TRANSFORMING ABSTRACT INTERPRETATIONS BY ABSTRACT INTERPRETATIONS

MODELLING SYSTEMS AS AI TRANSFORMERS

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SAS'08, Valencia July 2008

Abstract Interpretation

[Cousot & Cousot '79]

- A program P
- A domain of computation for P: C typically a complete lattice
- Semantic specification (interpreter): $[\![P]\!] : C \longrightarrow C$
- (Approximate) observable properties: $\rho \in uco(C)$
- DERIVE A SOUND APPROXIMATE SPECIFICATION $[\![P]\!]^\sharp$

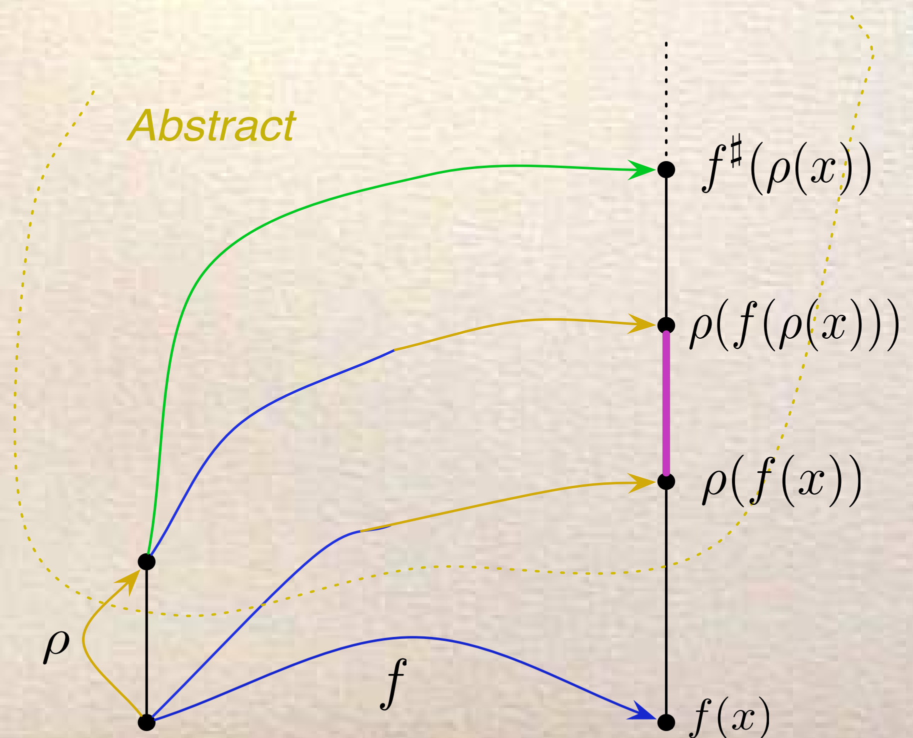
$$\rho([\![P]\!](x)) \leq [\![P]\!]^\sharp(x)$$

- THE LIMIT CASE: COMPLETENESS

$$\rho([\![P]\!](x)) = [\![P]\!]^\sharp(x) \text{ iff } \rho([\![P]\!](x)) = \rho([\![P]\!](\rho(x)))$$

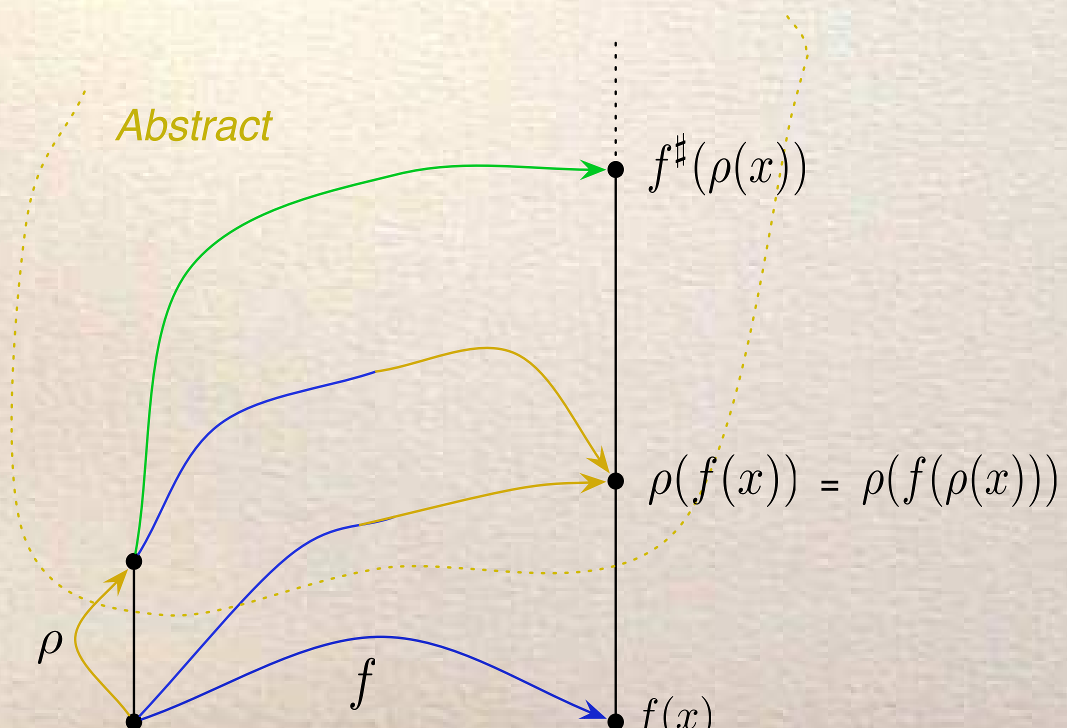
Completeness in Abstract Interpretation

- Backward Soundness: no information is lost by approximating the input/output
- $\rho \circ f \leq \rho \circ f \circ \rho$



COMPLETENESS IN ABSTRACT INTERPRETATION

- BACKWARD COMPLETENESS: NO LOSS OF PRECISION IS ACCUMULATED BY APPROXIMATING THE INPUT
- $\rho \circ f = \rho \circ f \circ \rho$



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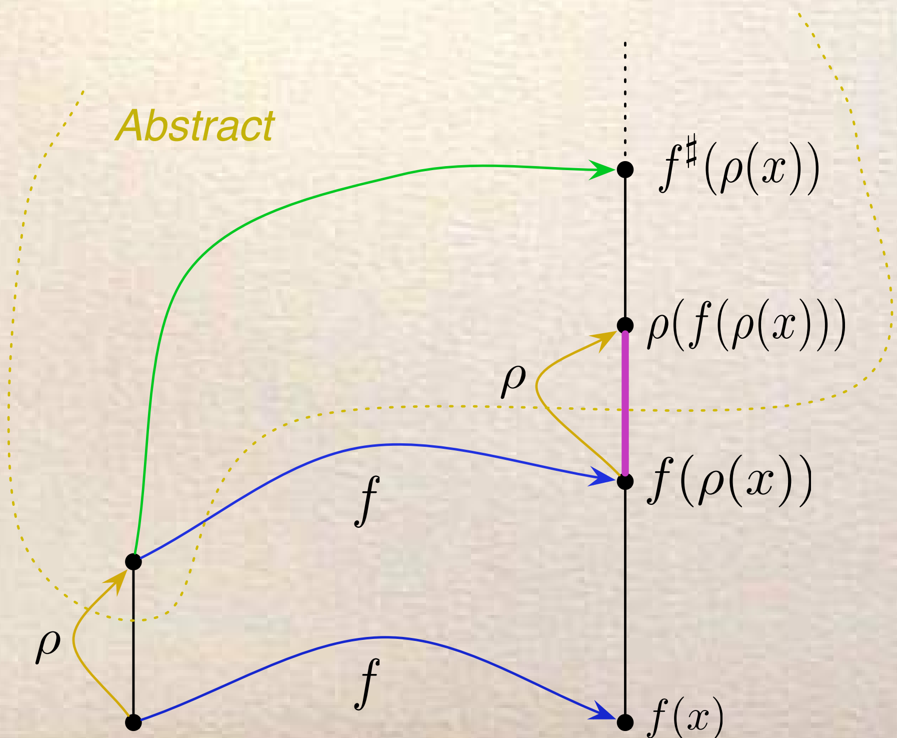

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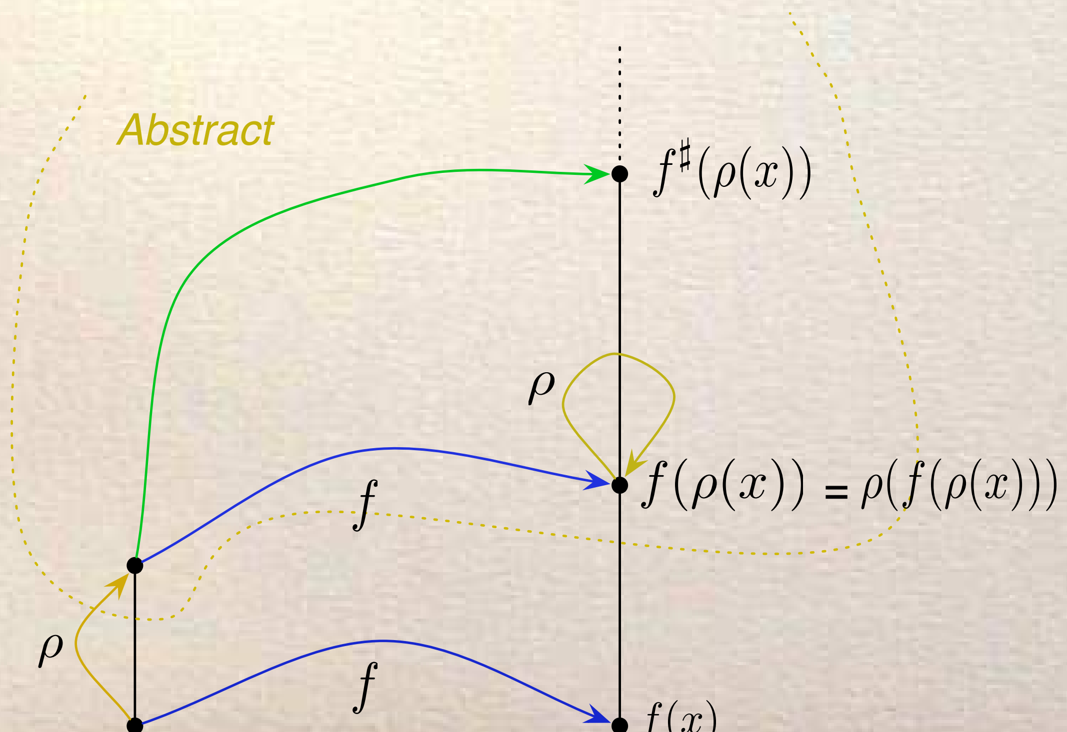

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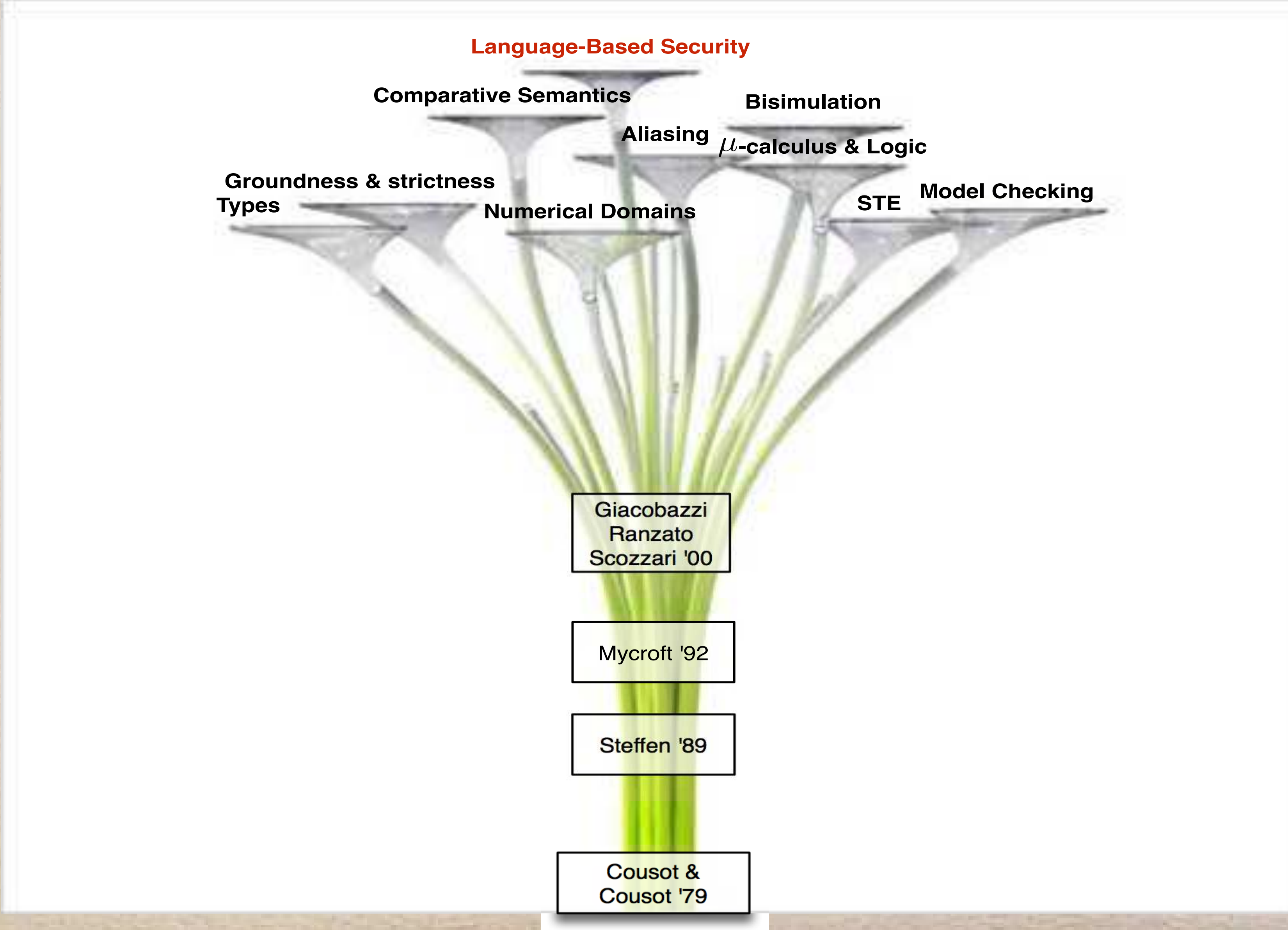
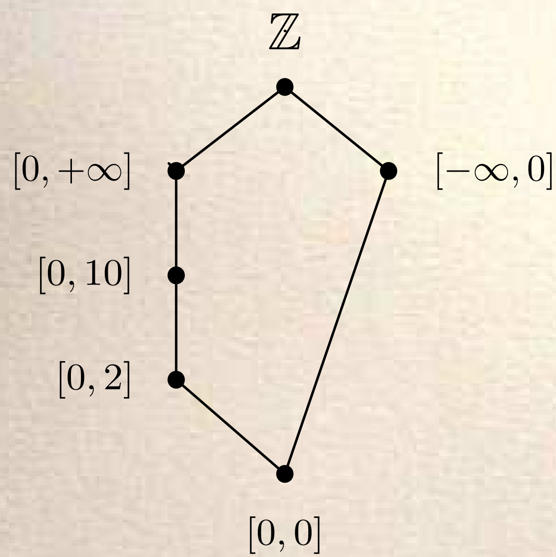


IN NUMERICAL DOMAINS

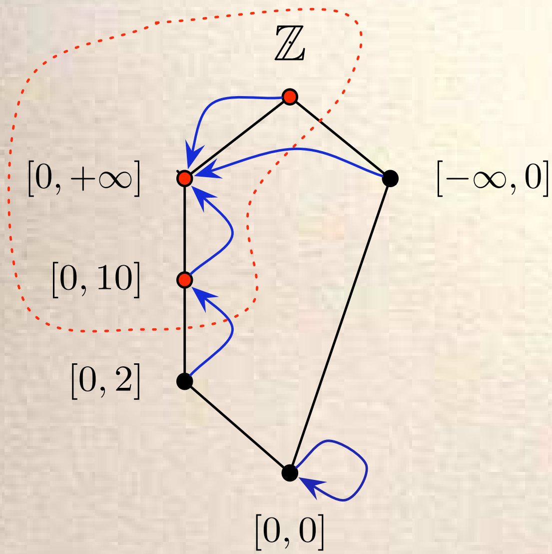
A SIMPLE EXAMPLE IN INTERVAL ANALYSIS



- A simple domain of intervals

In Numerical Domains

A simple example in Interval analysis



- A simple domain of intervals
- $sq(X) = \left\{ x^2 \;\middle|\; x \in X \right\}$
- $\{\mathbb{Z}, [0,+\infty], [0,10]\}$ is Forward but not Backward complete

IN NUMERICAL DOMAINS

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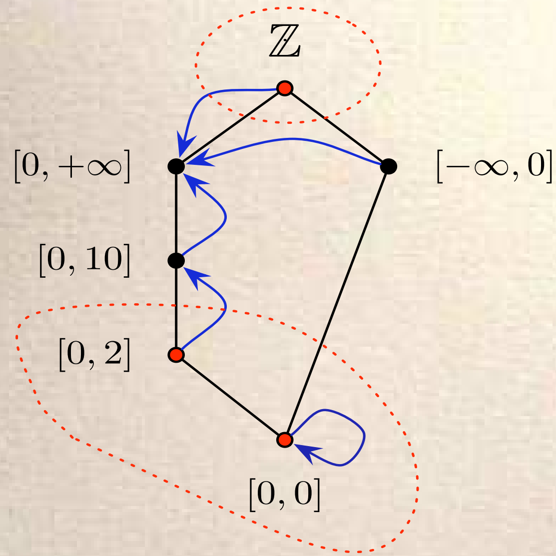

- A simple domain of intervals
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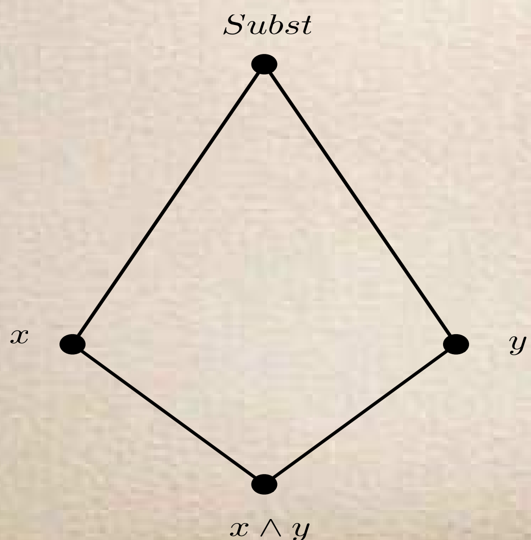
- $\{\mathbb{Z}, [0, +\infty], [0, 10]\}$ is Forward but not Backward complete
- $\{\mathbb{Z}, [0, 2], [0, 0]\}$ is Backward but not Forward complete

IN GROUNDNESS: HEYTING COMPLETION

GROUNDNESS ANALYSIS DETERMINES WHETHER A VARIABLE IS DEFINITIVELY INSTANTIATED

- $(\wp(Subst)^{\downarrow}, \cap)$ is a complete Heyting Algebra
- $\Theta_1 \cap \Theta_2 \leq \Theta_3 \iff \Theta_2 \leq \Theta_1 \xrightarrow{\cap} \Theta_3 = \bigcup \left\{ \Theta \;\middle|\; \Theta_1 \cap \Theta \leq \Theta_3 \right\}$
- $A \xrightarrow{\cap} B = \left\{ \Theta_1 \xrightarrow{\cap} \Theta_3 \;\middle|\; \Theta_1 \in A, \Theta_2 \in B \right\}$



$$X = \mathcal{G} \sqcap (X \xrightarrow{\cap} X)$$

In Groundness: Heyting completion

Groundness analysis determines whether a variable is definitively instantiated

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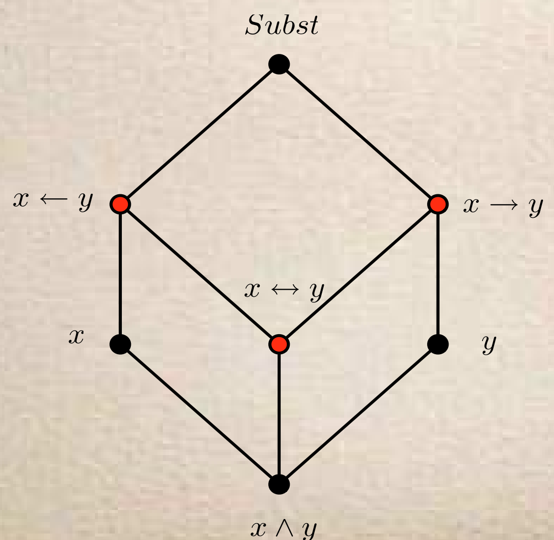

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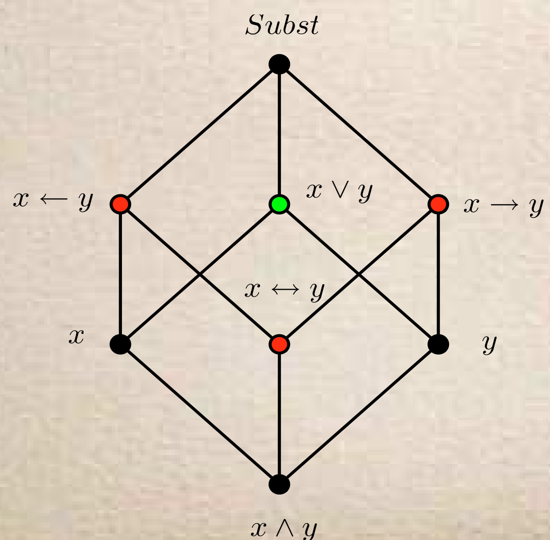


$$X = \mathcal{G} \sqcap (X \xrightarrow{\cap} X)$$

$\boxed{\Rightarrow \text{A COMPLETENESS PROBLEM}}$

[Giacobazzi & Scozzari '98]

In Comparative Semantics

Condensing generalises the lifting lemma from SLD-resolution to arbitrary semantics [Giacobazzi et al. '05]

$$[\![a \otimes b]\!] = a \otimes [\![b]\!]$$

$$\boxed{\begin{array}{llll} \textit{Program} & P & ::= & \varnothing \mid p(\overline{x}) \leftarrow A \mid P.P \\ \textit{Agent} & A & ::= & \theta \mid p(\overline{x}) \mid A \otimes A \mid \bigvee_{i=1}^{n} A_i \end{array}}$$

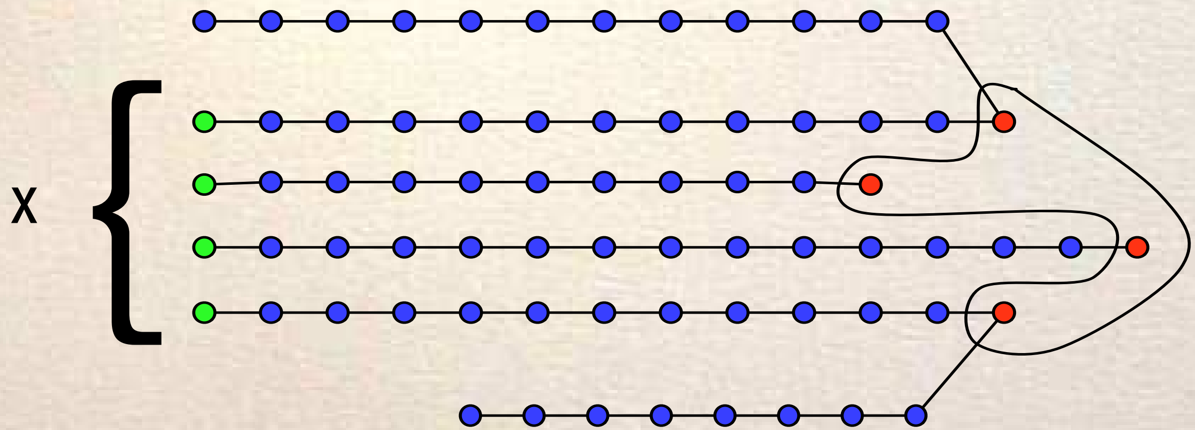
- $\otimes$ is a tensor operator (e.g. unification)
- $$a \otimes b \leq c \iff b \leq a \multimap c = \bigvee \left\{ b \in C \,\middle|\, a \otimes b \leq c \right\}$$
- $$A \xrightarrow{\otimes} B = \left\{ a \multimap b \in C \,\middle|\, a \in A,\ b \in B \right\}$$
- X is condensing iff $X = X \sqcap (X \xrightarrow{\otimes} X)$ iff X is complete for $F_X = \left\{ \lambda y.x \otimes y \,\middle|\, x \in X \right\}$.

