Algorithmic Graph Theory
Part V - Approximation Algorithms for Graph Problems

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Coping with NP-hardness

There are several approaches on how to deal with (the intractability of) NP-hard problems:

- polynomial algorithms for particular input instances
- approximation algorithms
- heuristics, local optimization
- “efficient” exponential algorithms
- randomized algorithms
- parameterized complexity
  (fixed-parameter tractable (FPT) algorithms)

What we'll do

1. Basic Definitions.
2. 2-Approximation Algorithm for Vertex Cover.
3. Approximation Algorithms for the Metric TSP.

BASICS OF APPROXIMATION ALGORITHMS.

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Approximation algorithms:

- guaranteed to run in polynomial time;
- guaranteed to find “high quality” solution, say within 1% of optimum;
- Obstacle:
  need to prove a solution’s value is close to optimum,
  without even knowing what the optimum value is!

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  $\mathrm{OPT}(I)$ is the optimal solution value.
- for maximization problems: $f_A(I) \ge \mathrm{OPT}(I)/\rho$.

Approaches to the Design of Approximation Algorithms

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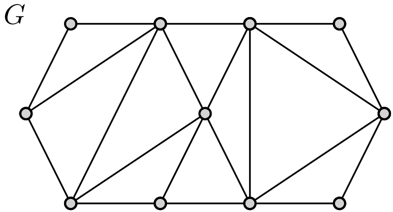
- algorithms based on linear programming,
- randomized algorithms,
- etc.

2-Approximation Algorithm for the Vertex Cover Problem

The Vertex Cover Problem

Recall:
vertex cover in a graph $G = (V, E)$:
a subset $C \subseteq V$ such that for all $e \in E$, $e \cap C \neq \emptyset$



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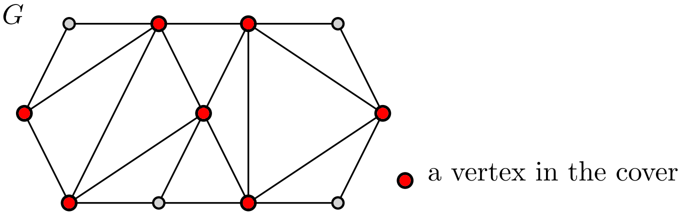

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Input: Graph $G = (V, E)$.

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For general graphs, the problem is NP-hard.

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return C.

The algorithm computes an inclusion-wise maximal matching M and returns the union of all edges in the matching.

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Claim

Approx-Cover *is a 2-approximation algorithm for the* MINIMUM VERTEX COVER *problem.*

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and consequently

$$|C| = 2|M| \leq 2 \cdot \mathrm{OPT}\,.$$

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Inapproximability of vertex cover:

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Inapproximability of vertex cover:

- If there exists a polynomial 1.36-approximation algorithm for MINIMUM VERTEX COVER, then P = NP (Dinur-Safra 2005).
- No ρ-approximation algorithm for MINIMUM VERTEX COVER is known with $\rho < 2$.

Approximation Algorithms for the Traveling Salesman Problem

The Traveling Salesman Problem

TRAVELING SALESMAN (TSP)

Input:	Graph $G = (V, E)$, a cost function $c : E \to \mathbb{R}+$.
Task:	Find a Hamiltonian cycle in G of smallest total cost.

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We will show how to decide in polynomial time, using *A*, whether a given graph *G* is Hamiltonian (which is an NP-complete problem).

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Let $G' = K_V$ (complete graph), $c : \binom{V}{2} \to \mathbb{R}_+$, where

$$c(e) = \begin{cases} 1, & \text{if } e \in E; \\ \rho|V| + 1, & \text{otherwise.} \end{cases}$$

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(a) If $c(\Gamma) \leq \rho|V|$, then $\Gamma \subseteq E$, hence G is Hamiltonian.

(b) If $c(\Gamma) > \rho|V|$, then $\mathrm{OPT} > |V|$, hence G is not Hamiltonian. □

A Heuristic for TSP

Approx-TSP(G, c)

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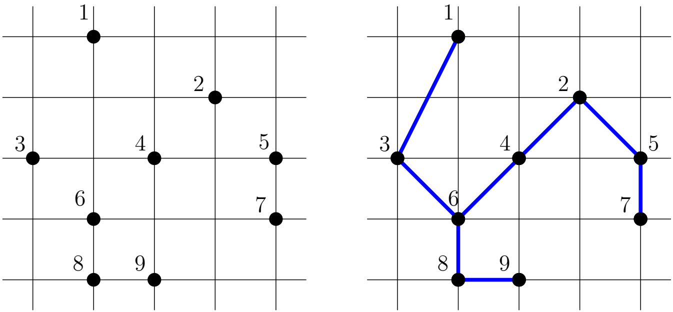

input instance

minimum spanning tree T

(The cost of an egde connecting two vertices is equal to the **Euclidean distance** between them.)

