion of probabilistic weak bisimulation can be defined as follows.

Definition 3. *A weak bisimulation is an equivalence relation $\sim_w$ on S such that for all $s \sim_w s'$ and all $\alpha \in A \setminus \{\tau\} \cup \varepsilon$ and all equivalence classes $C \in S/\sim_w$ we have:*

$$\mathcal{P}(s, \tau^*\alpha\tau^*, C) = \mathcal{P}(s', \tau^*\alpha\tau^*, C)$$

3.1 Reachability of States via α^*

The first step is to look at the reachability of a state from another state via a single trace. It is well known that iterating a transition matrix n times gives the probability of reaching state s from t in *exactly* n steps. This is sometimes known as the Chapman-Kolmogorov equations, e.g. [14, Thm 6.1.7].

Unfortunately — due to the recursive definition of $\mathcal{P}$ — we can't simply sum up all iterations of the transition matrix in order to compute $\mathcal{P}$, as this may lead to count some "reaching probabilities" (related to loops) too often. In order to obtain a correct result we have to compute the probability of reaching a state t the *first* time, i.e. along the minimal trace leading to t. This means we have to "block" out all contributions which come from paths which already passed through t before. We can achieve this by removing all transitions from t in the operator $\mathbf{M}_\alpha(p)$. We define a projection into t as a diagonal matrix which contains a single entry 1 at the position (t,t), and its "negation", i.e.

$$(\mathbf{P}_t)_{ij} = \begin{cases} 1 \text{ for } i = j = t \\ 0 \text{ otherwise} \end{cases} \qquad (\mathbf{P}_t^\perp)_{ij} = \begin{cases} 1 \text{ for } i = j \neq t \\ 0 \text{ otherwise} \end{cases}$$

If we now consider the modified transition operator

$$\mathbf{M}_{\alpha,\neg t}(p) = \mathbf{P}_t^\perp \mathbf{M}_\alpha(p)$$

we get the same transitions as in $\mathbf{M}_\alpha(p)$ except that all transitions from t are cancelled out — as the matrix $\mathbf{M}_{\alpha,\neg t}(p)$ is identical to $\mathbf{M}_a(p)$ except for the fact that the row t contains only zeros.

If we consider now the column t in $\mathbf{M}^n_{\alpha,\neg t}(p)$ we obtain for each state s the probability of reaching t in exactly n steps without passing through t, i.e. for the first time. We can extract this t column by multiplying with the projection $\mathbf{P}_t$, i.e.$(\mathbf{P}_t^\perp \mathbf{M}_\alpha(p))^n \cdot \mathbf{P}_t = \mathbf{M}_{\alpha,\neg t}(p) \cdot \mathbf{P}_t$. The probability of getting from any state s to t via the minimal trace in at most n steps is given by:

$$\sum_{i=0}^{n} (\mathbf{P}_t^\perp \mathbf{M}_\alpha(p))^i \cdot \mathbf{P}_t = \sum_{i=0}^{n} (\mathbf{M}_{\alpha,\neg t}(p))^i \cdot \mathbf{P}_t$$

Once we have a trace from a state s reaching state t the first time, all its extensions are ignored as in $\mathbf{M}_{\alpha,\neg t}(p)$ there is no transition which leaves the state t again.

Proposition 2. *Given the operator representations* $\mathbf{M}(p)$ *of a probabilistic transition system* $p = (S, A, \longrightarrow, s_0)$ *then for all* $\alpha \in A$*:*

$$\mathcal{P}(s, \alpha^*, \{t\}) = \left(\sum_{i=0}^{\infty} \left(\sum_{t \in S} \mathbf{M}^i_{\alpha,\neg t}(p) \mathbf{P}_t \right) \right)_{st} .$$

3.2 Reachability of States via $\tau^*\alpha\tau^*$

Definition 4. *Given the operator representation* $\mathbf{M}(p)$ *of a PTS* p *with* $A = \{a, b, \dots, \tau\}$*, then we define for all* $\alpha \in A$*:*

$$\mathbf{F}_\alpha(p)(n, m) = \sum_{t \in S} \mathbf{M}_\tau(p)^n \cdot \mathbf{M}_\alpha(p) \cdot (\mathbf{P}_t^\perp \mathbf{M}_\tau(p))^m \cdot \mathbf{P}_t.$$

We denote by $\mathbf{F}(p)(n, m)$ *the direct sum* $\bigoplus_{\alpha \in A} \mathbf{F}_\alpha(p)(n, m)$ *of all* $\mathbf{F}_\alpha(p)(n, m)$*.*