$\boxed{\Rightarrow \text{A completeness problem}}$

In Comparative Semantics

$$[\![P_1; P_2]\!]^A = [\![P_1]\!]^A \diamond [\![P_2]\!]^A$$

- *Forward termination*: $Pot^{\rightarrow ?}(X) = \left\{ \sigma \;\middle|\; \delta \in X^+ \wedge \sigma_\dashv = \delta_\dashv \right\}$



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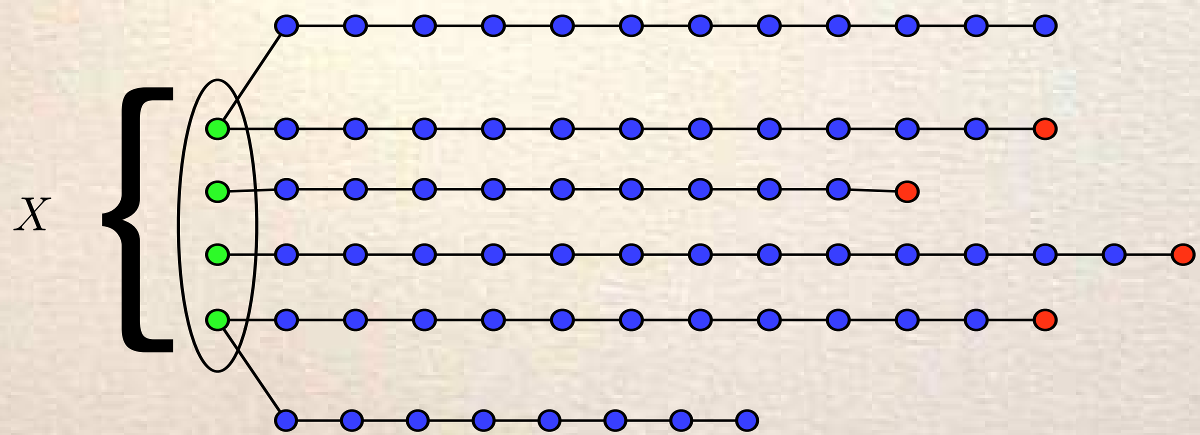

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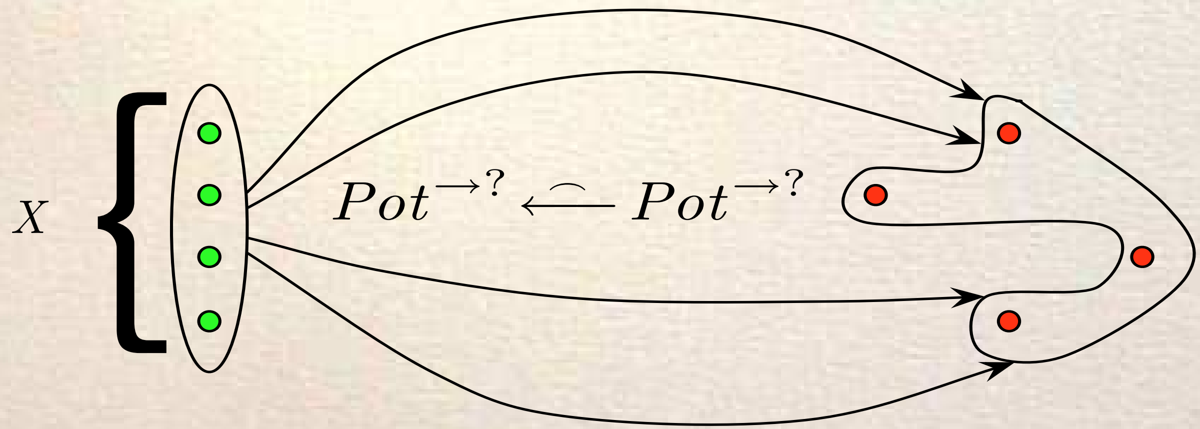
$$[\![P_1;P_2]\!]^A = [\![P_1]\!]^A \diamond [\![P_2]\!]^A$$

- $X = Pot^{\rightarrow ?} \sqcap (X \stackrel{\frown}{\longrightarrow} X)$ and the solution: $Pot^{\rightarrow ?} \stackrel{\frown}{\longleftarrow} Pot^{\rightarrow ?} = [\![\cdot]\!]$.



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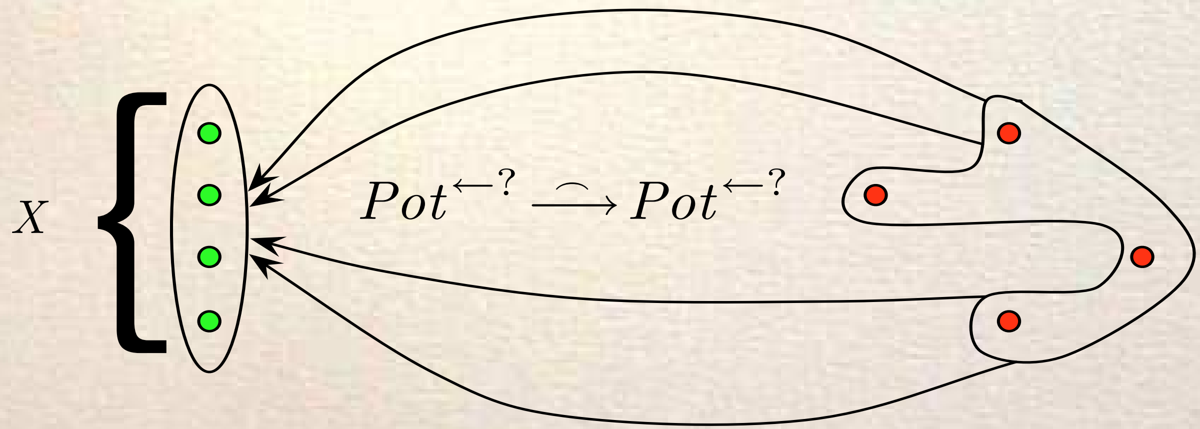
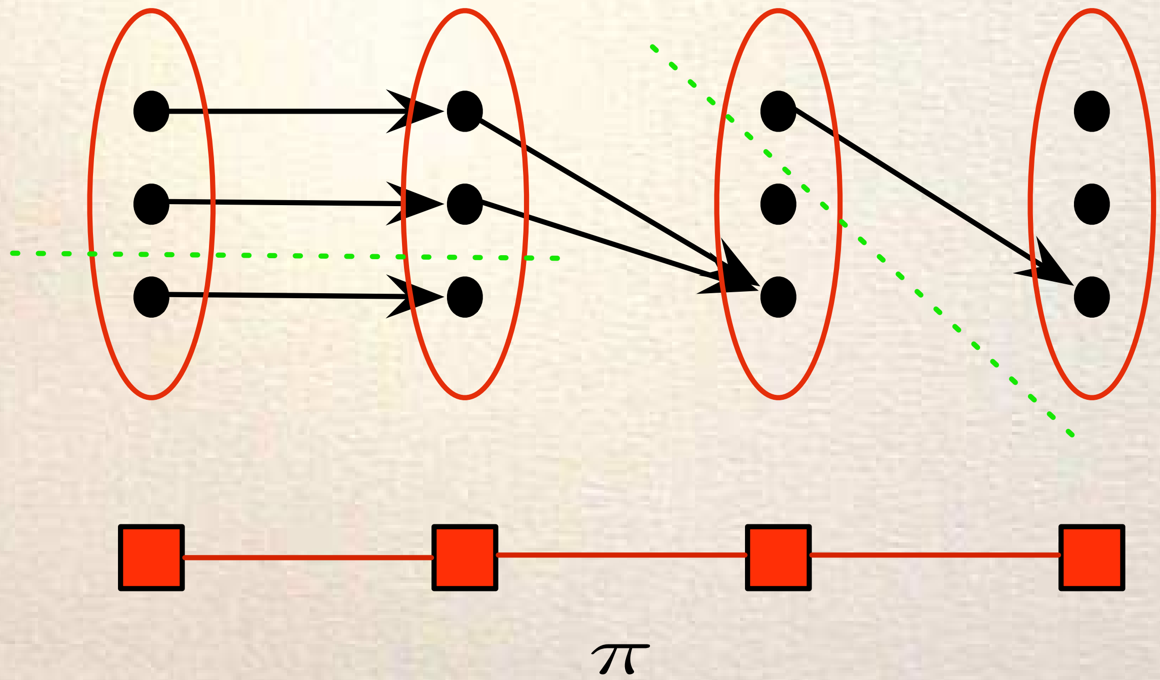


$\boxed{\Rightarrow \text{A COMPLETENESS PROBLEM}}$

[Giacobazzi & Mastroeni '05]

In Model Checking

Complete Abstract Model Checking: $M^A \models \Phi \iff M^C \models \Phi$



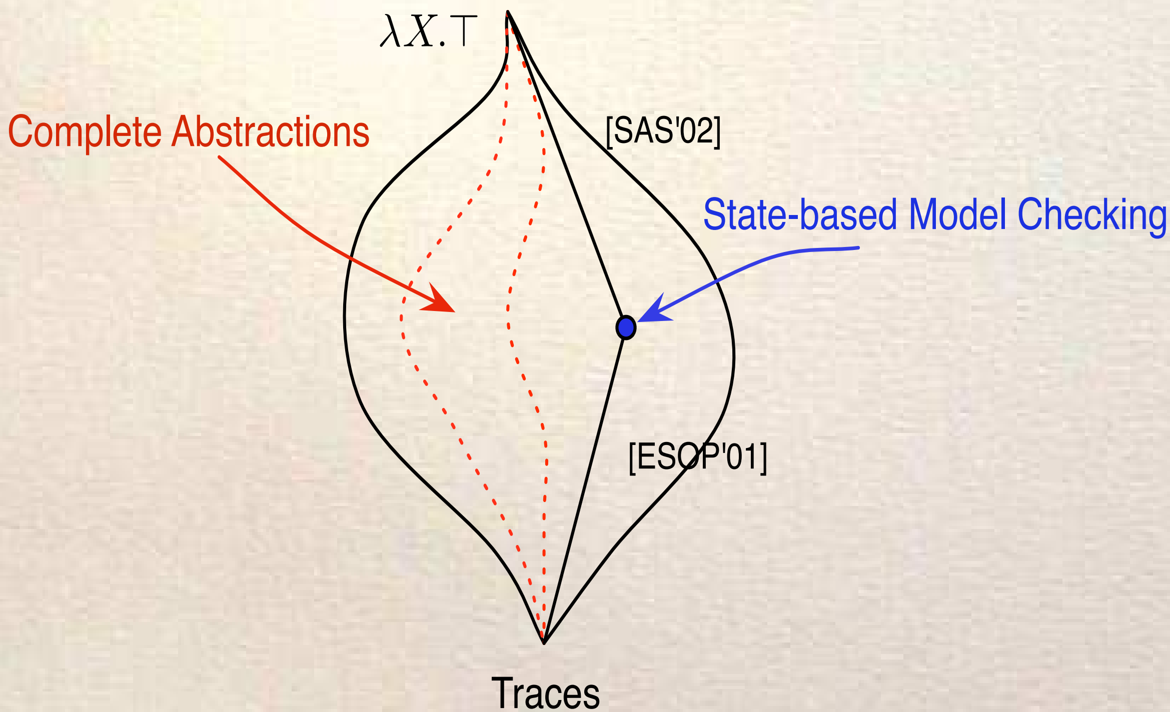
If π is spurious then the abstraction is incomplete for *post*

[Giacobazzi & Quintarelli '01, Ranzato & Tapparo '06, Cousot et al '07, Schmidt '08]

IN MODEL CHECKING

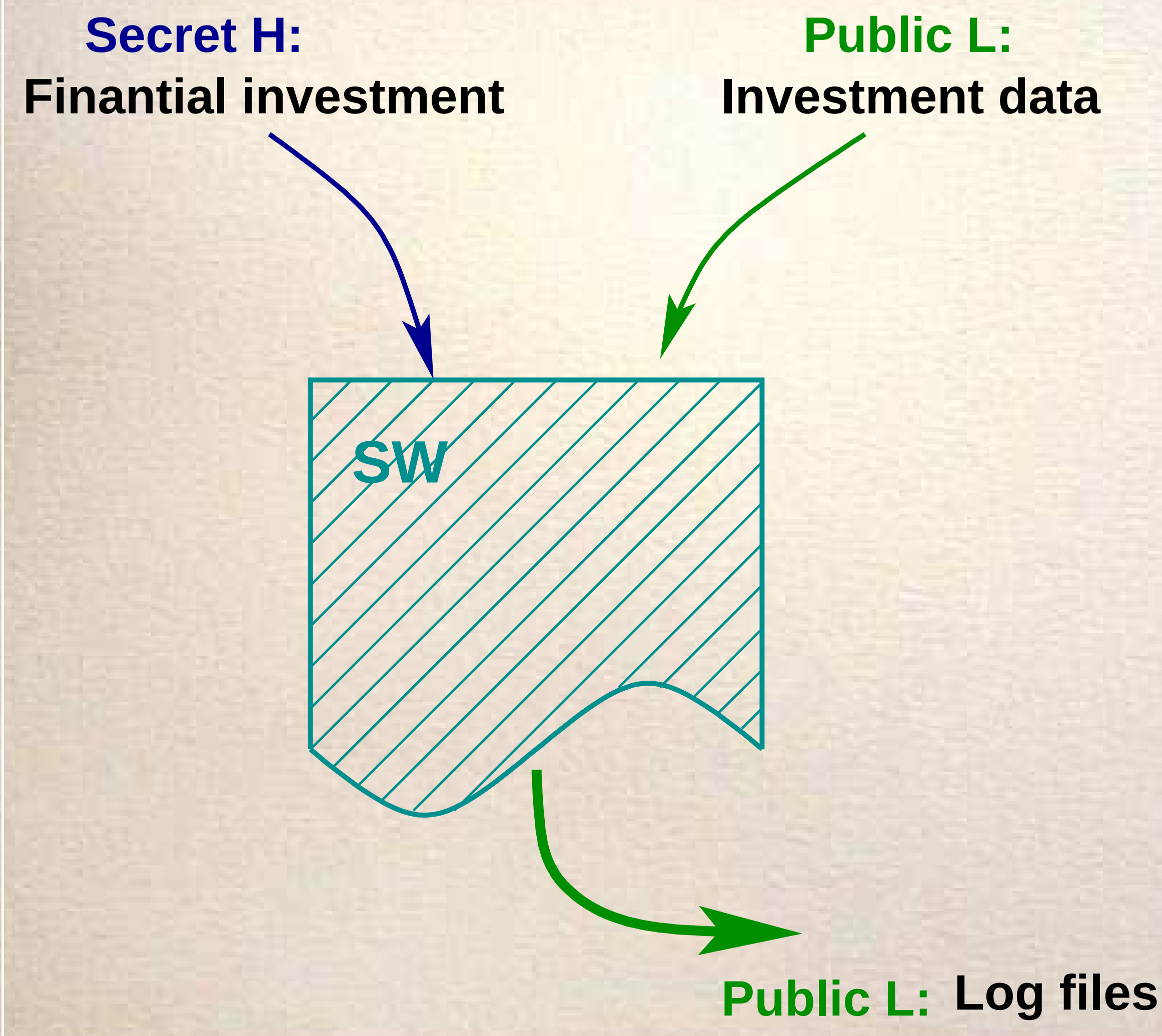
[Cousot & Cousot '00]

Let $\Phi \in \mu$-calculus: $[\![\Phi]\!]^{State} \subset \alpha^{State}([\![\Phi]\!]^{Trace})$



STATE-BASED MODEL CHECKING IS INTRINSICALLY INCOMPLETE FOR PROPERTIES OF TRACES!!

In Security: Non-Interference



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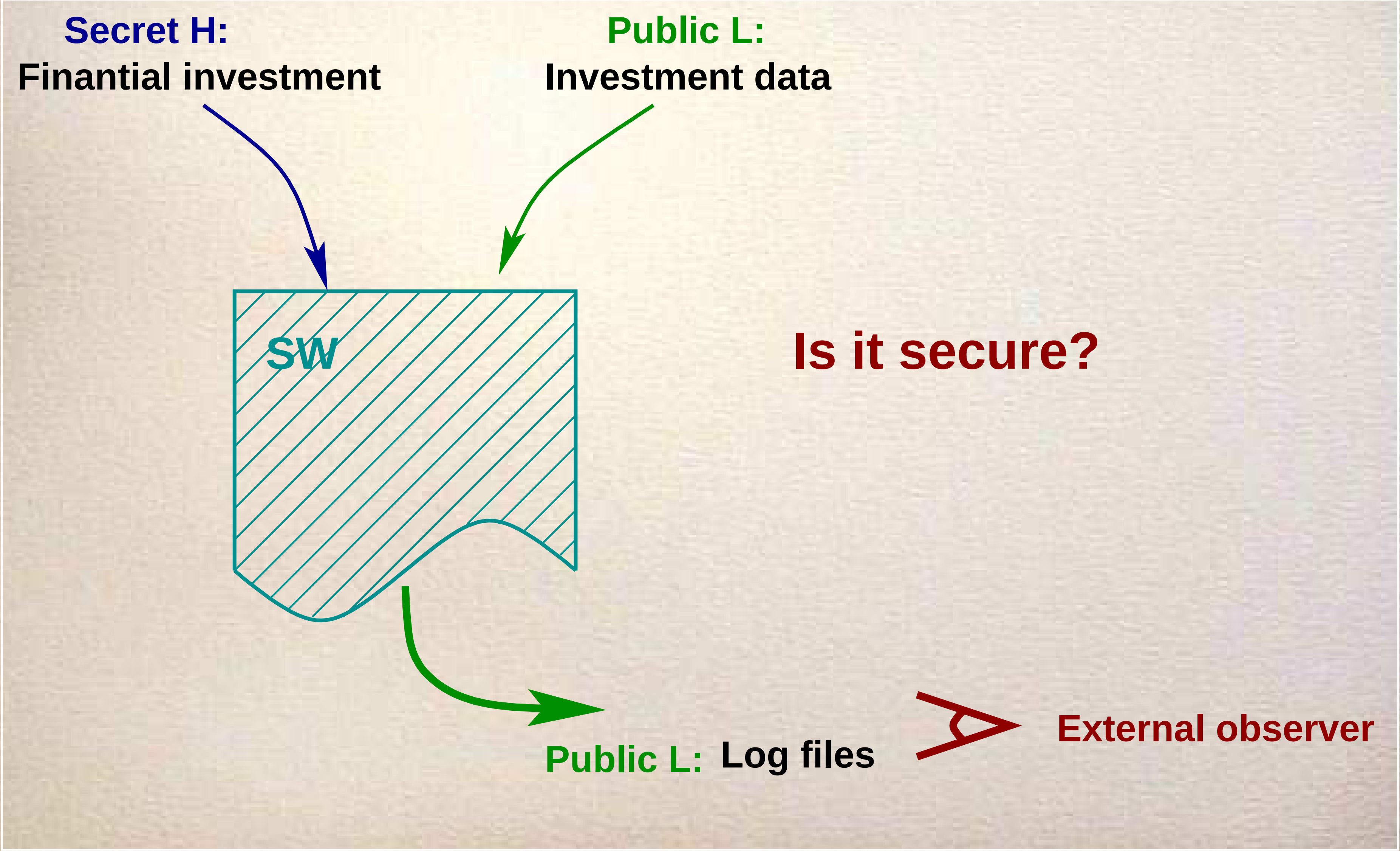
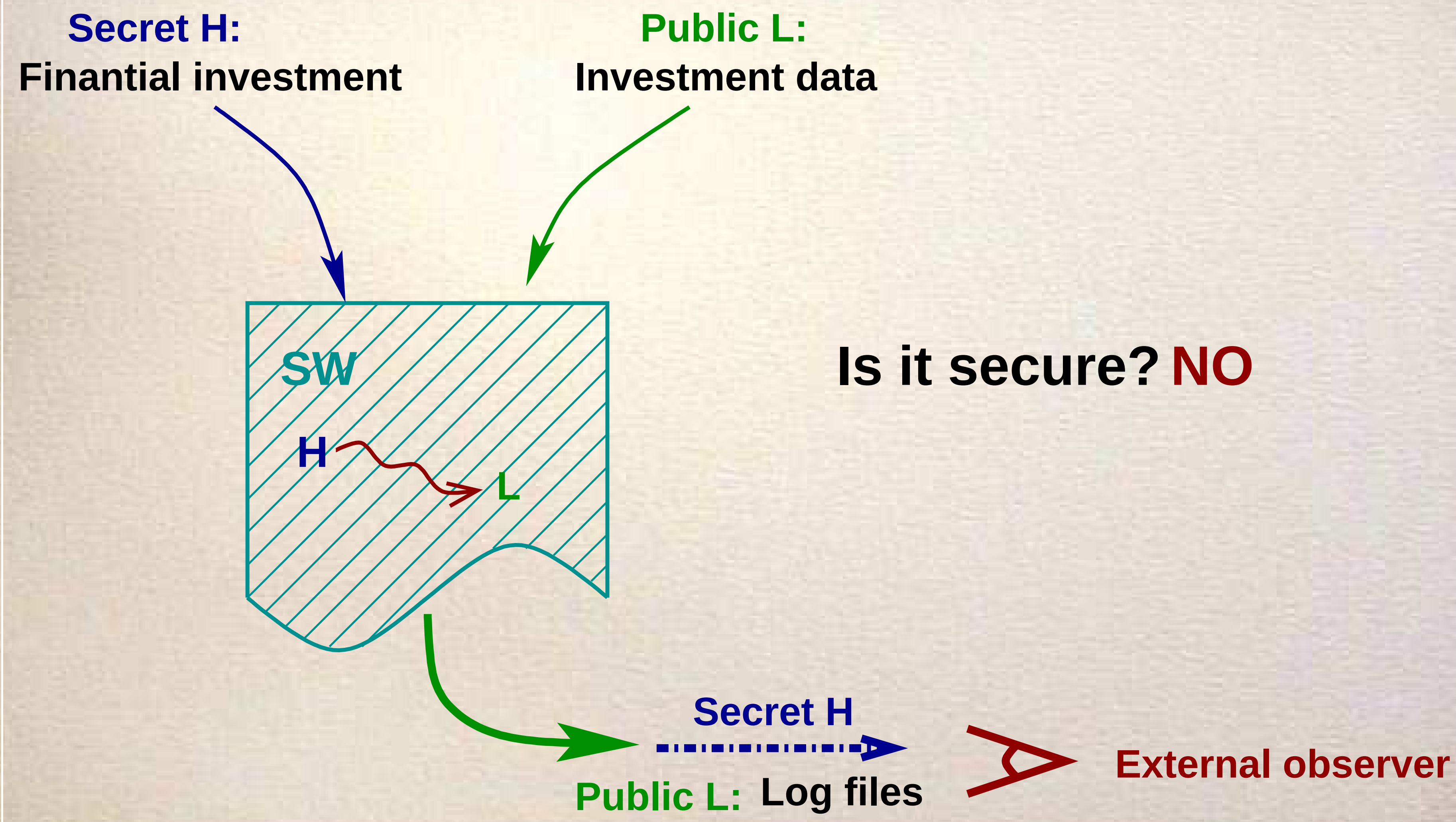
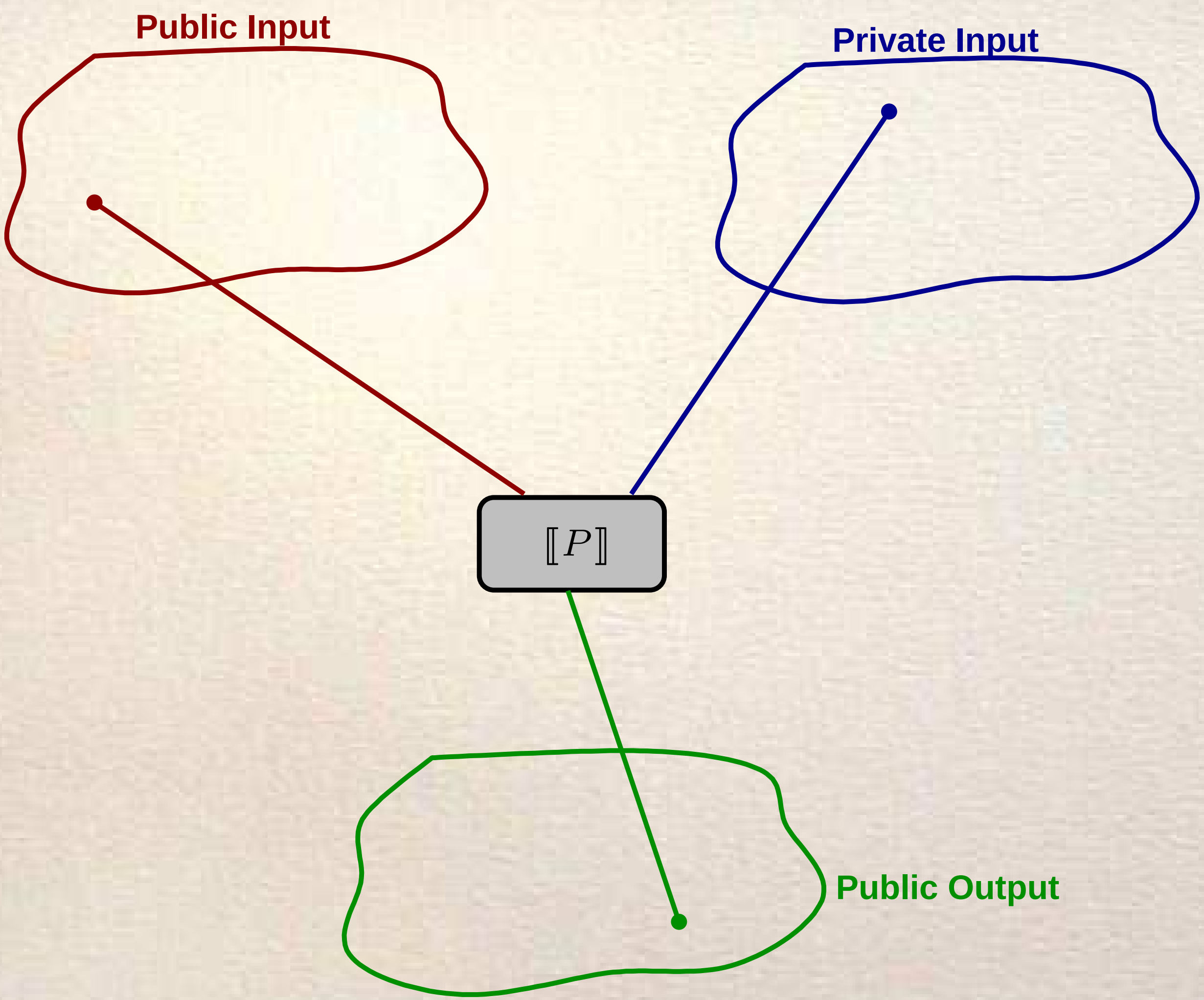