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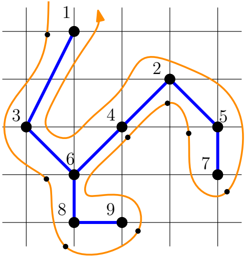
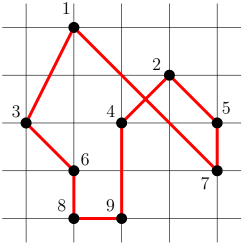


walk W
(obtained with DFS)



traveling salesman tour determined by W

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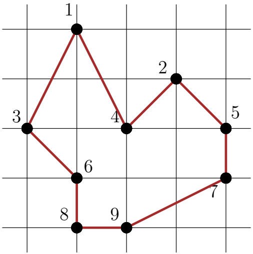
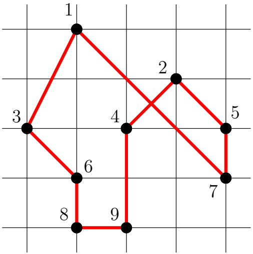


optimal solution

cost $\approx 13,95$



traveling salesman tour determined by W

cost $\approx 15,72$

The Metric TSP

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For all $u, v, w \in V$:

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A reduction from the HAMILTONIAN CYCLE problem shows:

Proposition

METRIC TSP *is NP-hard.*

The Metric TSP: a 2-approximation

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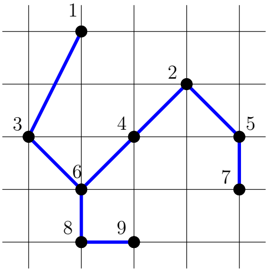
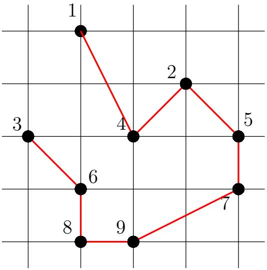


minimum spanning tree T



optimal solution H^* minus one edge

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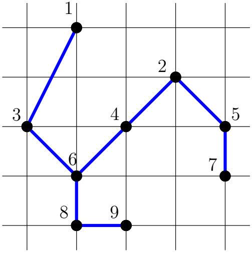

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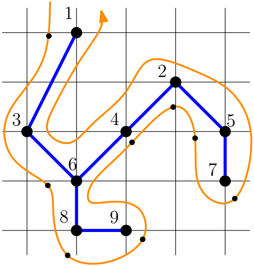

walk W

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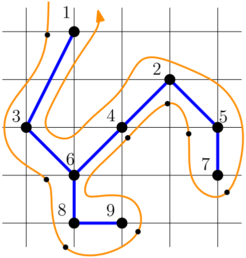
- $c(H) \leq \cdot c(W)$, due to the triangle inequality.



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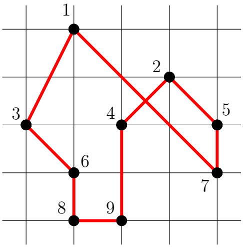

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The Metric TSP: Christofides' Algorithm

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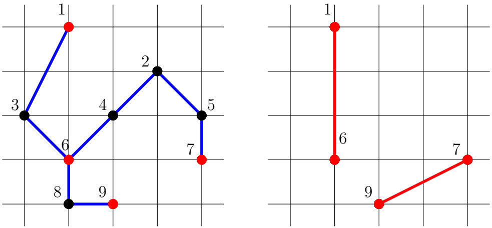
- Find a minimum cost perfect matching M connecting vertices of odd degree in T.



minimum spanning tree T
vertices of odd degree are red

matching M

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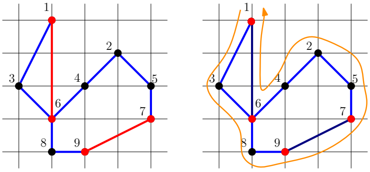
- Find a minimum cost perfect matching M connecting vertices of odd degree in T.
- $G' \leftarrow T \cup M$
- $E \leftarrow$ Euler tour in G' (graph G' is Eulerian, since it is connected and has all vertices of even degree).



G' $\quad$ E = Euler tour in G'

The Metric TSP: Christofides' Algorithm

Proof:

Let H^* be an optimal tour. We need to show: $c(H) \leq 1.5 \cdot c(H^*)$.

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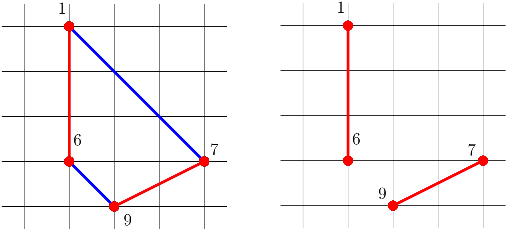
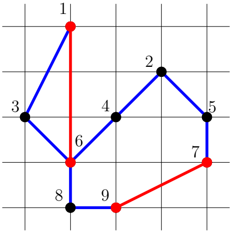
  - The first inequality follows since Γ^* is the union of two perfect matchings.
  - The second inequality follows from the triangle inequality.



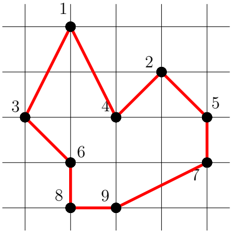
cycle Γ^* matching M

Proof:

- $c(T) \leq c(H^*)$, as before
- $c(M) \leq (1/2) \cdot c(\Gamma^*) \leq (1/2) \cdot c(H^*)$, where Γ^* is an optimal cycle on the odd vertices of T.
- Due to the triangle inquality:
  $c(H) \leq c(M) + c(T) \leq (3/2) \cdot c(H^*)$.



G'



H = traveling salesman tour

What we did – Week 1

1. Tue March 5: Review of basic notions in graph theory, algorithms and complexity ✓
2. Wed March 6: Graph colorings ✓
3. Thu March 7: Perfect graphs and their subclasses, part 1 ✓
4. Fri March 8: Perfect graphs and their subclasses, part 2 ✓

What we'll do – Week 2

1. Tue March 19: Further examples of tractable problems, part 1 ✓
2. Wed March 20:
   Further examples of tractable problems, part 2 ✓
   Approximation algorithms for graph problems ✓
3. Thu March 21: Lectio Magistralis lecture, *"Graph classes: interrelations, structure, and algorithmic issues"*

Questions?

Thank you for your attention!

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