The operator $\mathbf{F}_\alpha(p)(n, m)$ encodes the probabilities of reaching a state by the trace $\tau^n\alpha\tau^m$, for some fixed $n, m \in \mathbb{N}$. The extension to the language $\tau^*\alpha\tau^*$ can be achieved by the operator

$$\overline{\mathbf{F}}_\alpha(p) = \sum_{n,m=0}^{\infty} \mathbf{F}_\alpha(p)(n, m)$$

which gives us the probabilities for any string in $\tau^*\alpha\tau^*$. More precisely we have:

Proposition 3. *Given the operator representation* $\mathbf{M}(p)$ *of a probabilistic transition system* $p = (S, A, \longrightarrow, s_0)$ *then for all* $\alpha \in A$*:*

$$\mathcal{P}(s, \tau^*\alpha\tau^*, \{t\}) = (\overline{\mathbf{F}}_\alpha(p))_{st}$$

3.3 Reachability of Classes via $\tau^*\alpha\tau^*$

The last step towards the definition of a linear operator representing the probabilistic weak bisimulation equivalence in Definition 3 is to introduce projection operators on classes of states. Let $C \subseteq S$ be a set of states, then the projection on C and its negation are defined by

$$(\mathbf{P}_C)_{ij} = \begin{cases} 1 \text{ for } i = j \wedge i \in C \\ 0 \text{ otherwise} \end{cases} \qquad (\mathbf{P}_C^\perp)_{ij} = \begin{cases} 1 \text{ for } i = j \wedge i \notin C \\ 0 \text{ otherwise} \end{cases}$$

As recalled in Section 2.2, an equivalence relation $\mathcal{R}$ has a linear representation given by a classification matrix $\mathbf{K}_\mathcal{R}$. If $\mathbf{K}$ is the classification matrix associated to a probabilistic weak bisimulation equivalence on a state space S, then we can use it to construct the projection operators $\mathbf{P}_{C_i}$ and $\mathbf{P}_{C_i}^\perp$ for all classes C_i in the partition of the state space $S = \bigcup_i C_i$ induced by that relation. We denote by $\mathbf{K}_{.,i}$ the ith column of $\mathbf{K}$, corresponding to class C_i. Then $\mathbf{P}_{C_i}$ can be constructed as the diagonal matrix $diag(\mathbf{K}_{.,i})$ with the ith column of $\mathbf{K}$ as diagonal, and $\mathbf{P}_{C_i}^\perp$ as $\mathbf{I} - \mathbf{P}_{C_i} = \mathbf{I} - diag(\mathbf{K}_{.,i})$ with $\mathbf{I}$ the identity matrix.

Definition 5. *Given the operator representation* $\mathbf{M}(p)$ *of a process* $p = (S, A, \longrightarrow, s_0)$ *with* $A = \{a, b, \dots, \tau\}$*, and a partition* $\mathcal{C} = \{C_i\}_i$ *of* S *represented by a classification matrix* $\mathbf{K}$ *then we define for all* $\alpha \in A$*:*

$$\mathbf{F}_\alpha(p, \mathbf{K})(n, m) = \sum_{C_i \in \mathcal{C}} \mathbf{M}_\tau(p)^n \cdot \mathbf{M}_\alpha(p) \cdot (\mathbf{P}_{C_i}^\perp \mathbf{M}_\tau(p))^m \cdot \mathbf{P}_{C_i}.$$

We denote by $\mathbf{F}(p, \mathbf{K})(n, m)$ *the direct sum* $\bigoplus_{\alpha \in A} \mathbf{F}_\alpha(p, \mathbf{K})(n, m)$ *of all* $\mathbf{F}_\alpha(p, \mathbf{K})(n, m)$*. Furthermore,*

$$\overline{\mathbf{F}}_\alpha(p, \mathbf{K}) = \sum_{n,m=0}^{\infty} \mathbf{F}_\alpha(p, \mathbf{K})(n, m).$$

This operators "blocks" out all repeated visits to the same class in essentially the same way as discussed in Section 3.2 for states. We therefore have, as expected, the following result:

Proposition 4. *Given the operator representations* $\mathbf{M}(p)$ *of a probabilistic transition systems* $p = (S, A, \longrightarrow, s_0)$ *and a partition* $\mathcal{C} = \{C_i\}_i$ *of* S *represented by a classification matrix* $\mathbf{K}$ *then for all* $\alpha \in A$*:*

$$\mathcal{P}(s, \tau^*\alpha\tau^*, C) = (\overline{\mathbf{F}}_\alpha(p, \mathbf{K}) \cdot \mathbf{K})_{sC}.$$

4 Measuring Process Behaviour

The probabilistic weak bisimulation semantics introduced before gives us a criterion to compare the behaviour of two processes. The following proposition gives a necessary and sufficient condition for two processes being probabilistic weakly bisimilar.