IN SECURITY: NON-INTERFERENCE



Is it secure? NO

STANDARD NON-INTERFERENCE



$$\forall l : \mathsf{L}, \forall h_1, h_2 : \mathsf{H}.\ [\![P]\!](h_1, l)^{\mathsf{L}} = [\![P]\!](h_2, l)^{\mathsf{L}}$$

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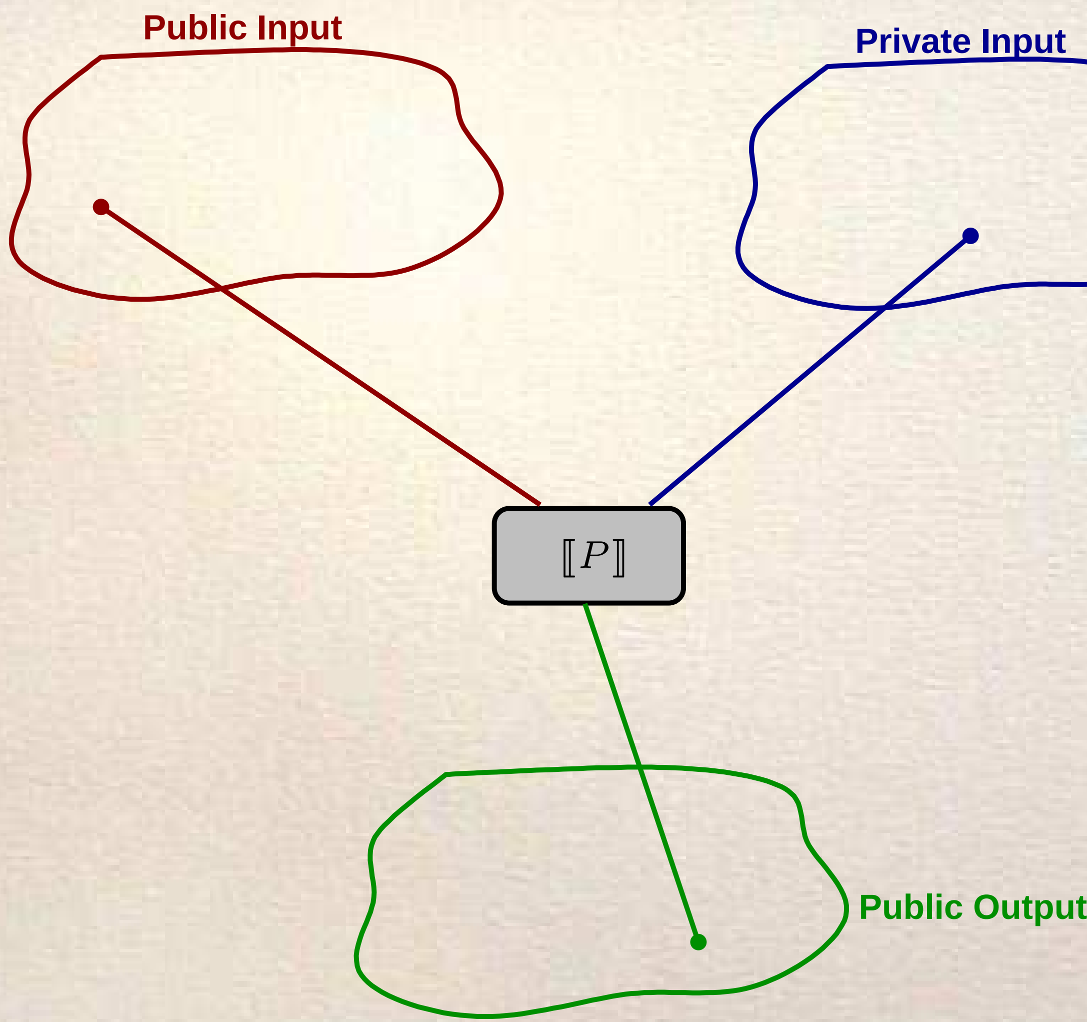

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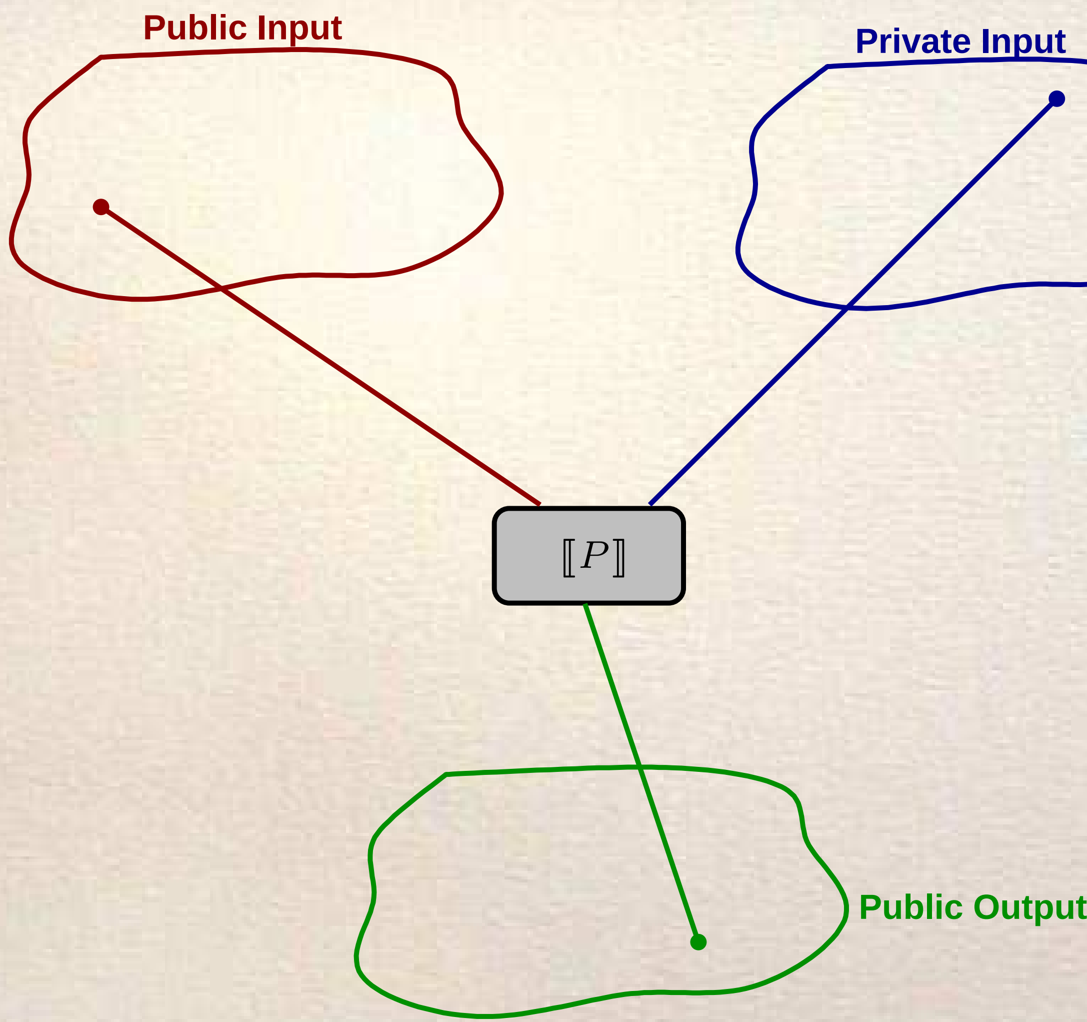

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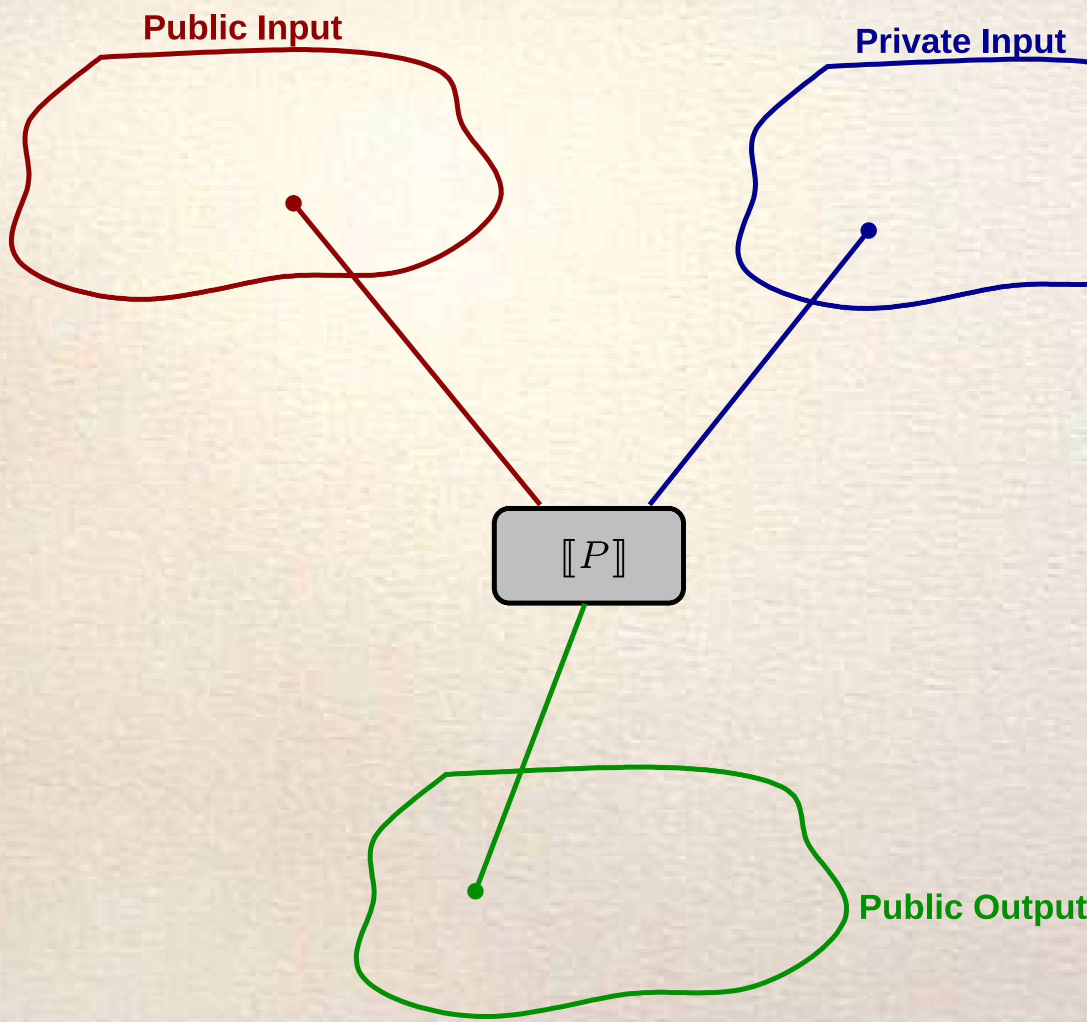

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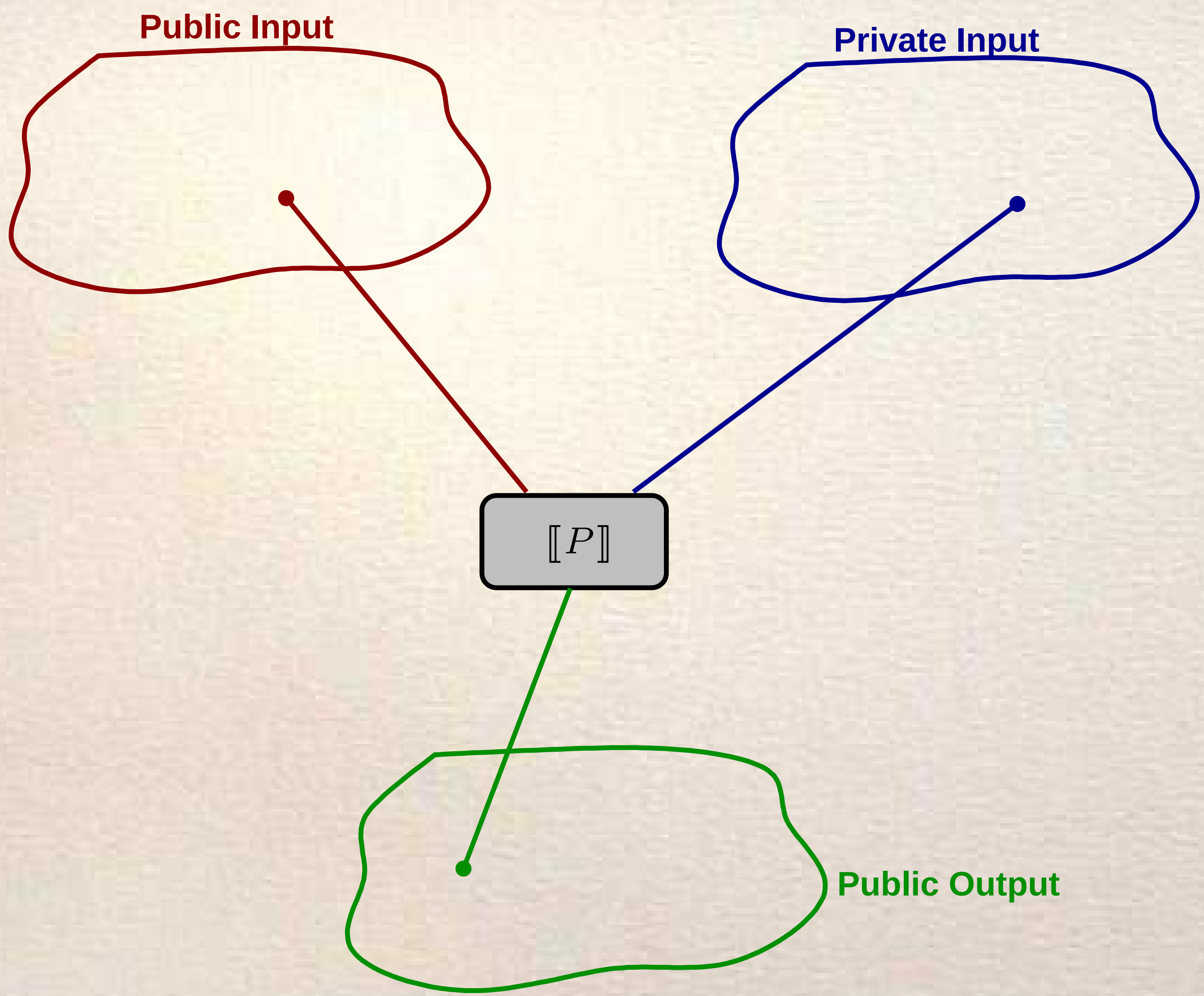

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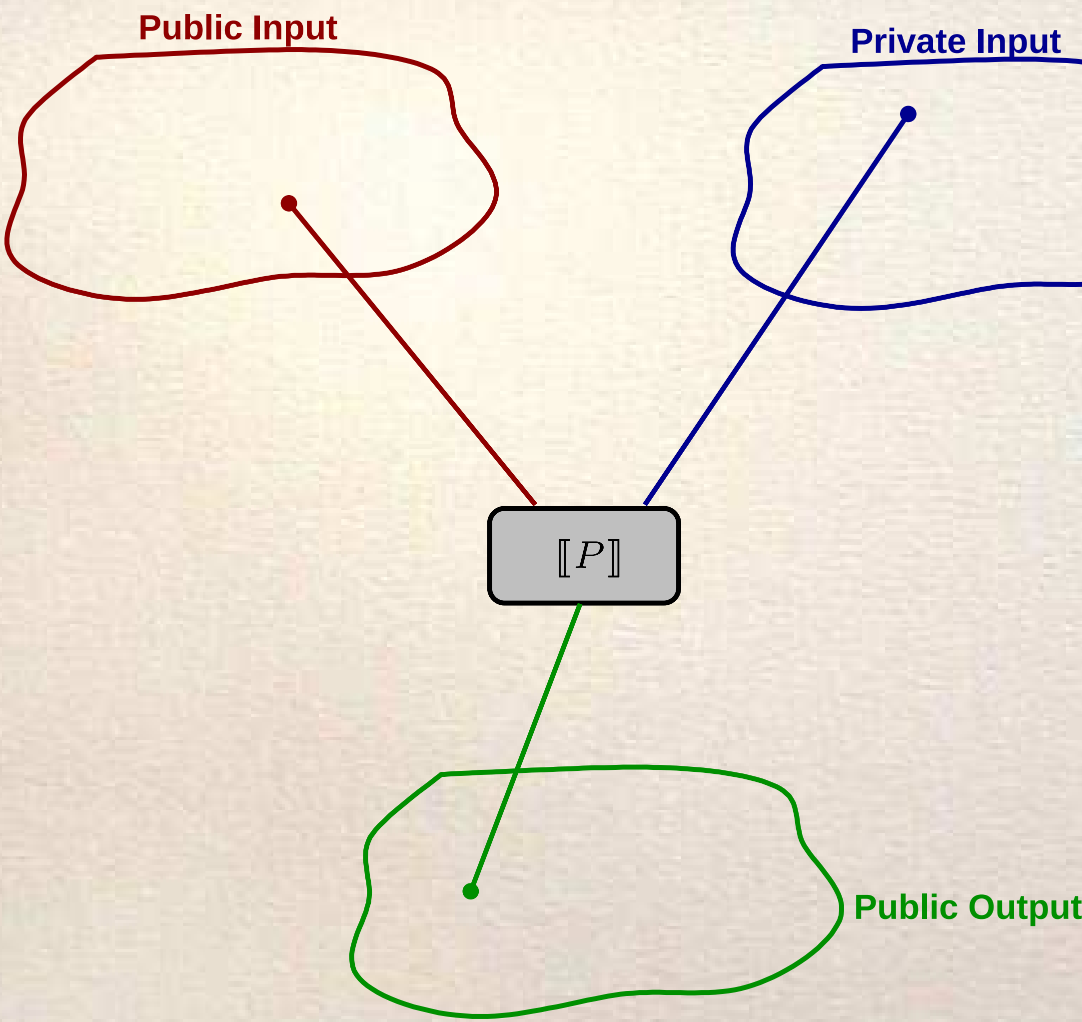

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NI: A COMPLETENESS PROBLEM

Recall that [Joshi & Leino'00]

$$P \text{ is } \textit{secure} \quad \text{iff} \quad \mathtt{HH}\,;\, P;\, \mathtt{HH} \doteq P\,;\, \mathtt{HH}$$

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Let $X = \langle X^{\mathsf{H}}, X^{\mathsf{L}}\rangle \Rightarrow \mathcal{H}(X) \stackrel{\text{def}}{=} \langle \top^{\mathsf{H}}, X^{\mathsf{L}}\rangle \in \mathit{uco}(\wp(\mathbb{V}))$

$$\begin{array}{ccc} \mathsf{HH}\,;\,P;\,\mathsf{HH} & \doteq & P\,;\,\mathsf{HH} \\ & \Downarrow & \\ \mathcal{H}\circ[\![P]\!]\circ\mathcal{H} & = & \mathcal{H}\circ[\![P]\!] \end{array}$$

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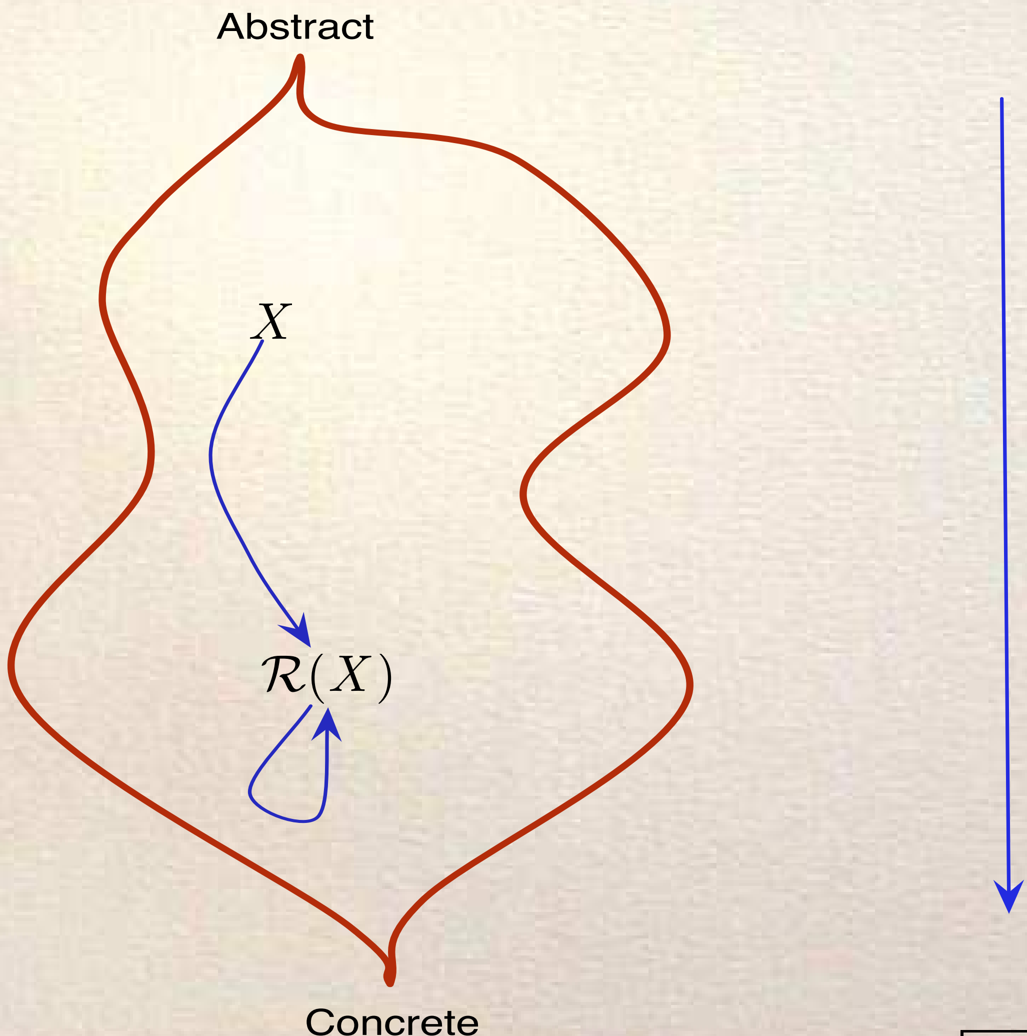
$\boxed{\Rightarrow \text{A COMPLETENESS PROBLEM}}$

