$[\log n = \log_2 n]$

⑥ (a) $5, \log\log n, \log n, \sqrt{n}, 2^{\log_2 n}, n \cdot \log n, n^2, n^3+n+15,$
$n^{42}, 2^n, n!$

Note : $2^{\log_2 n} = n$, since : $\log_a x = y \Leftrightarrow a^y = x$

(b) • $5n \in O(n)$, $n \in O(5n)$ (also written like this : $5n \in \Theta(n)$)
$5n$ and n are in the same class.

- $4^n \in O(n!)$ but $n! \notin O(4^n)$
  4^n grows slower than $n!$
- $\ln n = \log_e n$, that's the same class as $\log_2 n$
- n^3 and n^3+n+15 : same class
- $37n$ and n and $5n$: all same class
- 37 and 5 : same class
- $2^{n+1} = \underset{\substack{\uparrow \\ \text{constant}}}{2} \cdot 2^n \in O(2^n)$
- $2^{2n} = 4^n$

So altogether we have, writing $f(n) \simeq g(n)$ if
$f(n) \in O(g(n))$ and $g(n) \in O(f(n))$ (i.e. if $f(n) \in \Theta(g(n))$):

$$\underset{\substack{\wr\wr \\ 37}}{5}, \log\log n, \underset{\substack{\wr\wr \\ \ln n}}{\log n}, \sqrt{n}, \overset{2^{\log n}}{\overset{\shortparallel}{\underset{\substack{\wr\wr \quad \wr\wr \\ 5n \quad 37n}}{n}}}, n \cdot \log n, n^2, \underset{\substack{\wr\wr \\ n^3}}{n^3+n+15}, n^{42},$$

$$\underset{\substack{\wr\wr \\ 2^{n+1}}}{2^n}, 4^n, n!, n^n$$

<u>NB</u>: (b) is a bit tricky but (a) you have to know!