Proposition 5. *Given the operator representations* $\mathbf{M}(p)$ *and* $\mathbf{M}(q)$ *of two probabilistic transition systems* $p = (S, A, \longrightarrow, s_0)$ *and* $q = (S', A, \longrightarrow', s'_0)$ *then* p *and* q *are probabilistic weak bisimilar iff there exist classification matrices* $\mathbf{K}_p \in \mathcal{C}(|S|, n)$ *and* $\mathbf{K}_q \in \mathcal{C}(|S'|, n)$ *for some* $n \geq 1$ *such that*

$$\mathbf{K}_p^\dagger \cdot \overline{\mathbf{F}}(p, \mathbf{K}_p) \cdot \mathbf{K}_p = \mathbf{K}_q^\dagger \cdot \overline{\mathbf{F}}(q, \mathbf{K}_q) \cdot \mathbf{K}_q,$$

i.e. for all $\alpha \in A$ *we have* $\mathbf{K}_p^\dagger \cdot \overline{\mathbf{F}}_\alpha(p, \mathbf{K}_p) \cdot \mathbf{K}_p = \mathbf{K}_q^\dagger \cdot \overline{\mathbf{F}}_\alpha(q, \mathbf{K}_q) \cdot \mathbf{K}_q$*.*

An approximative version of this notion allows us to capture how close two processes are to being weakly bisimilar. In other words, when it is impossible to construct a weak bisimulation relation which identifies two processes p and q we can still ask for a weak bisimulation in which the "distance" between the two processes is minimal (or below a certain threshold). The linear setting we have chosen allows us to use an appropriate *norm* to measure such a distance.

The idea is to find classification operators $\mathbf{K}_p$ and $\mathbf{K}_q$ which result in two abstractions $\mathbf{K}_p^\dagger \cdot \overline{\mathbf{F}}(p, \mathbf{K}_p) \cdot \mathbf{K}_p$ and $\mathbf{K}_q^\dagger \cdot \overline{\mathbf{F}}(q, \mathbf{K}_q) \cdot \mathbf{K}_q$ of p and q such that the distance or difference between these two abstractions is minimal.

Definition 6. *Given the operator representations* $\mathbf{M}(p)$ *and* $\mathbf{M}(q)$ *of two probabilistic transition systems* $p = (S, A, \longrightarrow, s_0)$ *and* $q = (S', A, \longrightarrow', s'_0)$*, we say that* p *and* q *are* probabilistic ε-weak bisimilar*, denoted by* $p \sim_w^\varepsilon q$*, if*

$$\inf_{\mathbf{K}_p, \mathbf{K}_q \in \mathcal{C}} \|\mathbf{K}_p^\dagger \cdot \overline{\mathbf{F}}(p, \mathbf{K}_p) \cdot \mathbf{K}_p - \mathbf{K}_q^\dagger \cdot \overline{\mathbf{F}}(q, \mathbf{K}_q) \cdot \mathbf{K}_q\| = \varepsilon$$

where $\|.\|$ *denotes an appropriate norm.*

For $\varepsilon = 0$ we recover the original notion of strict (probabilistic) weak bisimulation:

Proposition 6. *For finite probabilistic transition systems, a* ε*-weak bisimulation for* $\varepsilon = 0$*, i.e.* $\sim_w^0$*, is a probabilistic weak bisimulation.*

5 Conclusions

We have re-casted the weak bisimulation semantics for probabilistic processes of [13] in terms of linear operators on a vector space construction over the set of states. Our main motivation was the definition of a related similarity notion according to which two processes can be considered indistinguishable up to a given tolerance factor. This factor can be calculated as the norm of a linear operator representing the operational behavioural difference of the two processes. This norm defines a distance on the set of processes. Other approaches to the definition of such a distance have been proposed in the literature — usually based on some notion of metric or pseudo-metric — starting from the work by Giacalone et al. [15] to the approach by van Breugel and Worrell [16] and the work of Desharnais et al. [17–19]. Contrary to these approaches, we can provide the quantity measuring the distance between two processes with a statistical meaning expressing the number of tests an external observer needs to perform in order to distinguish them [2].

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