MAKING
ABSTRACT INTERPRETATIONS COMPLETE

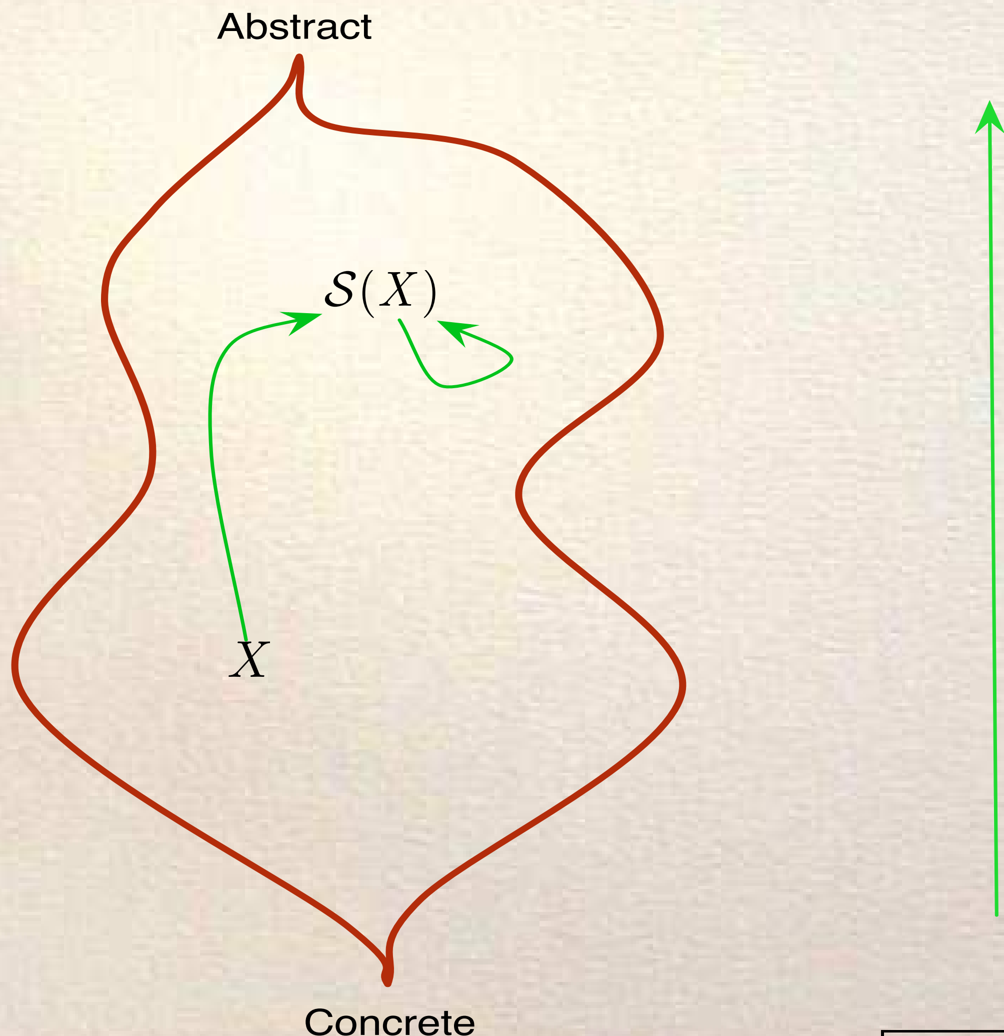
10 YEARS AFTER

THE GEOMETRY OF AI TRANSFORMERS



lco – REFINEMENT

THE GEOMETRY OF AI TRANSFORMERS



$\boxed{uco - \text{SIMPLIFICATION}}$

The Geometry of AI transformers

Can we use abstract interpretation for transforming abstract interpretations?

- Refinements: $X \subseteq \mathcal{R}(X)$ (improving precision – lower closure)
- Simplification: $\mathcal{S}(X) \subseteq X$ (reducing precision – upper closure)

[Janowitz '67]

$$(1)\ \eta \in uco(C) \Leftrightarrow \eta^+ \in lco(C) \Leftrightarrow \begin{cases} \eta \circ \eta^+ = \eta^+ \\ \eta^+ \circ \eta = \eta \end{cases}$$

$$(2)\ \eta \in uco(C) \Leftrightarrow \eta^- \in lco(C) \Leftrightarrow \begin{cases} \eta \circ \eta^- = \eta \\ \eta^- \circ \eta = \eta^- \end{cases}$$

THE GEOMETRY OF AI TRANSFORMERS

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[Janowitz '67]

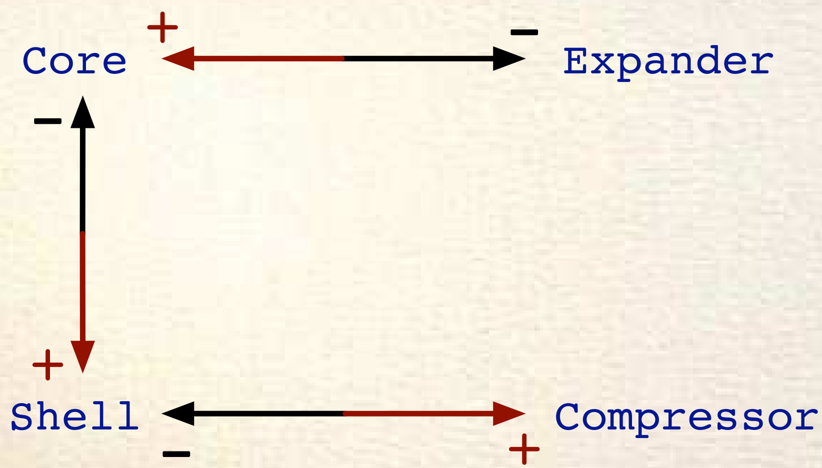
$$(1)\ \ \mathcal{S} \text{ simplification } \Leftrightarrow \mathcal{S}^{+} \text{refinement } \Leftrightarrow \begin{cases} \mathcal{S} \circ \mathcal{S}^{+} = \mathcal{S}^{+} \\ \mathcal{S}^{+} \circ \mathcal{S} = \mathcal{S} \end{cases}$$

Shell/Core of a given property

$$(2)\ \ \mathcal{S} \text{ simplification } \Leftrightarrow \mathcal{S}^{-} \text{refinement } \Leftrightarrow \begin{cases} \mathcal{S} \circ \mathcal{S}^{-} = \mathcal{S} \\ \mathcal{S}^{-} \circ \mathcal{S} = \mathcal{S}^{-} \end{cases}$$

Expander/Compressor for a given property

THE GEOMETRY OF DOMAIN TRANSFORMERS



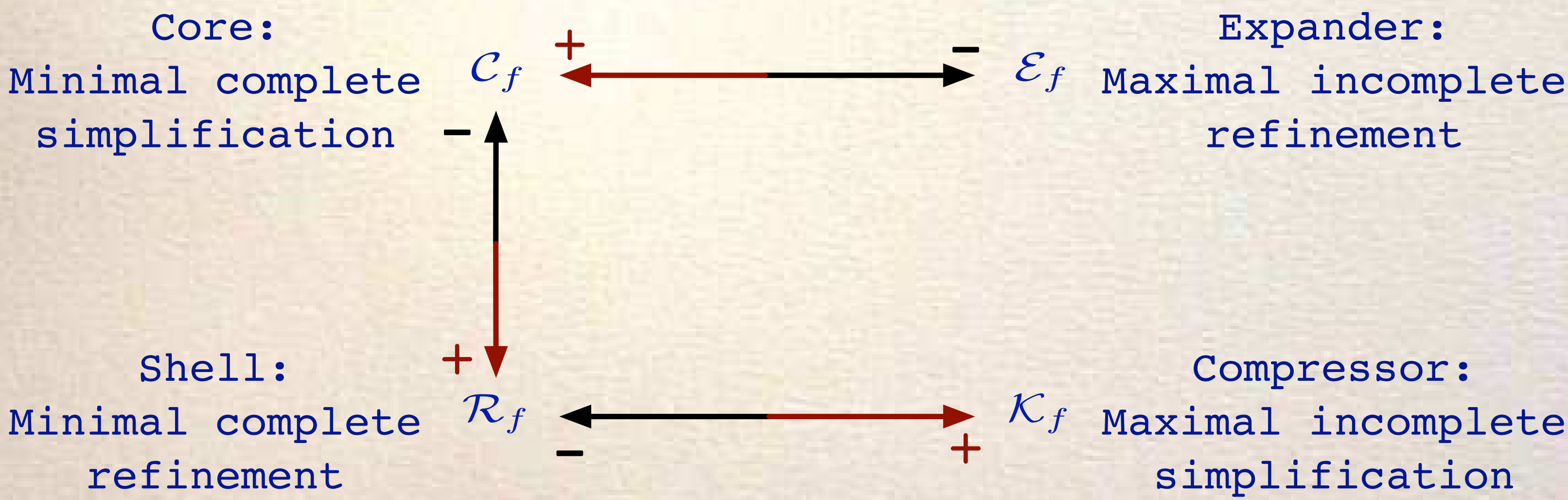
- Shell/Core minimally transform domains in order to achieve a given property

- Expander/Compressor maximally transform domains in order to achieve a given property

WHAT IS THE MEANING OF SHELL/CORE AND EXPANDER/COMPRESSOR FOR THE COMPLETENESS PROPERTY?

THE GEOMETRY OF DOMAIN TRANSFORMERS

Basic abstract domain transformers

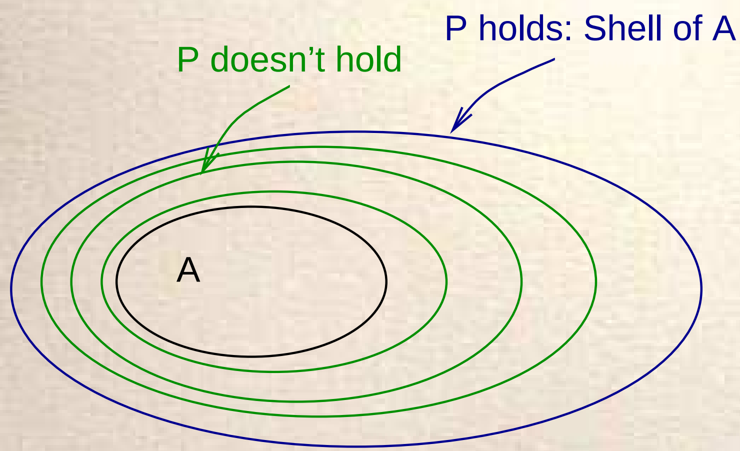


[Giacobazzi et al.'00]

[SAS'08]

SHELL/CORE

Let P be completeness



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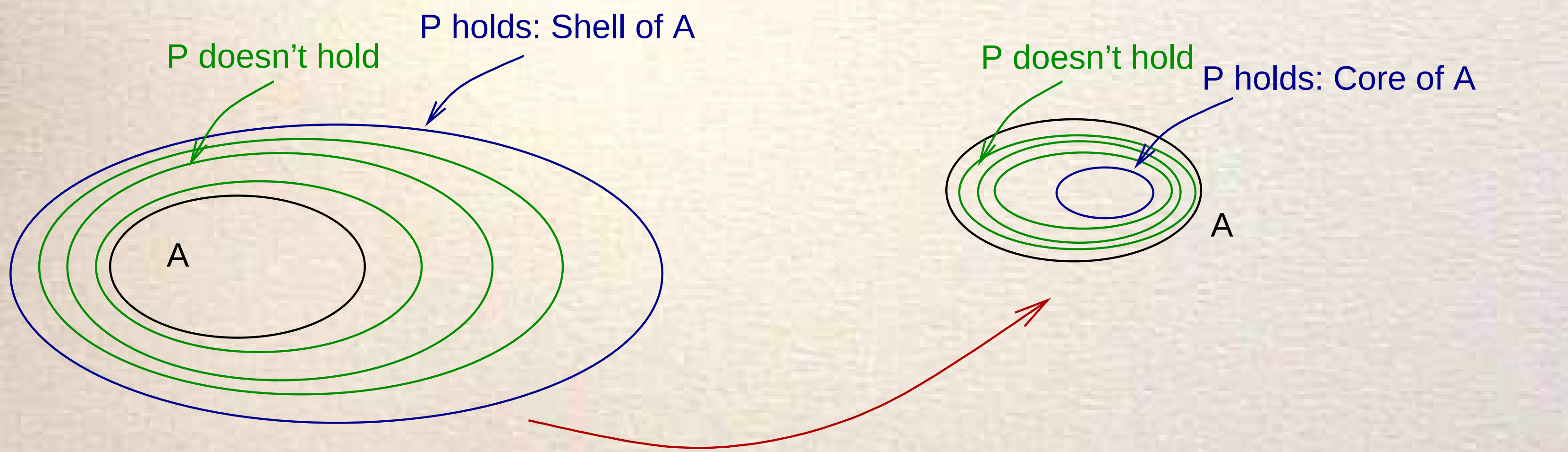
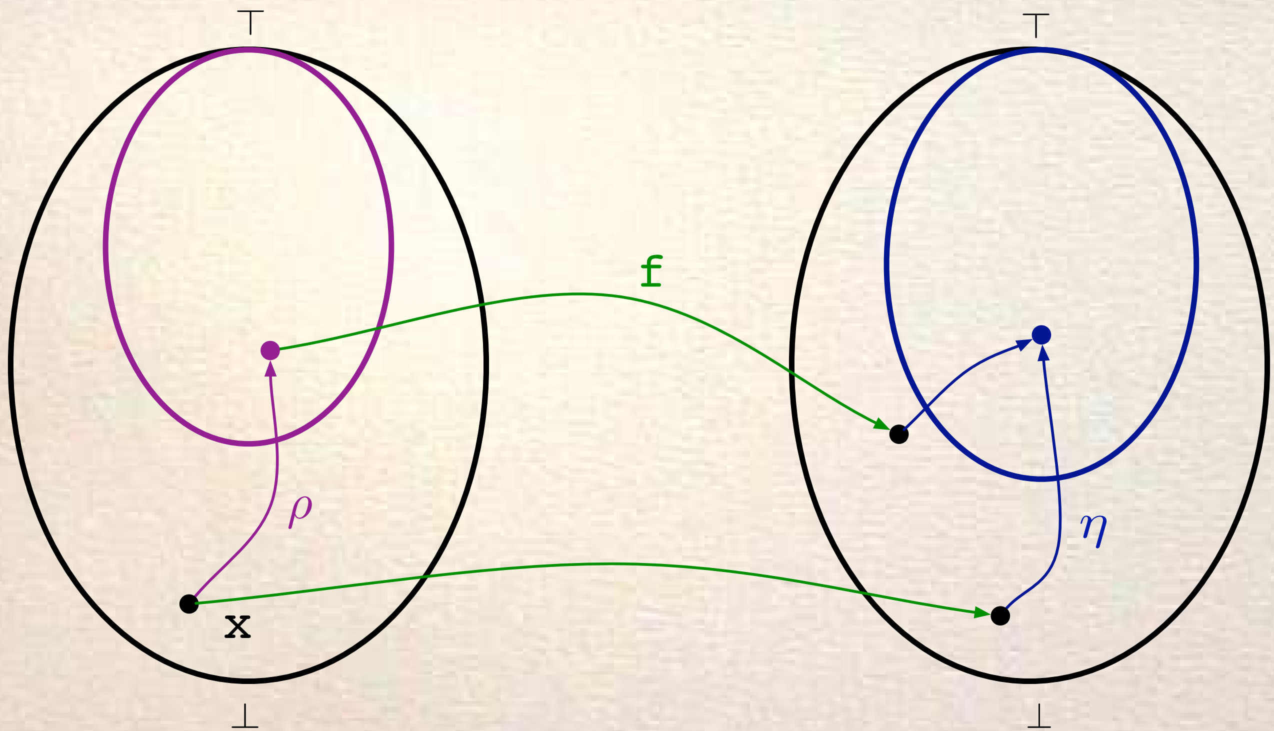


DOMAIN COMPLETENESS: SHELL/CORE



BACKWARD COMPLETENESS: $\eta \circ f \circ \rho = \eta \circ f$

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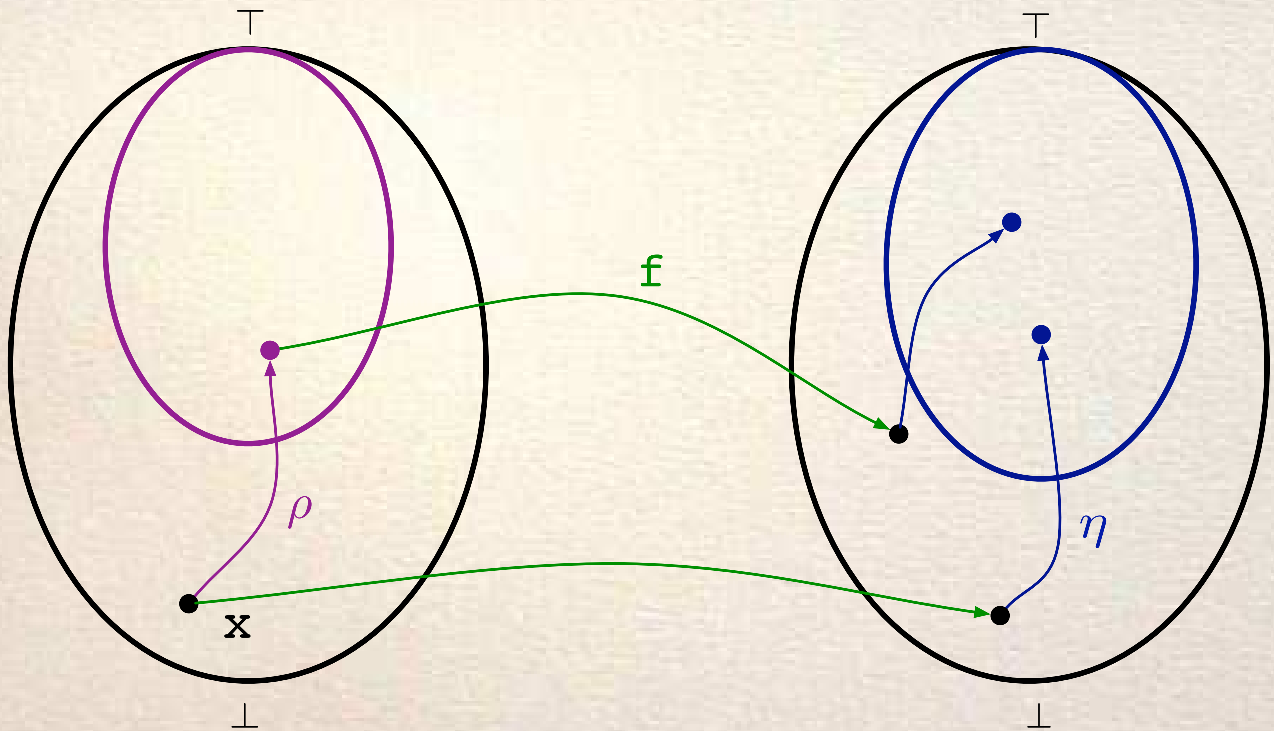

BACKWARD IN-COMPLETENESS: $\eta \circ f \circ \rho \geq \eta \circ f$

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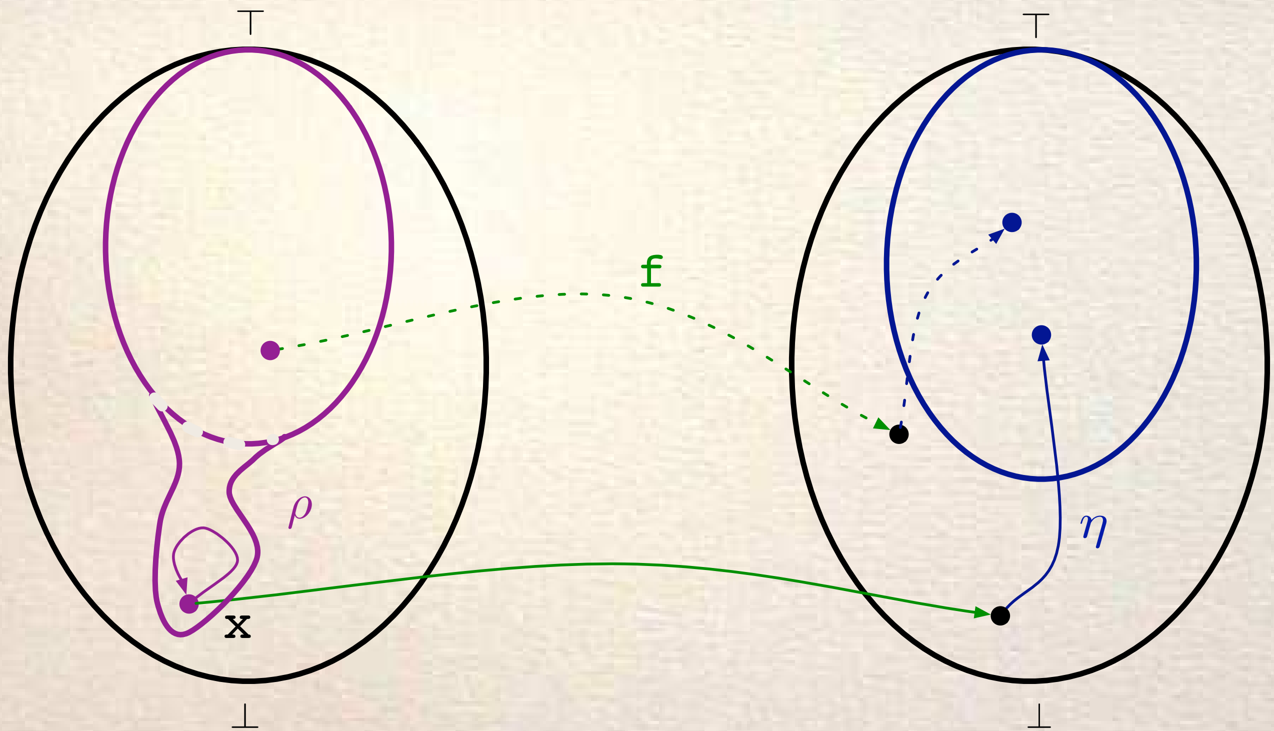
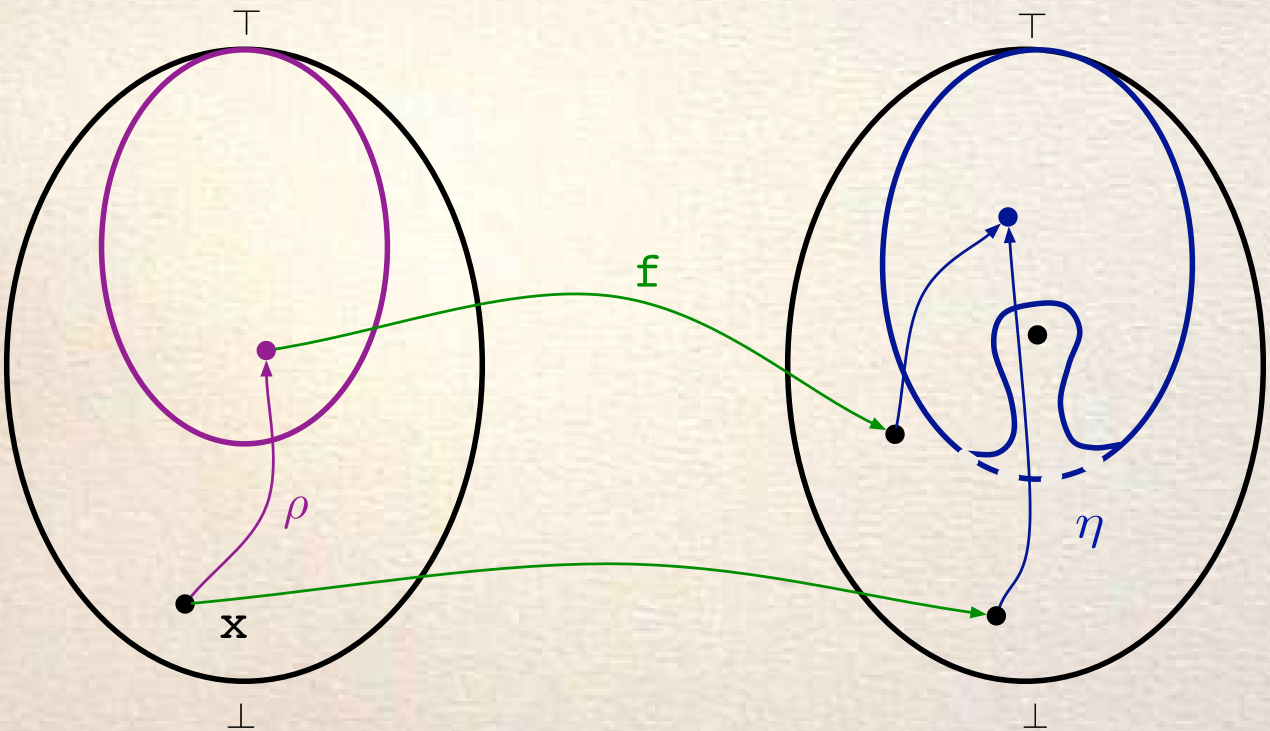


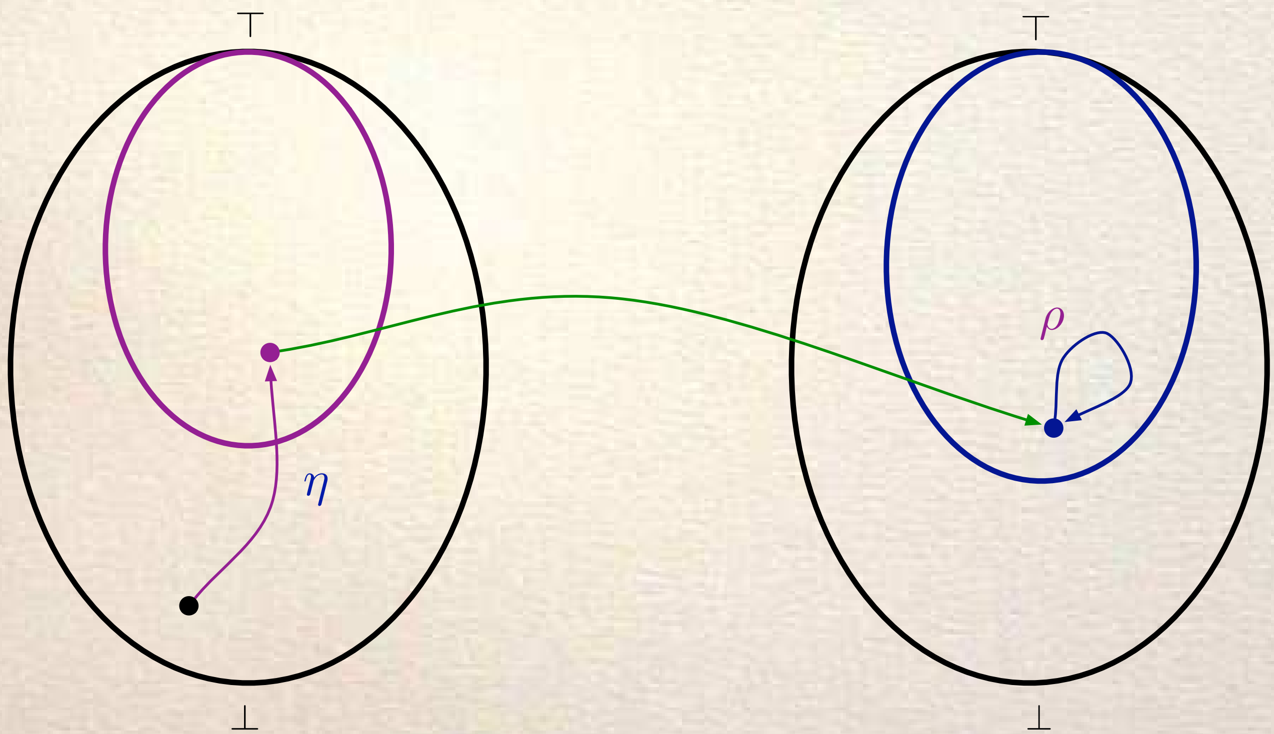
Making BACKWARD COMPLETE: Refining input domains [GRS'00]

Domain Completeness: Shell/Core



Making BACKWARD COMPLETE: Simplifying output domains [GRS'00]

DOMAIN COMPLETENESS: SHELL/CORE



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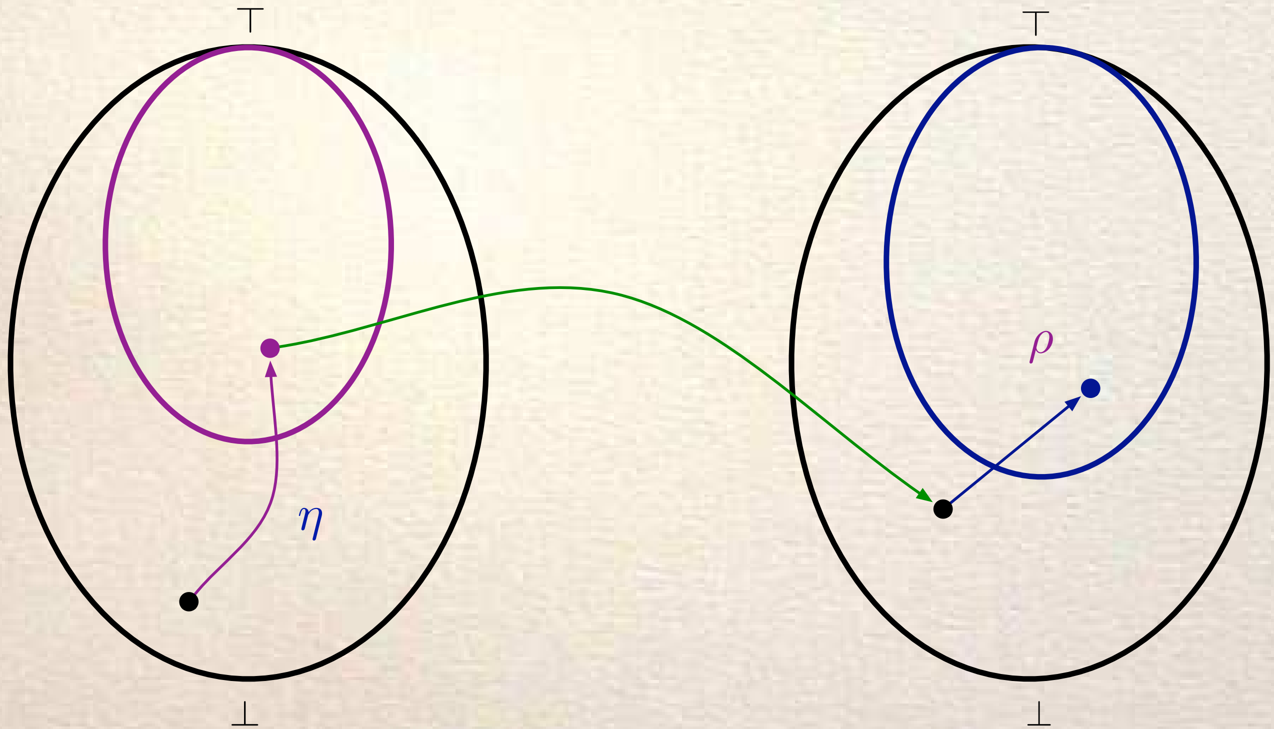
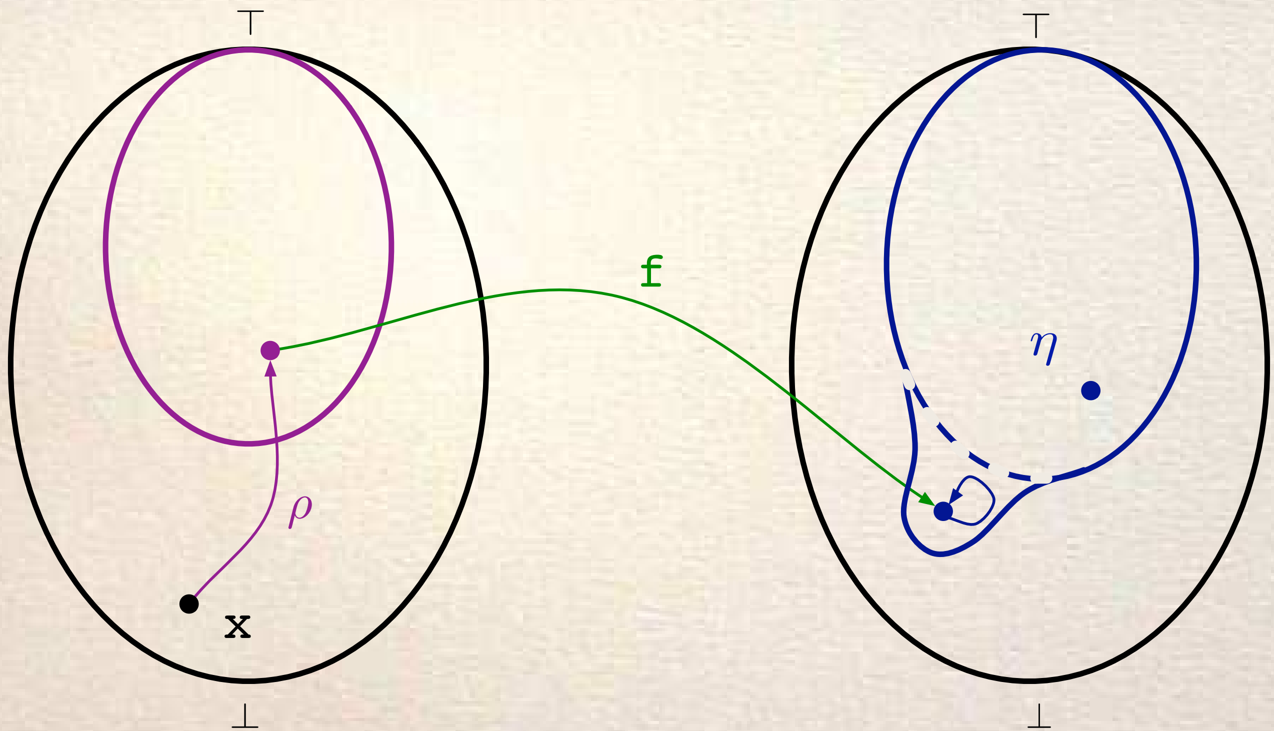


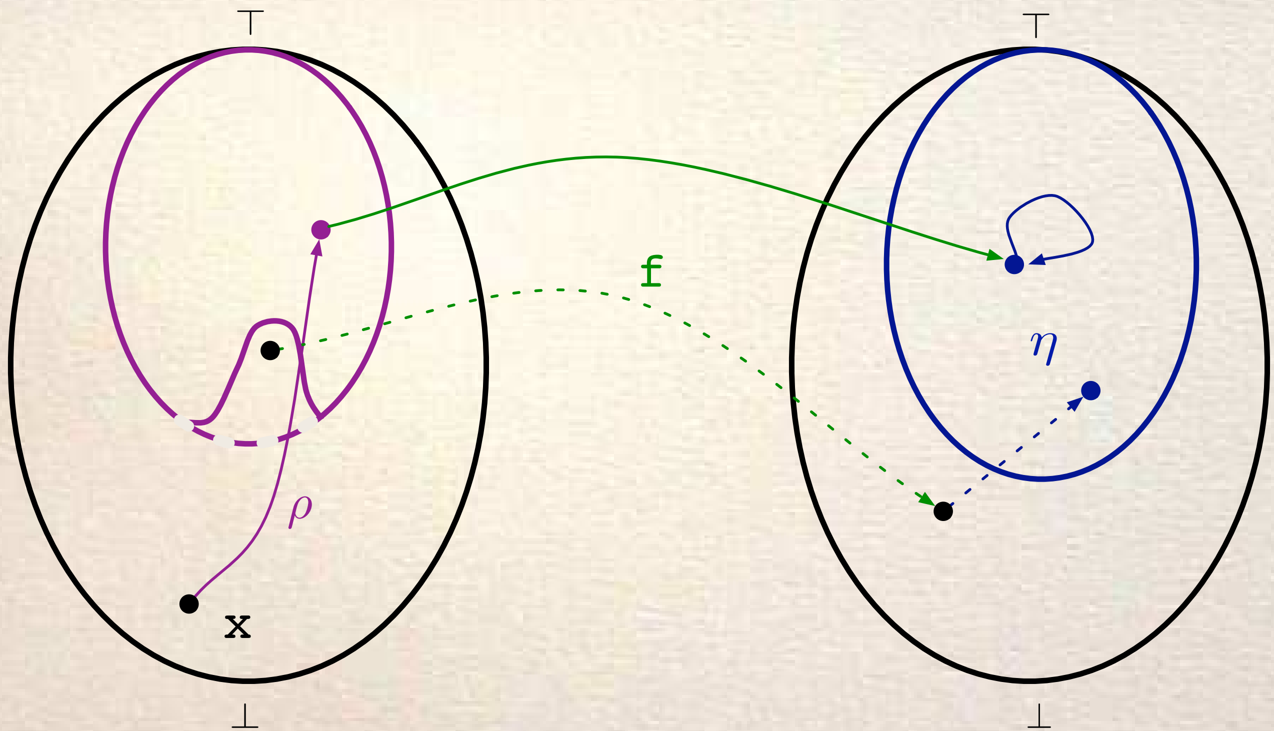
FORWARD IN-COMPLETENESS: $\eta \circ f \circ \rho \geq f \circ \rho$

DOMAIN COMPLETENESS: SHELL/CORE



Making FORWARD COMPLETE: Refining output domains [GQ'01]

DOMAIN COMPLETENESS: SHELL/CORE



Making FORWARD COMPLETE: Simplifying input domains [GQ'01]

BACKWARD VS FORWARD

- A domain is *backward complete* wrt f iff it is *forward complete* wrt $f^+ = \lambda X.\bigcup\left\{ Y \,\middle|\, f(Y) \subseteq X \right\}$;
- A (not trivial) partition is *backward stable* wrt f iff it is *forward stable* wrt $f^{-1} = \lambda X.\left\{ y \,\middle|\, f(y) \in X \right\}$;
- If f is injective, a (not trivial) partition is *forward stable* wrt f iff it is *backward stable* wrt f^{-1};

Backward vs Forward

- A domain is *backward complete* wrt f iff it is *forward complete* wrt $f^{+} = \lambda X. \bigcup \left\{ Y \,\middle|\, f(Y) \subseteq X \right\}$;

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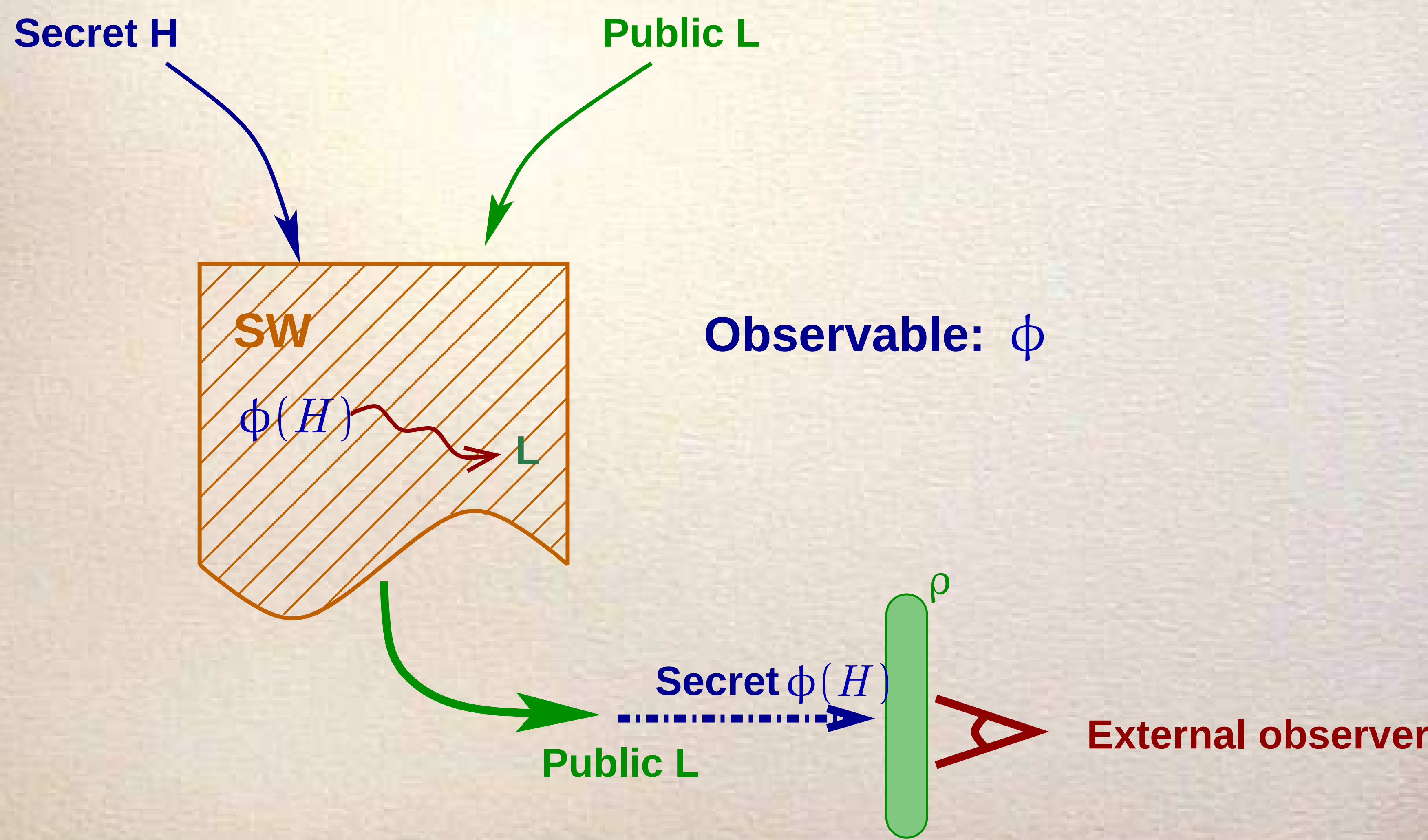
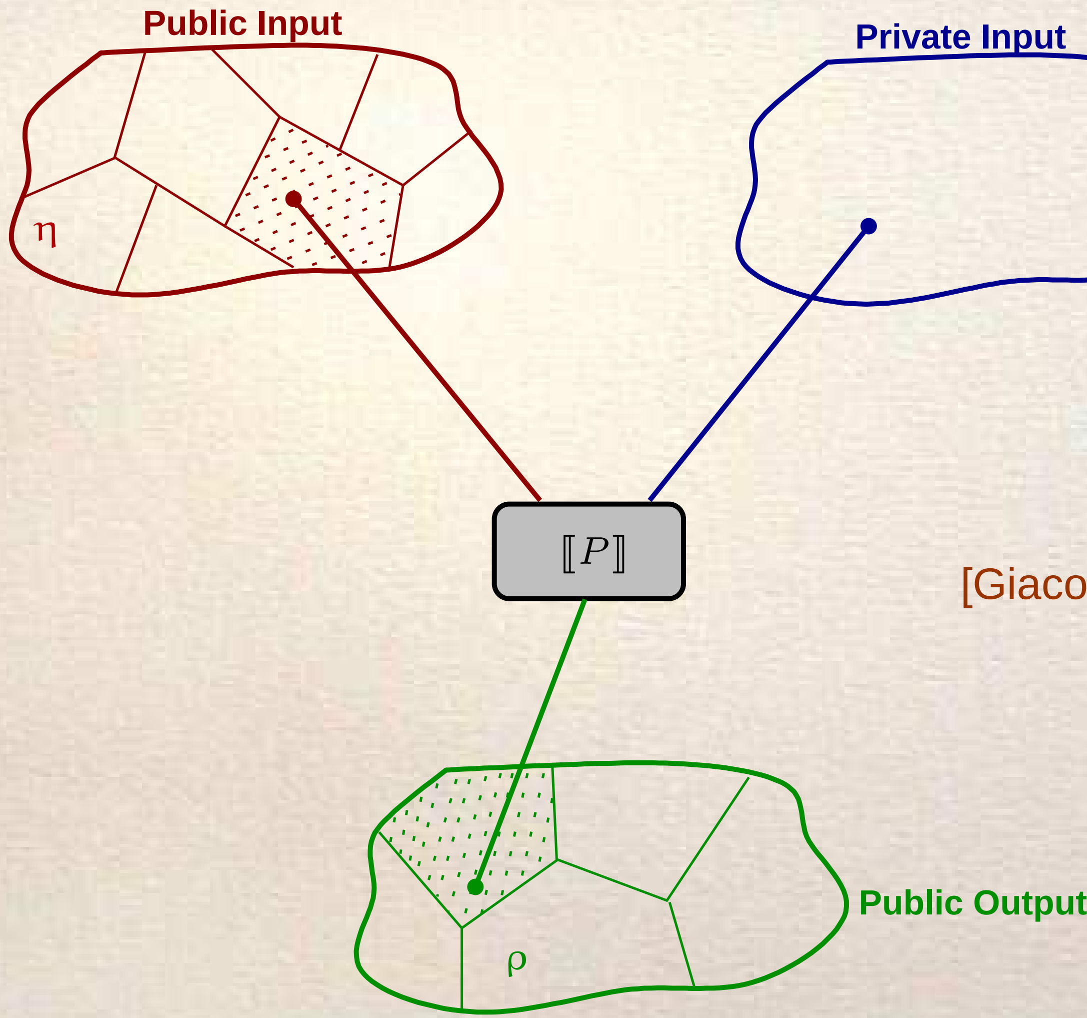
A backward problem can always be transformed in a forward one, but the viceversa is not always possible!

NEW PERSPECTIVES IN LANGUAGE-BASED SECURITY



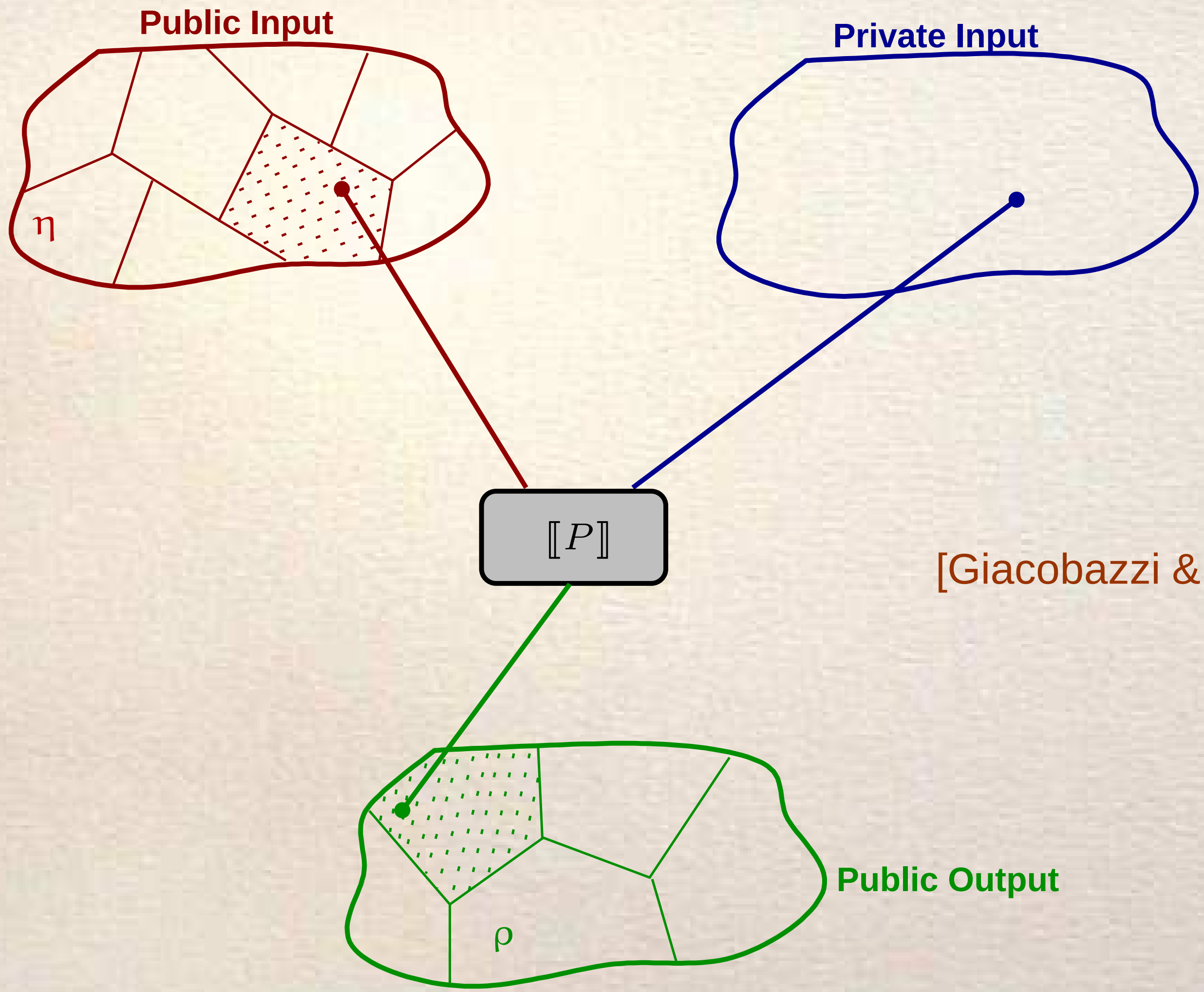
ABSTRACT NON-INTERFERENCE (NARROW)



[Giacobazzi & Mastroeni '04]

$$\rho, \eta \in uco(\wp(\mathbb{V}^{\mathsf{L}})):\ [\eta]P(\rho):\ \eta(l_1) = \eta(l_2) \Rightarrow \rho(\llbracket P \rrbracket(h_1, l_1)^{\mathsf{L}}) = \rho(\llbracket P \rrbracket(h_2, l_2)^{\mathsf{L}})$$

Abstract Non-Interference (Narrow)



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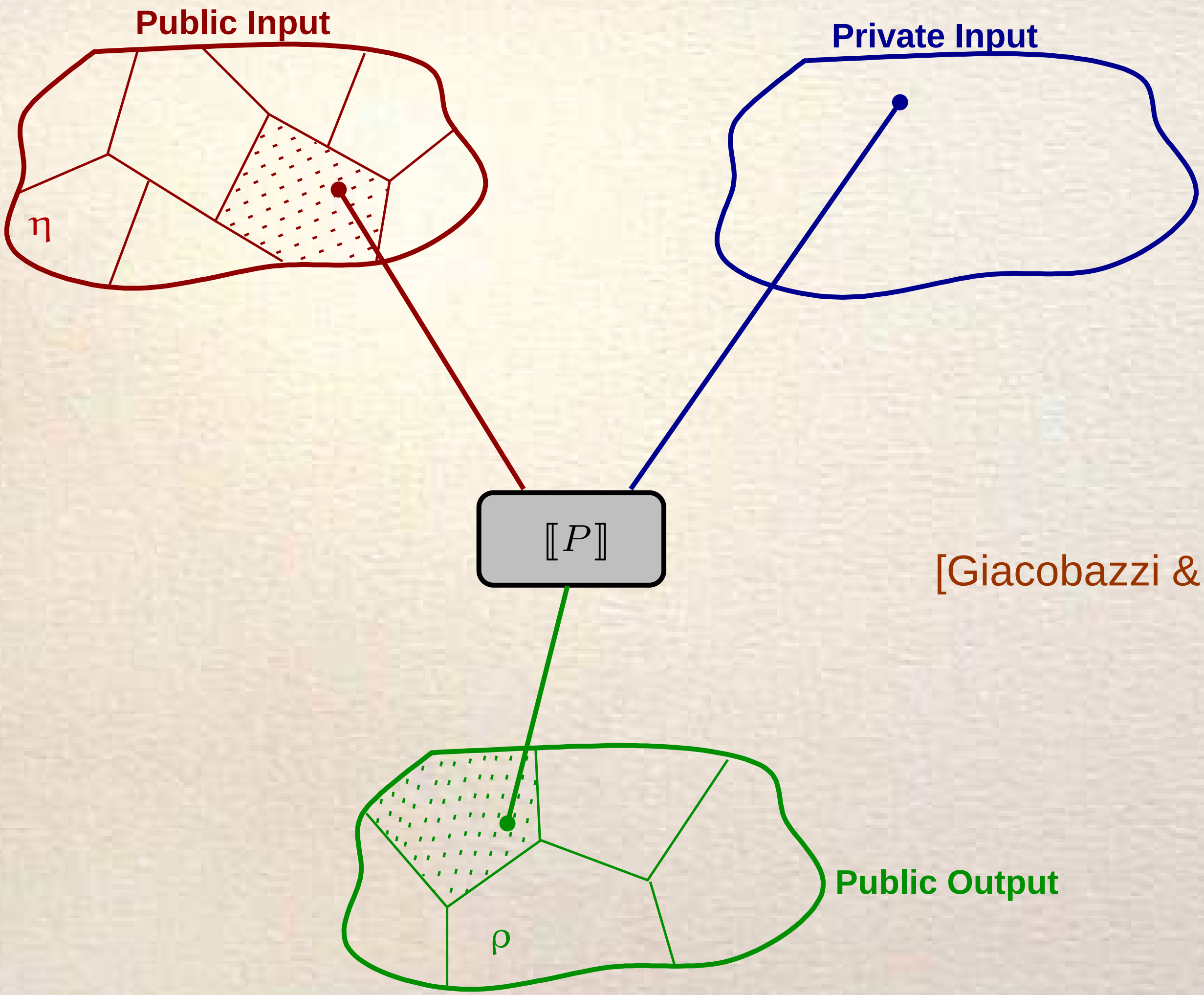

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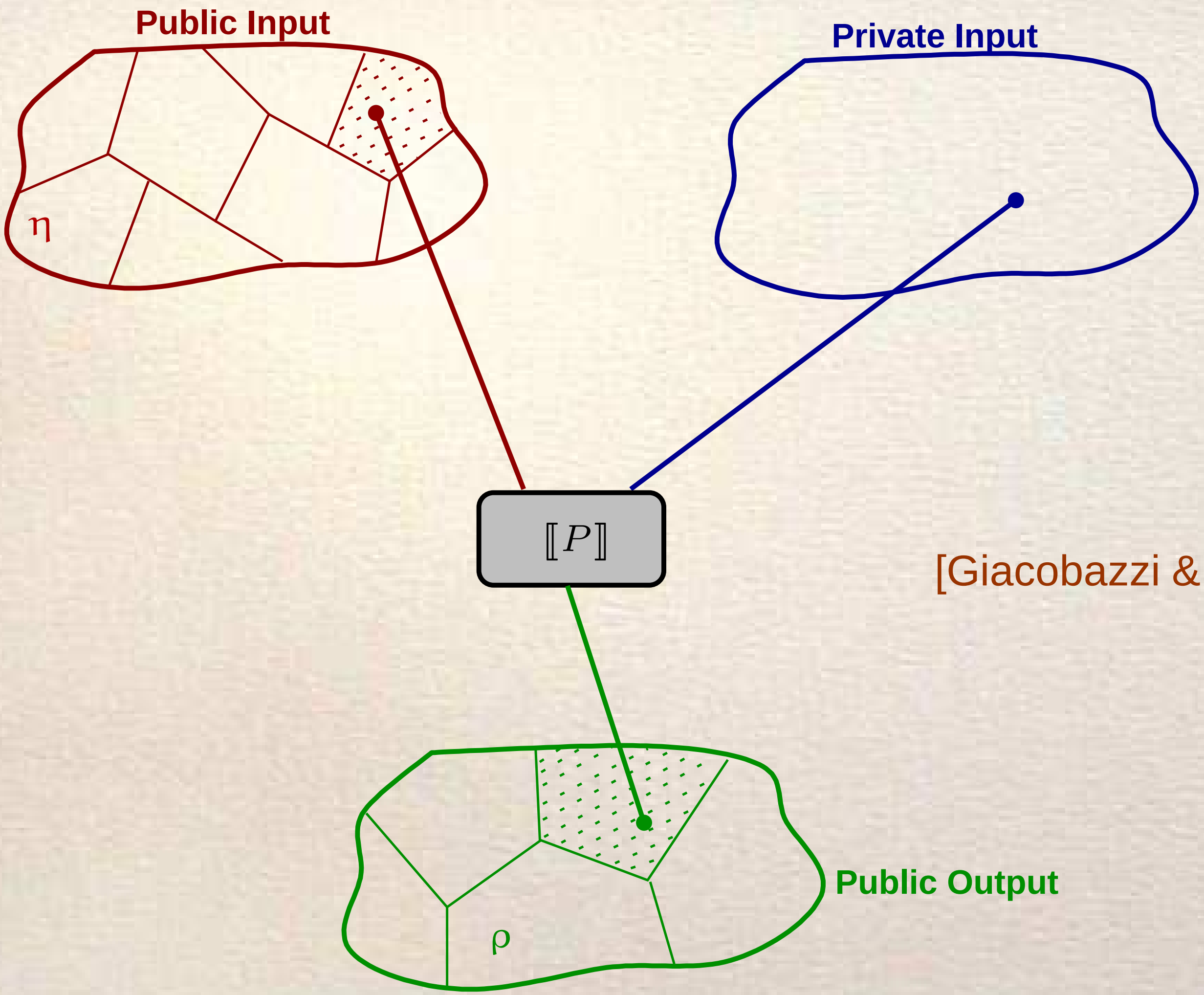

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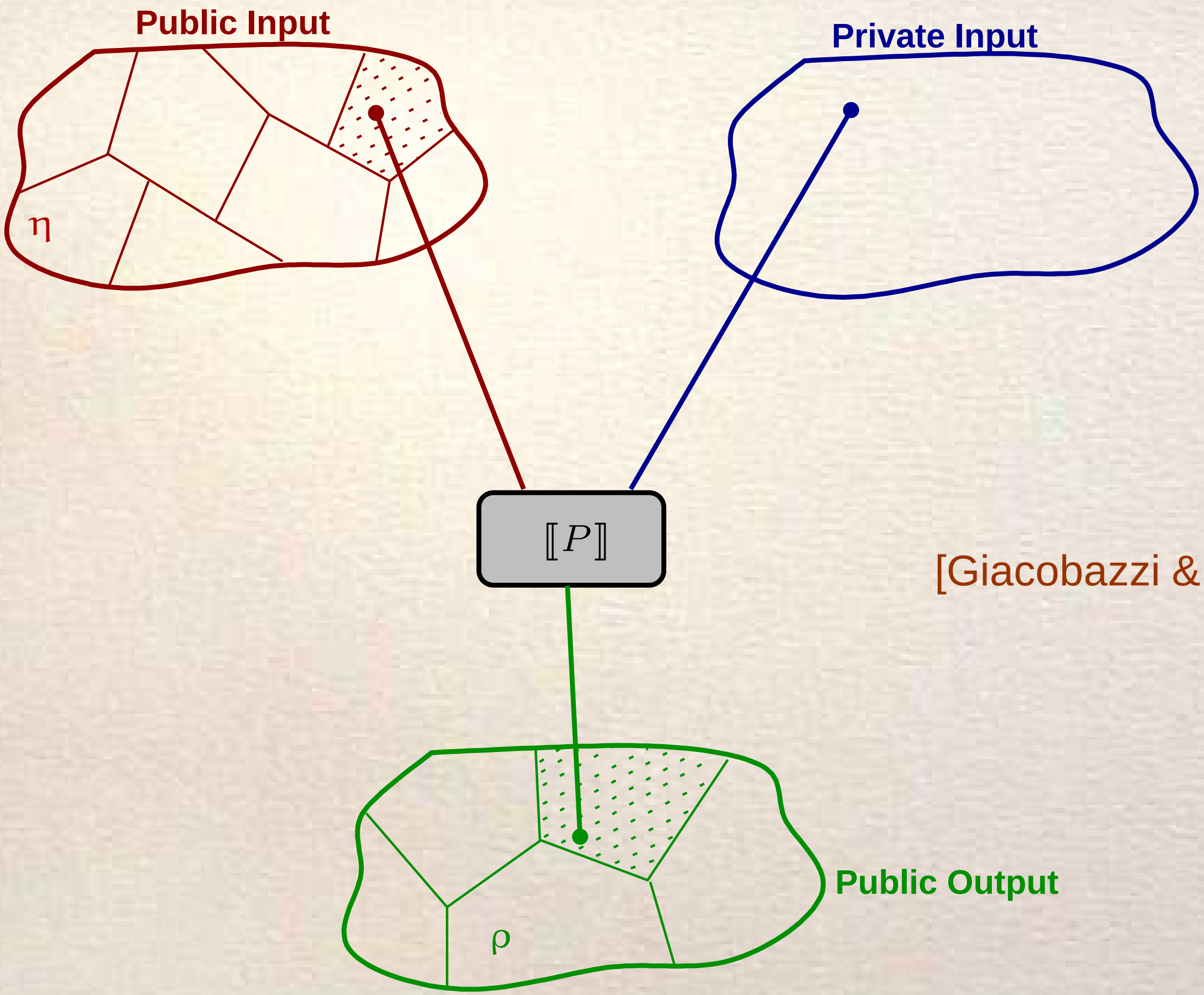

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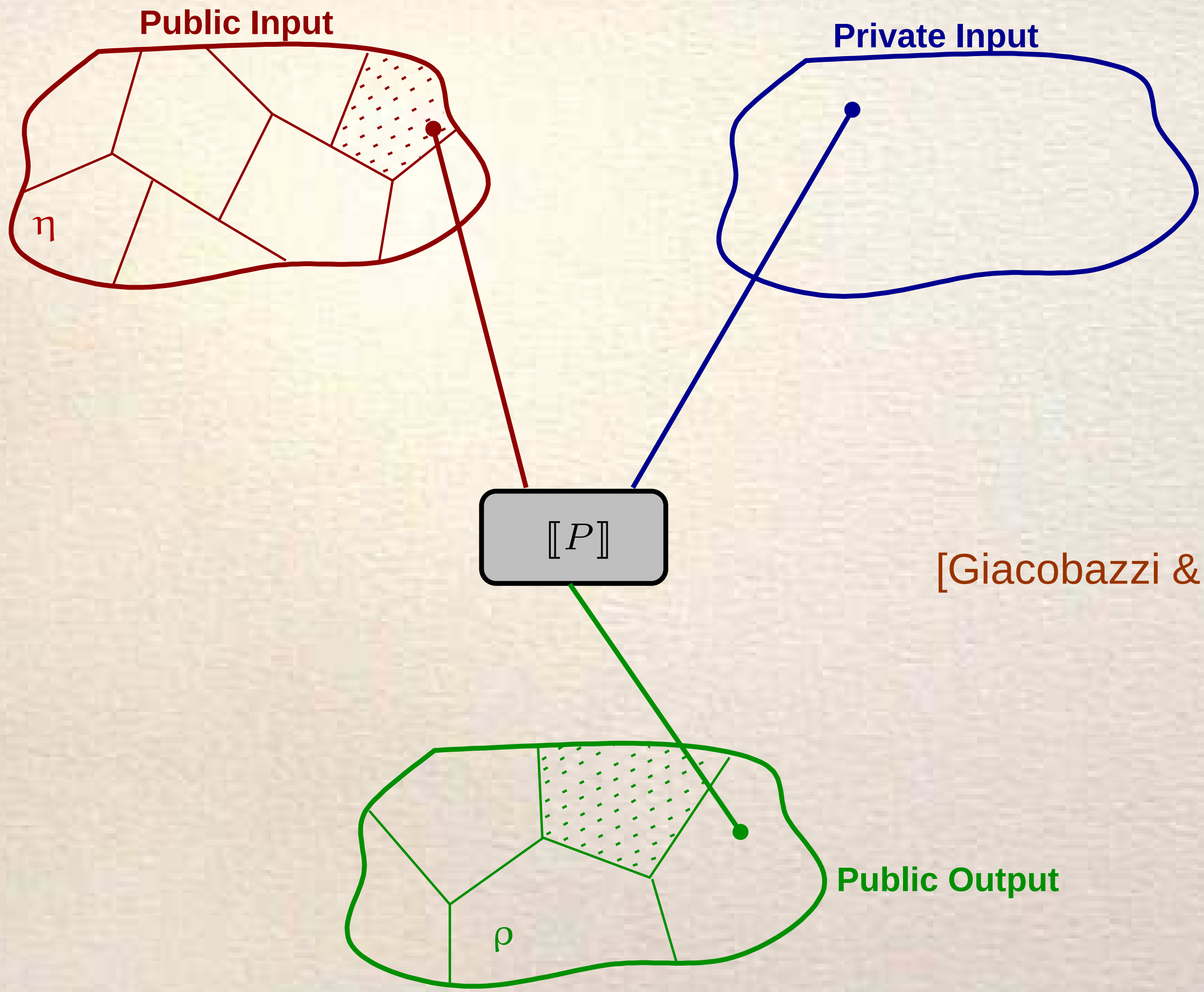

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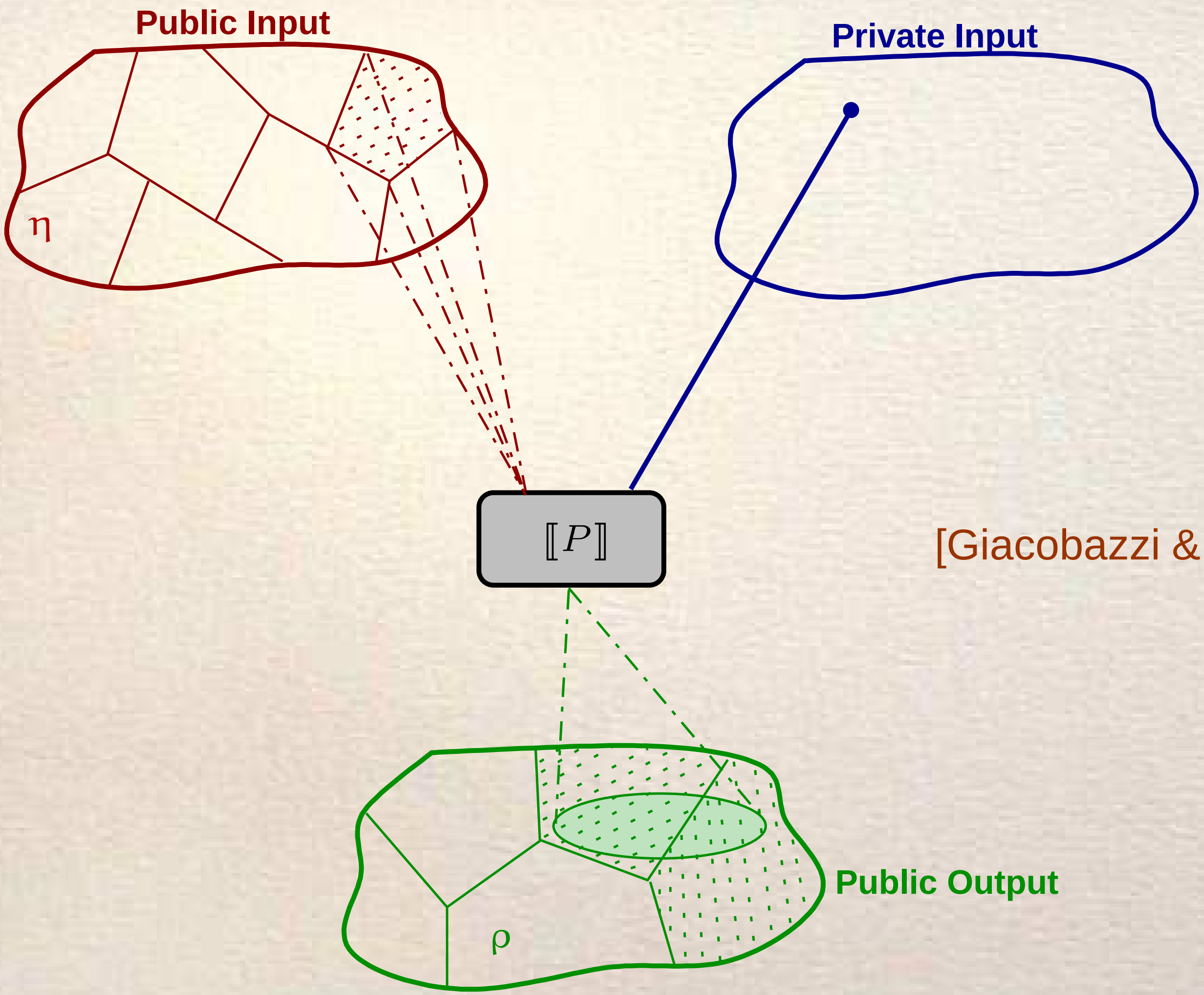

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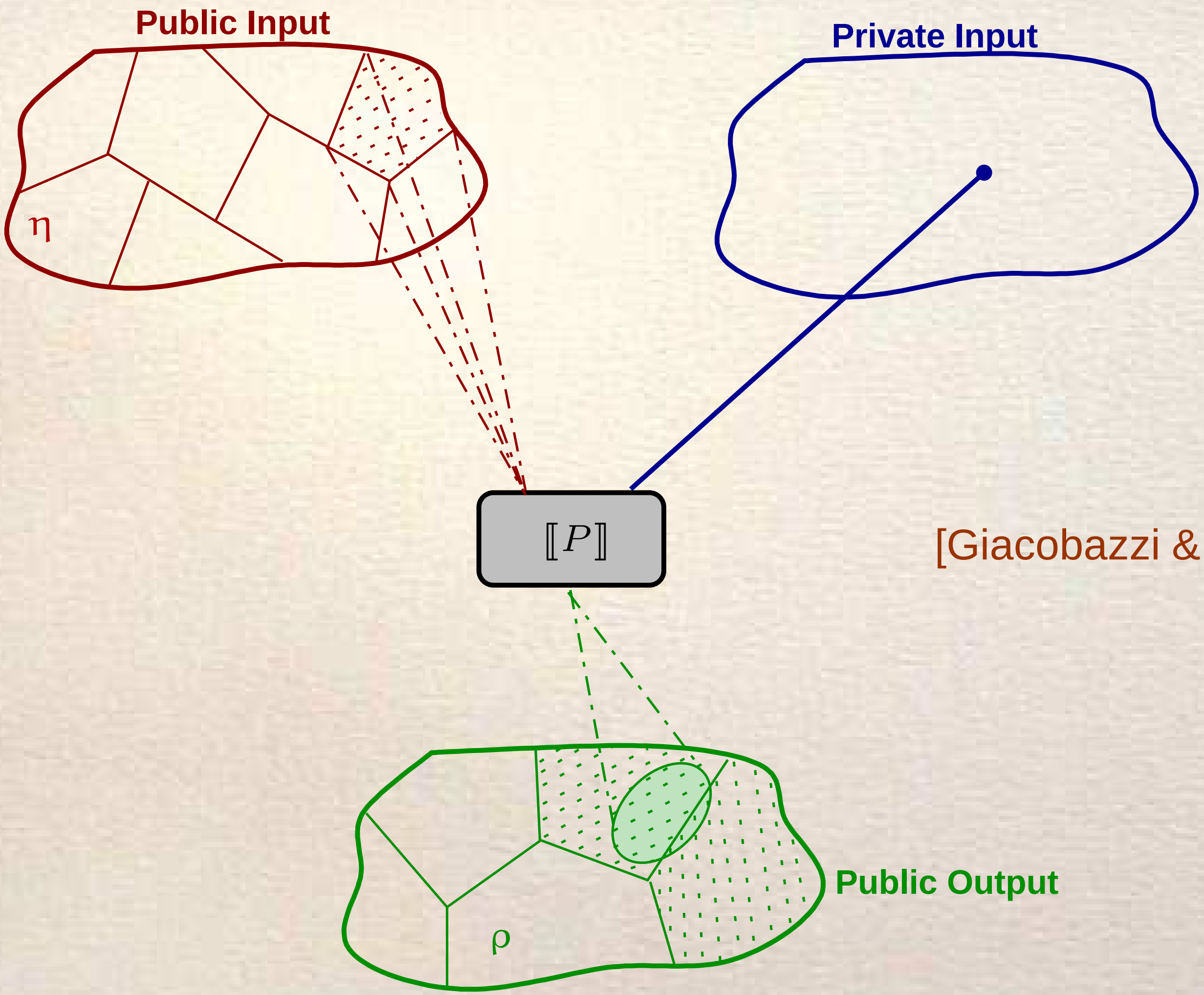

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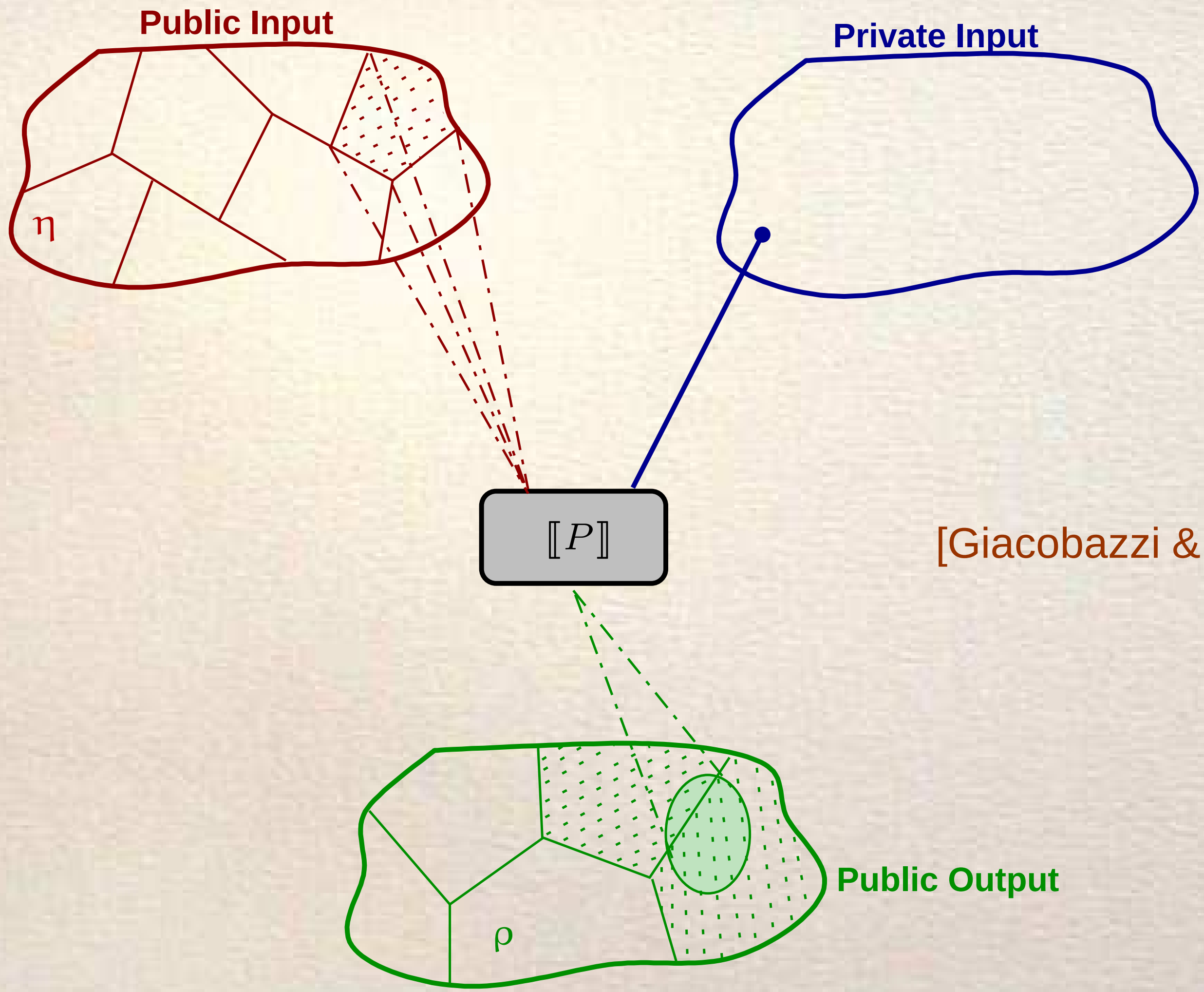

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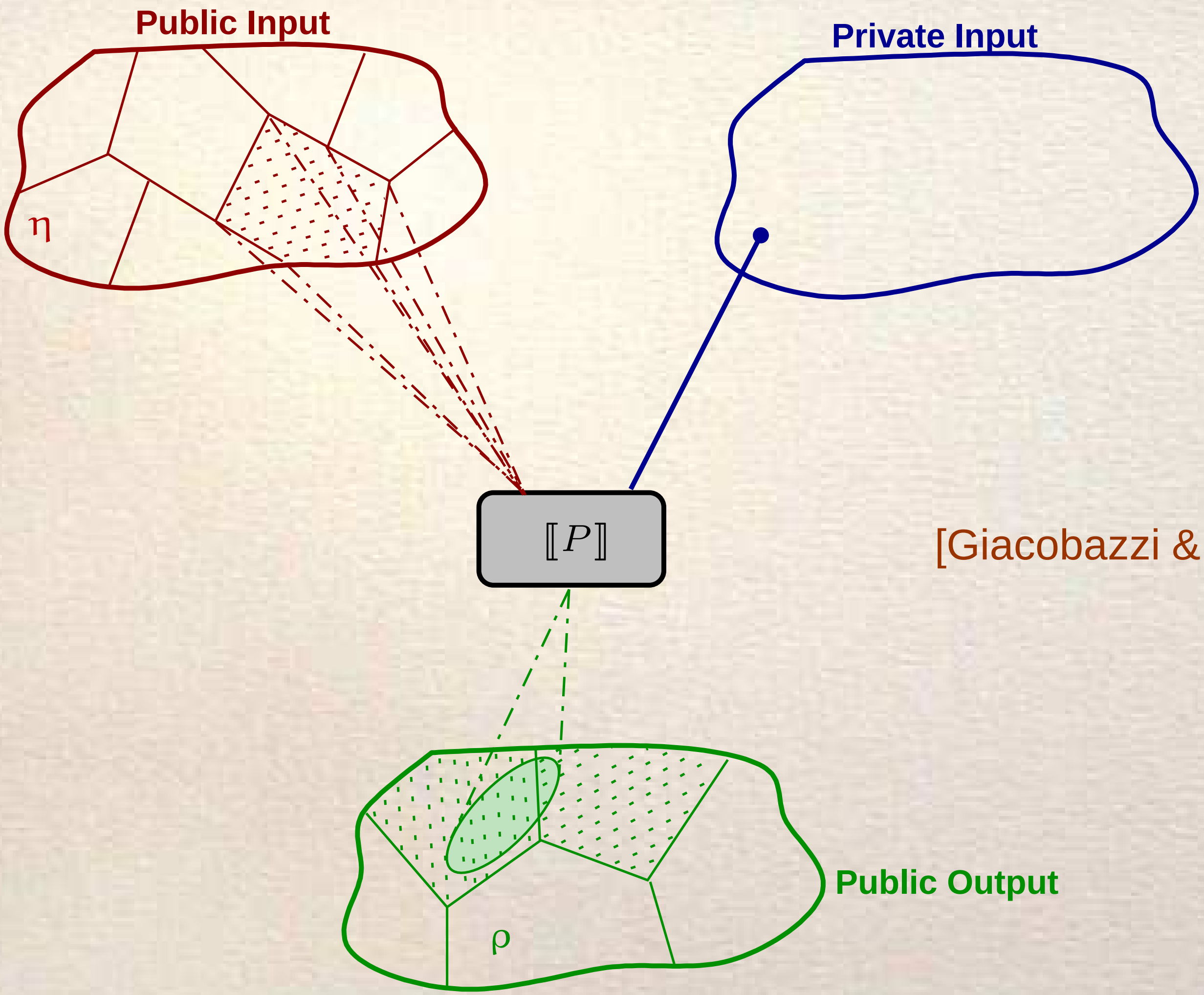

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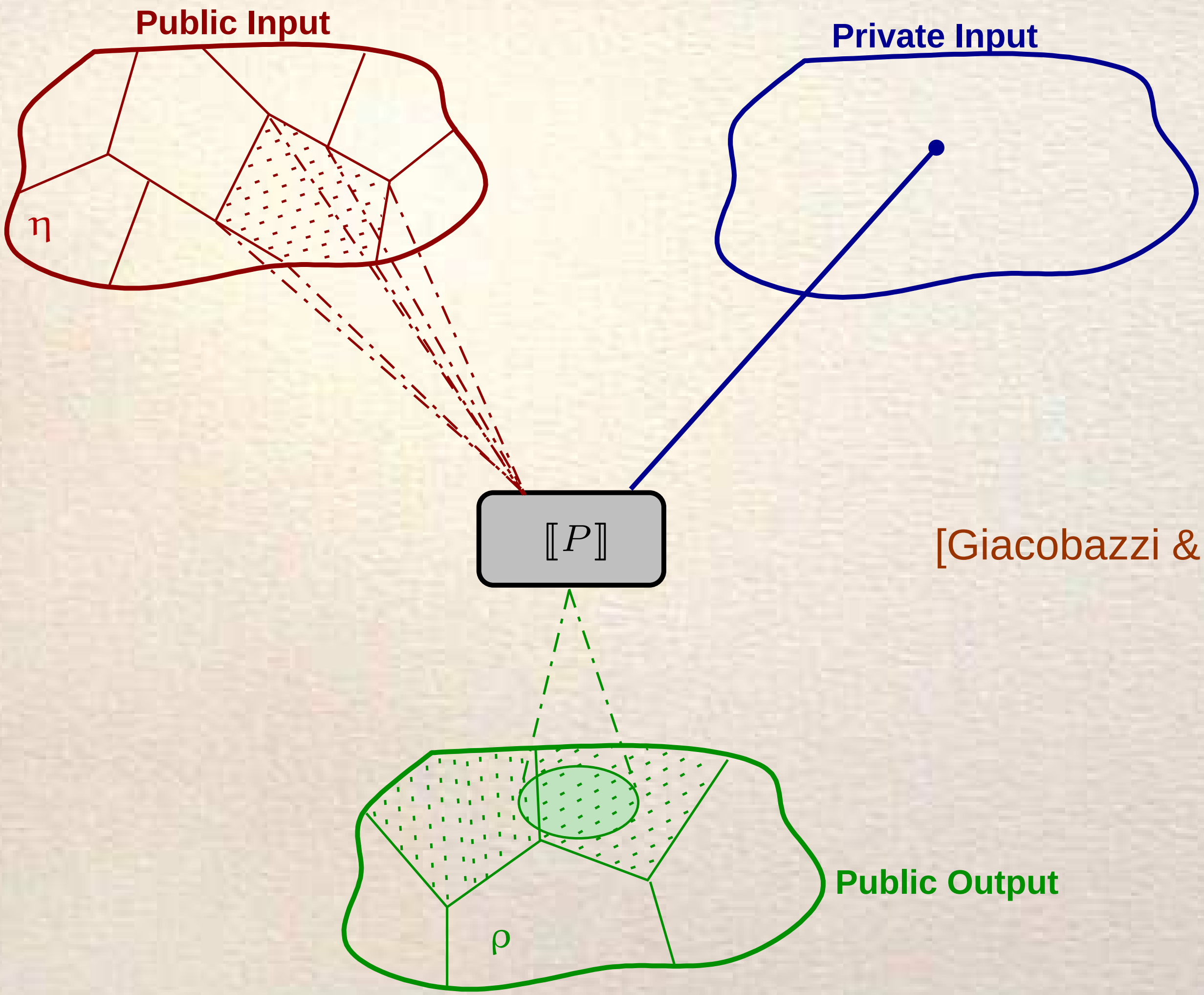

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EXAMPLES

EXAMPLE I:

$$\textbf{while } h \textbf{ do } (l := l + 2;\ h := h - 1).$$

$$\boxed{\text{Standard Non-Interference} \equiv [id]P(id)}$$

$$h = 0,\ l = 1 \rightsquigarrow l = 1$$
$$h = 1,\ l = 1 \rightsquigarrow l = 3$$
$$h = n,\ l = 1 \rightsquigarrow l = 1 + 2n$$

EXAMPLES

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$$\Downarrow$$

$$\boxed{[id]P(Par)}$$

$$h = 0,\ l = 1 \ \rightsquigarrow\ Par(l) = \text{odd}$$
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Examples

Example II:

$$P = \quad l := 2 * l * h^2.$$

$$\boxed{[Par]P(Sign)}$$

$$h = 1,\ l = 4\ (Par(4) = \text{even}) \ \rightsquigarrow\ Sign(l) = +$$
$$h = 1,\ l = -4\ (Par(-4) = \text{even}) \ \rightsquigarrow\ Sign(l) = -$$

$$\boxed{\text{Deceptive Flow}}$$

EXAMPLES

EXAMPLE II:

$$P = \quad l := 2 * l * h^2.$$

$$\boxed{[Par]P(Sign)}$$

$$h = 1,\ l = 4\ (Par(4) = \text{even}) \leadsto Sign(l) = +$$
$$h = 1,\ l = -4\ (Par(-4) = \text{even}) \leadsto Sign(l) = -$$

$$\boxed{\text{DECEPTIVE FLOW}}$$
$$\Downarrow$$
$$\boxed{(Par)P(Sign)}$$

$$h = -3,\ Par(l) = \text{even} \leadsto Sign(l) = \text{I don't know}$$
$$h = 1,\ Par(l) = \text{even} \leadsto Sign(l) = \text{I don't know}$$

Examples

Example III:

$$P = \quad l := l * h^2.$$

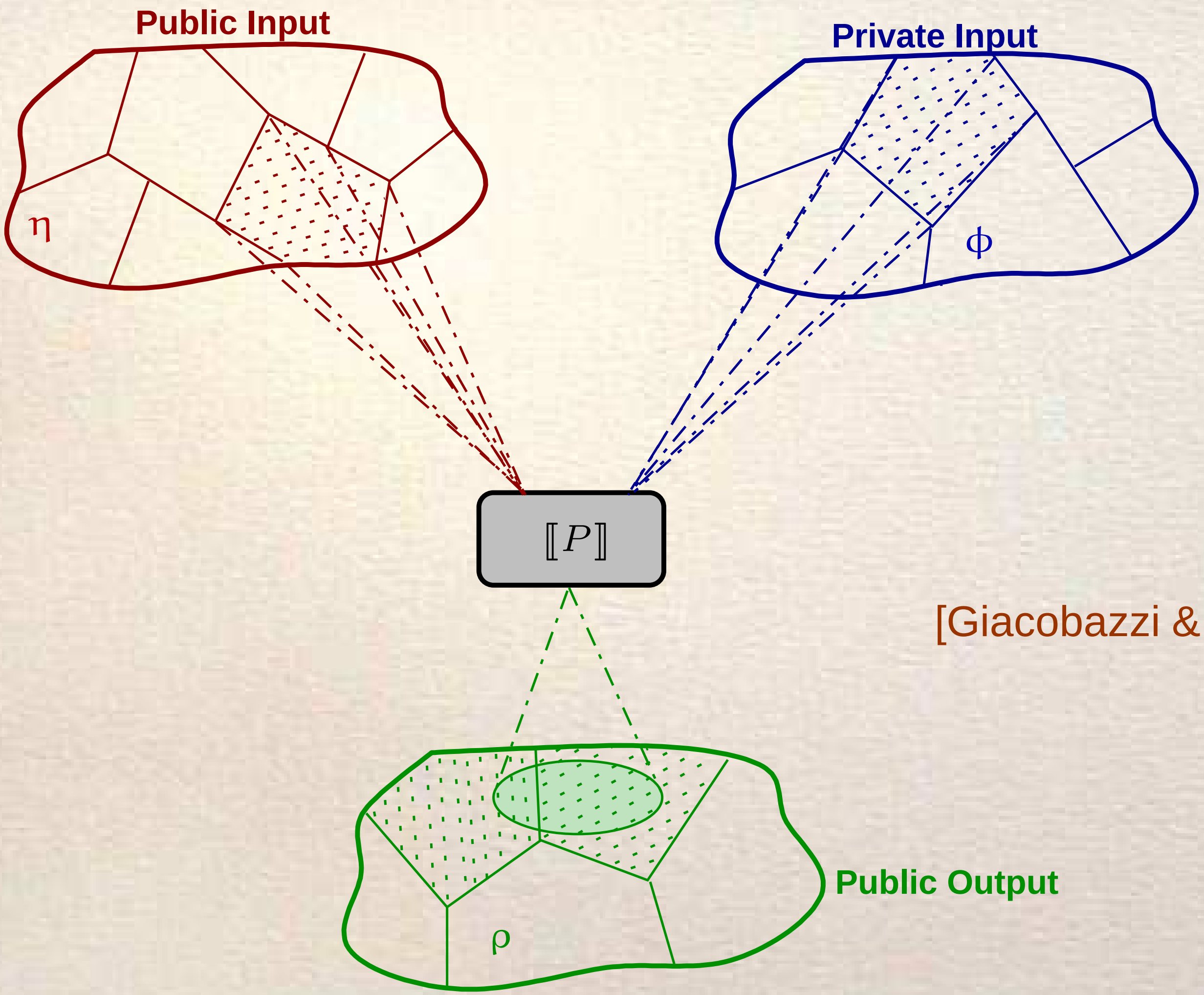
$$\boxed{(id)\,P\,(Par)}$$

$h = 2,\ l = 1 \ \leadsto\ Par(l) = \text{even}$

$h = 3,\ l = 1 \ \leadsto\ Par(l) = \text{odd}$

$h = n,\ l = 1 \ \leadsto\ Par(l) = Par(n)$

DECLASSIFIED ANI VIA BLOCKING



[Giacobazzi & Mastroeni '04]

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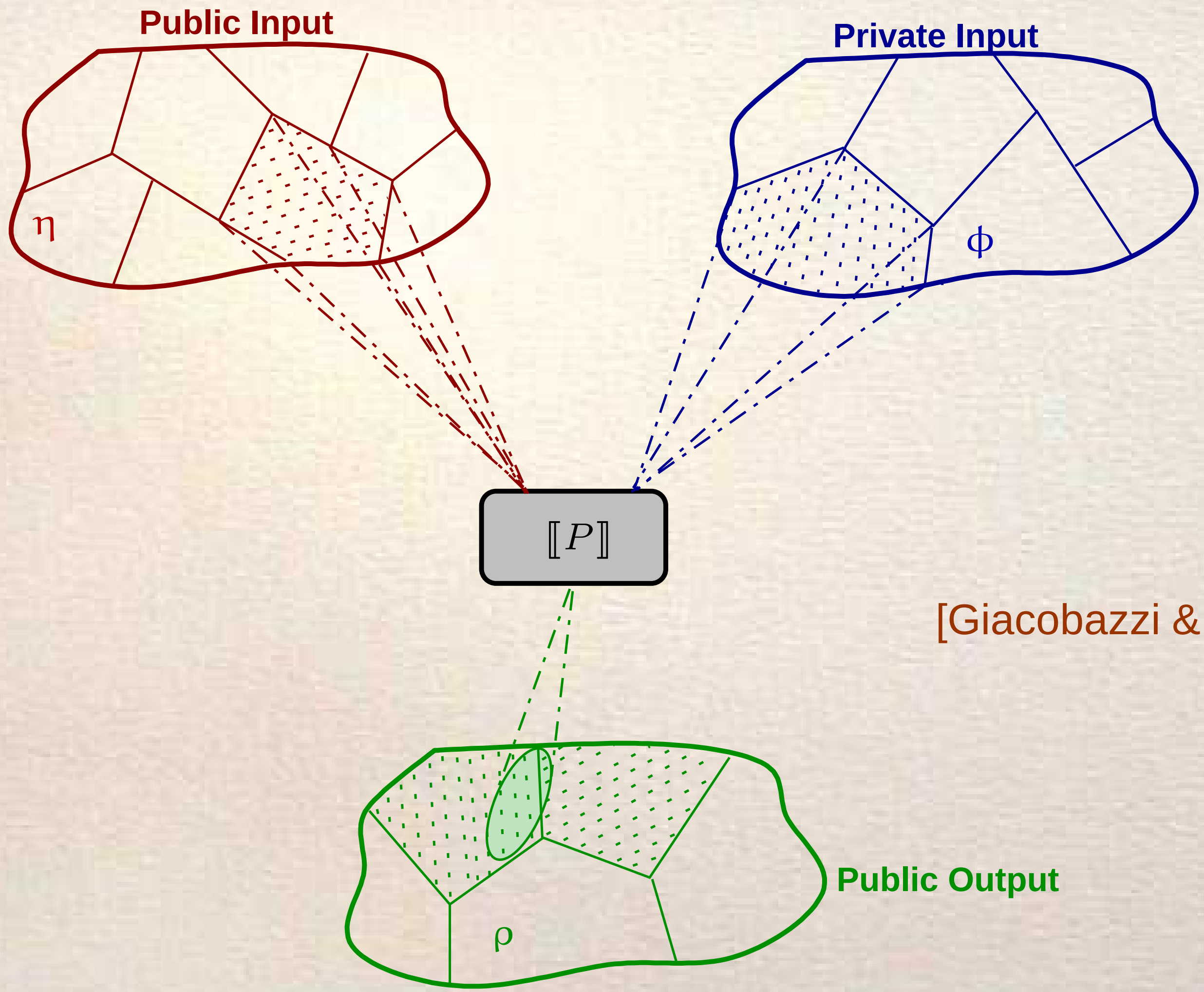

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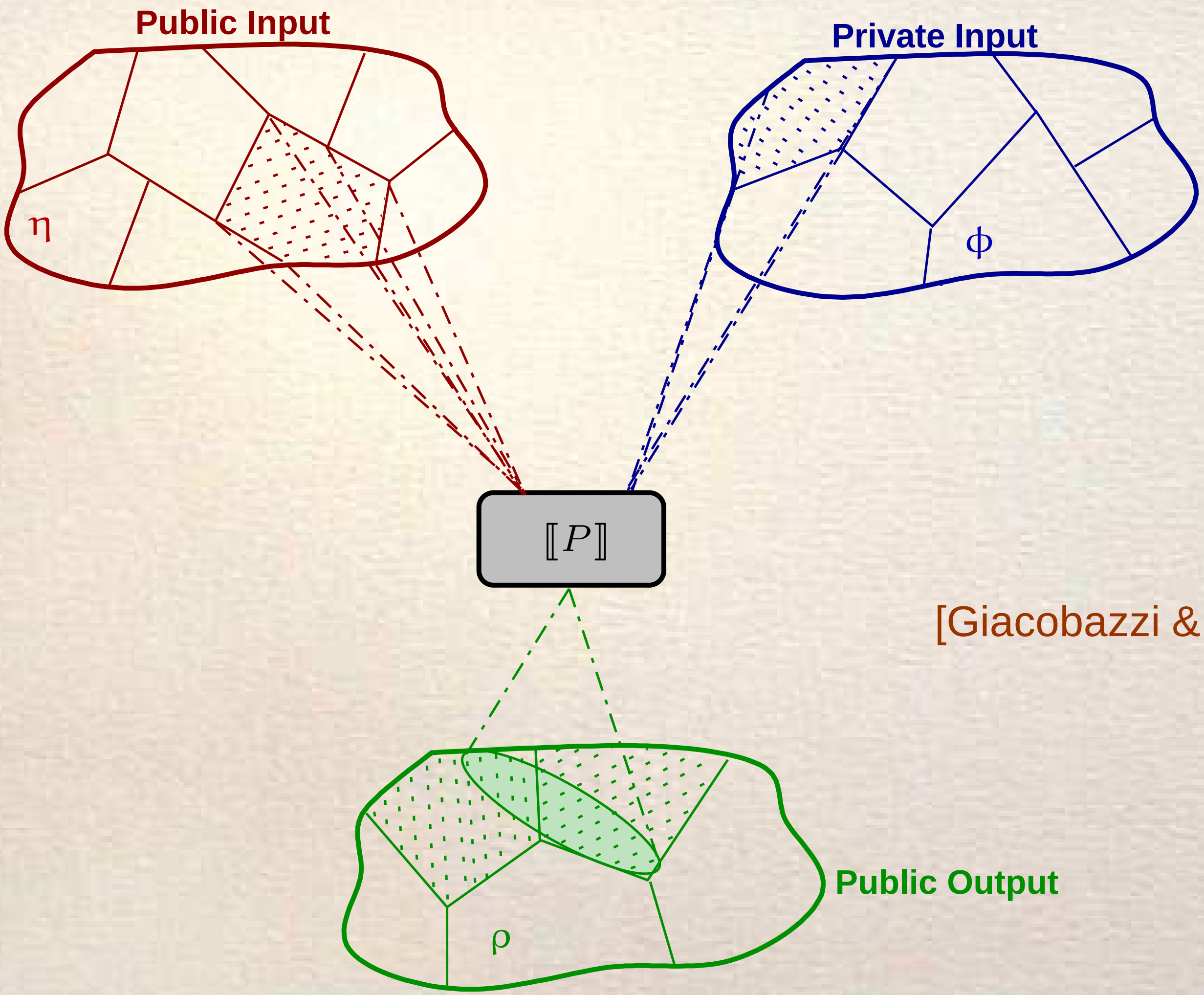

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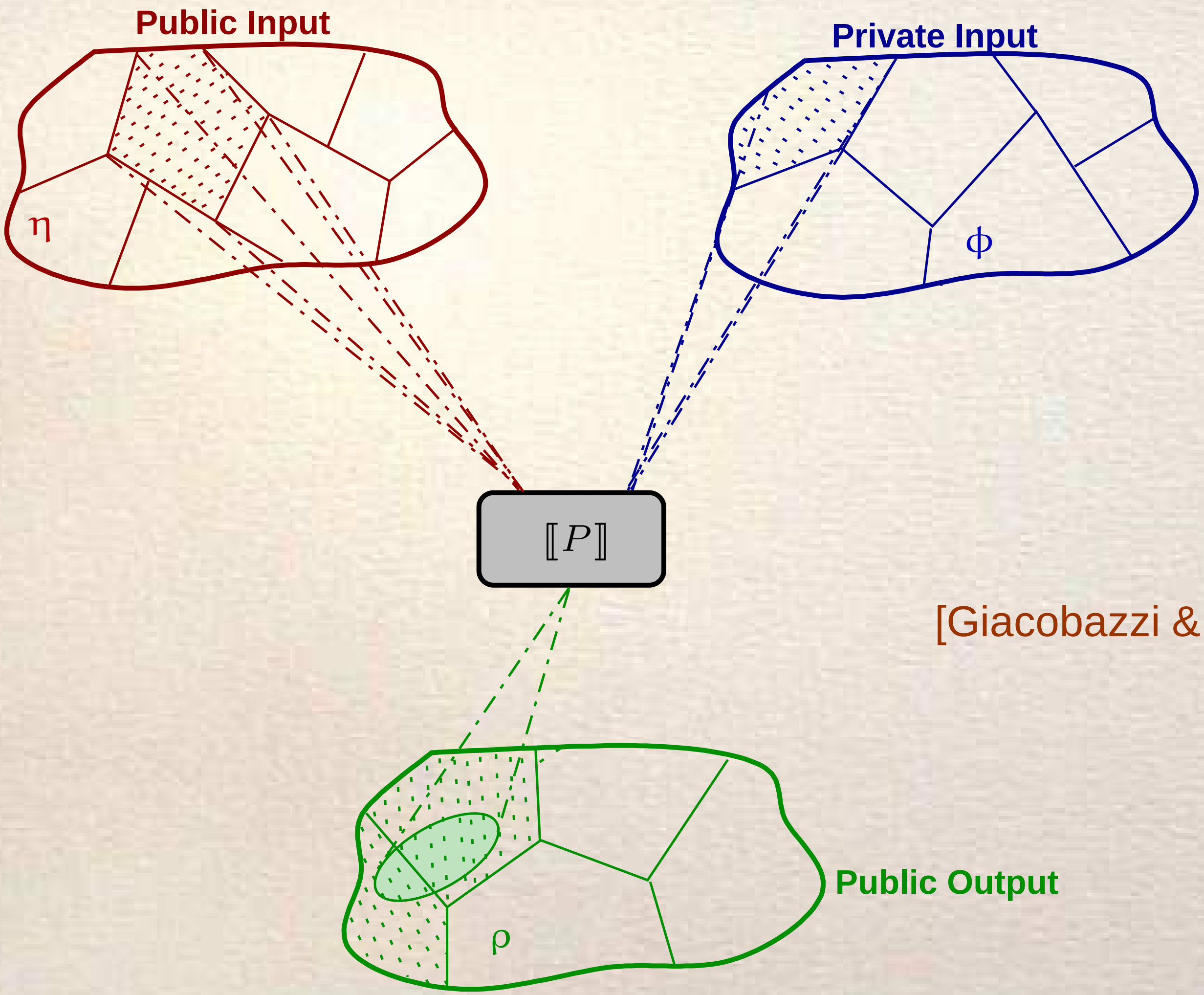

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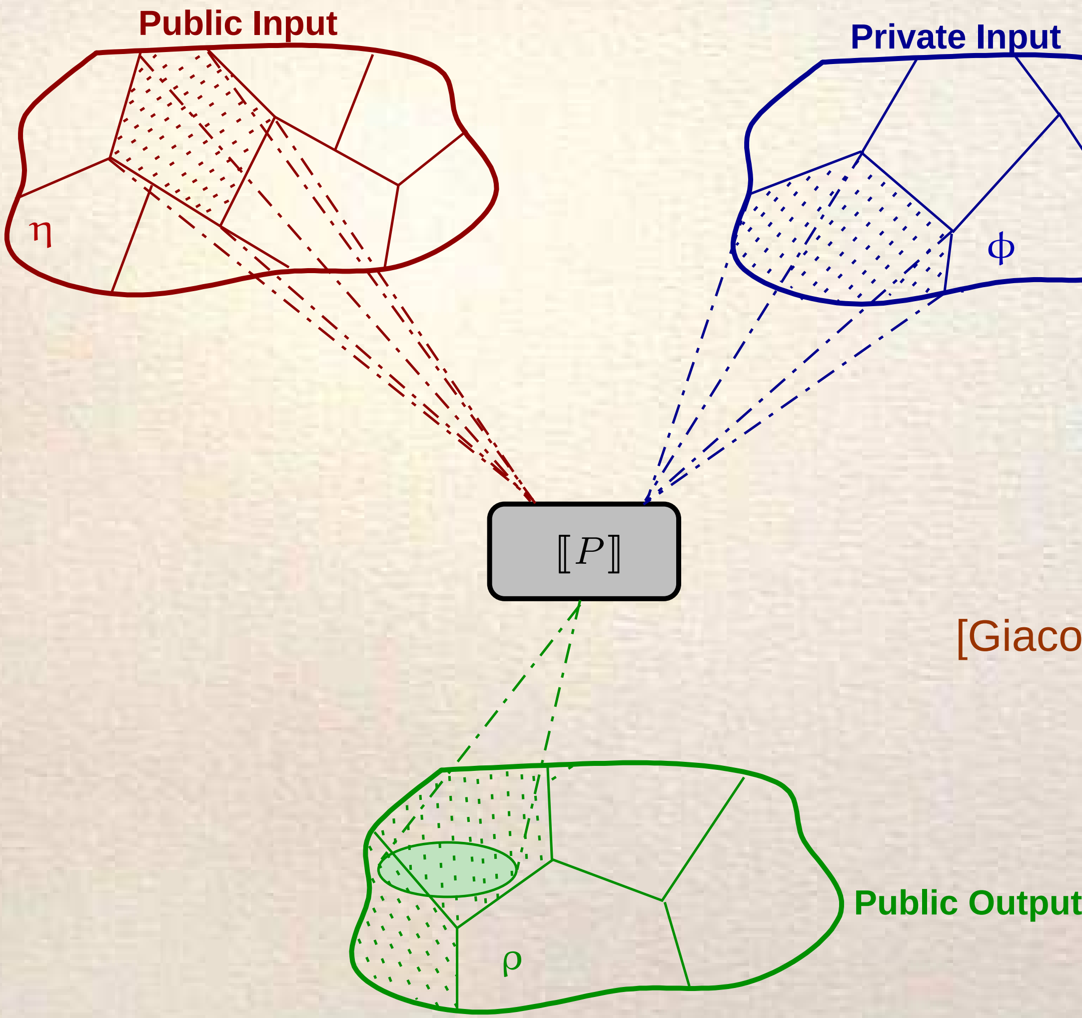

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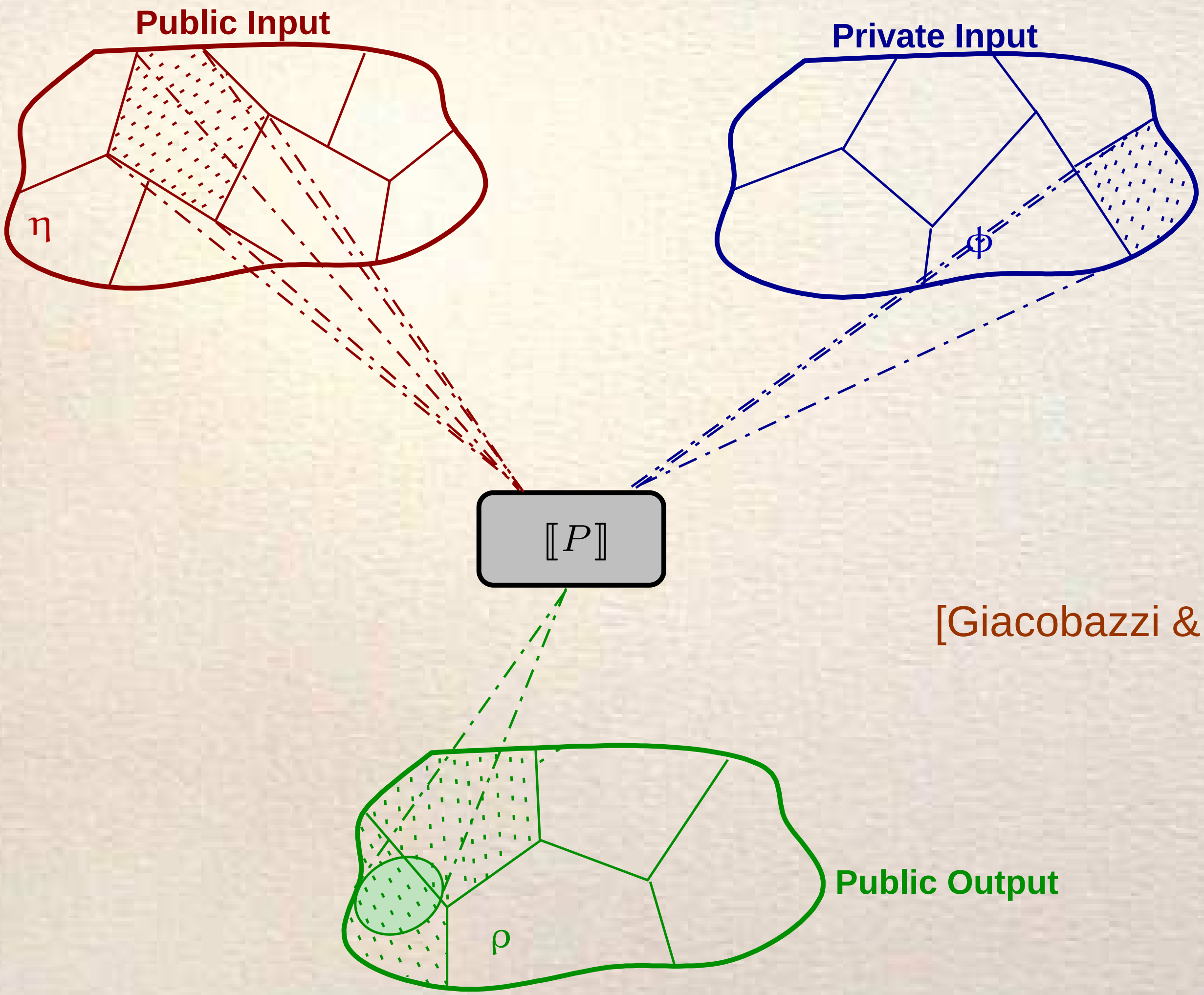

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Example

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$$\Downarrow$$

$$\boxed{(id)P(Sign \rightsquigarrow\!\!\| Par)}$$

$$Sign(h) = +,\ l = 1 \rightsquigarrow Par(l) = \text{I don't know}$$
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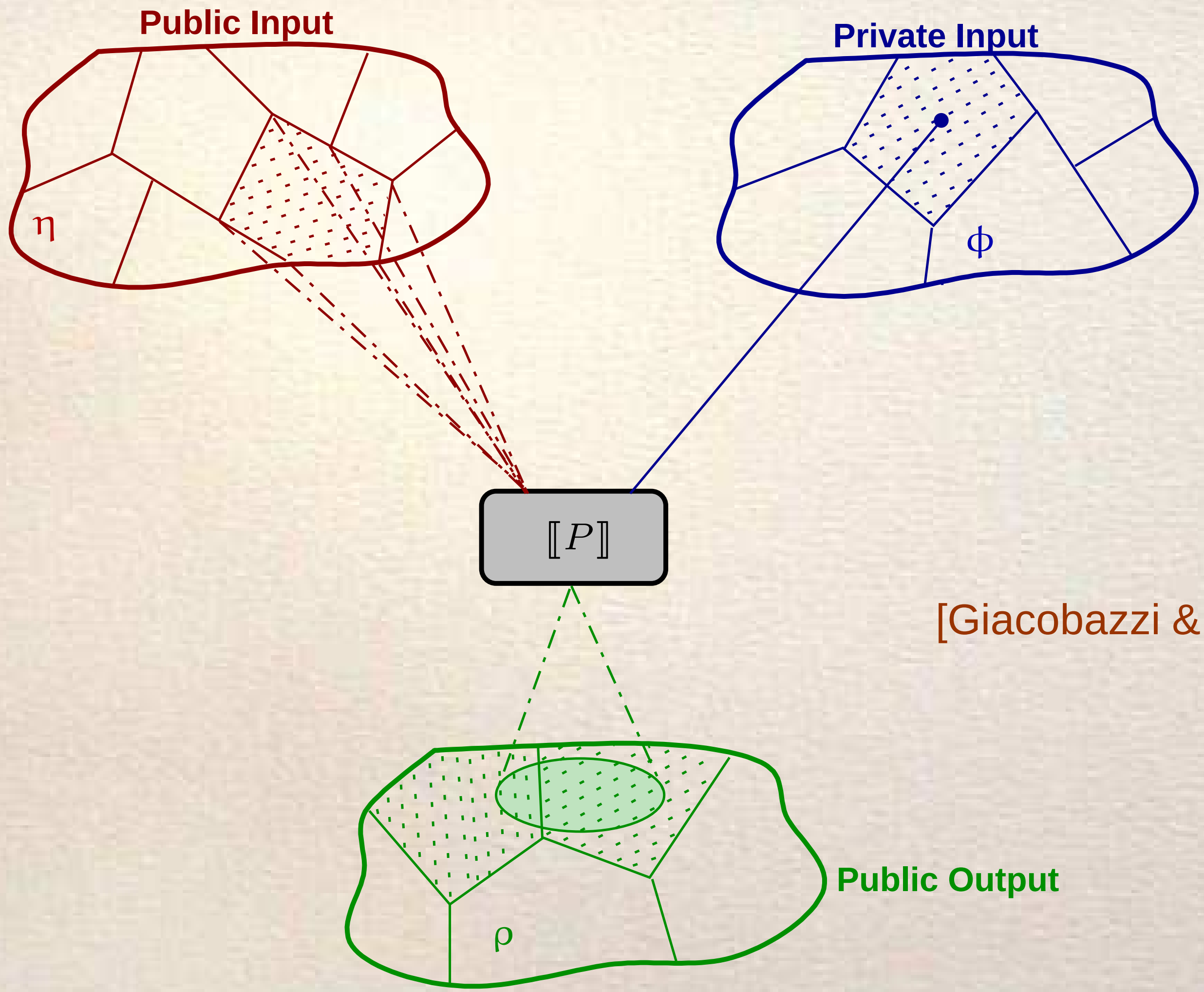
DECLASSIFIED ANI (VIA ALLOWING)



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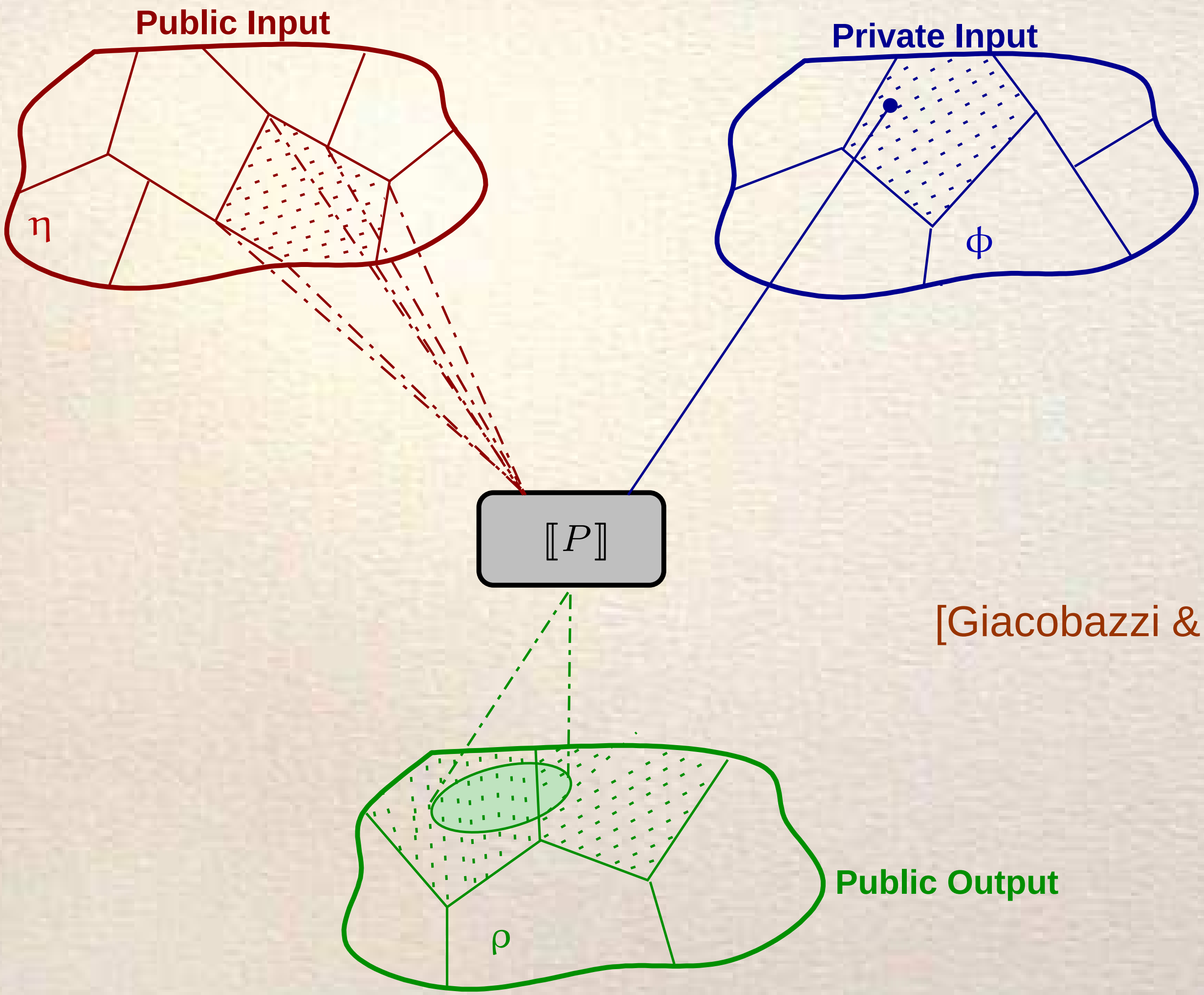

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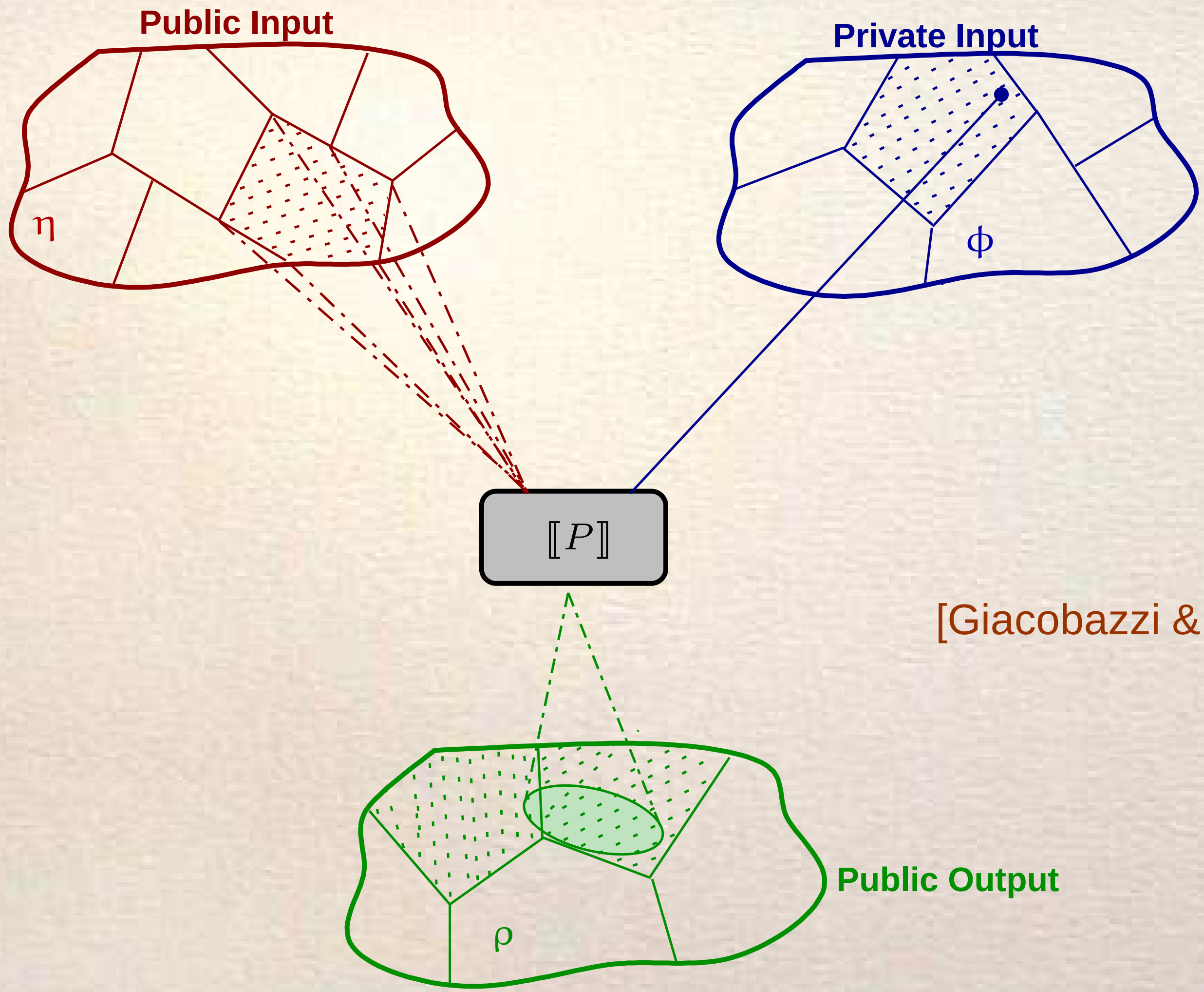

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Observer vs Observable

Modelling Attackers as Domain Transformers

Consider $\models (\eta)P(\phi \rightsquigarrow\!\!\rrbracket \rho)$: *In order to preserve non-interference...*

Observer vs Observable

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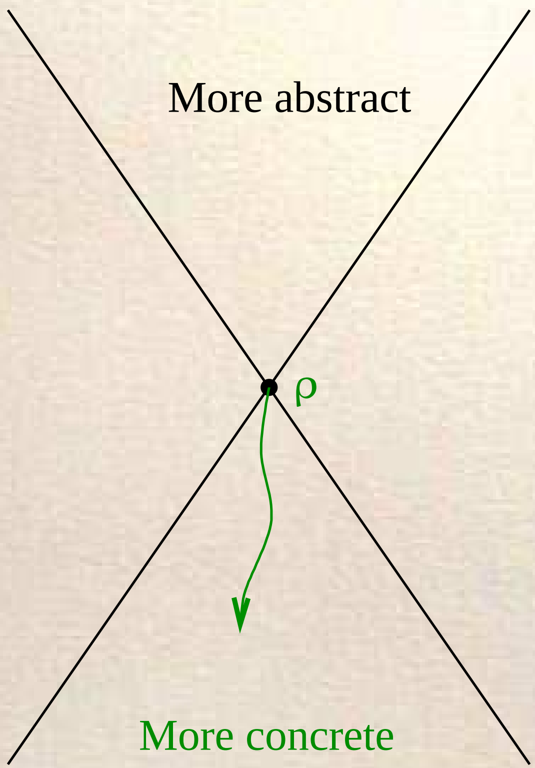
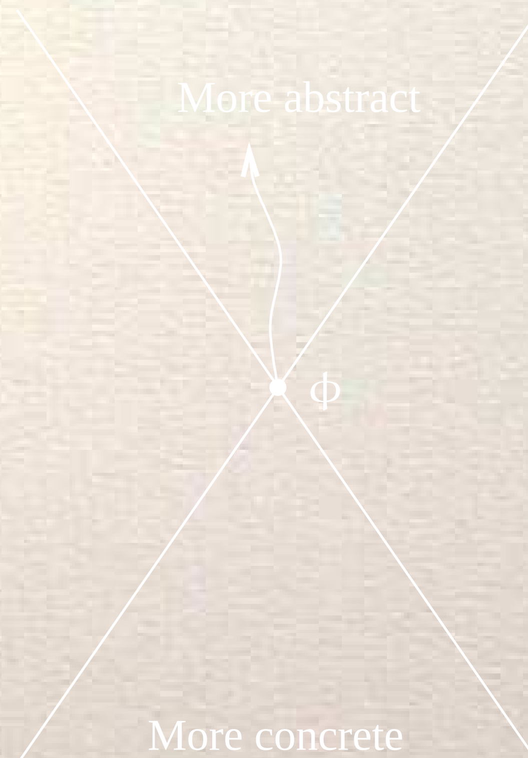


$uco(\wp(\mathbb{V}^{\mathsf{L}}))$

AND



$uco(\wp(\mathbb{V}^{\mathsf{H}}))$

Observer vs Observable

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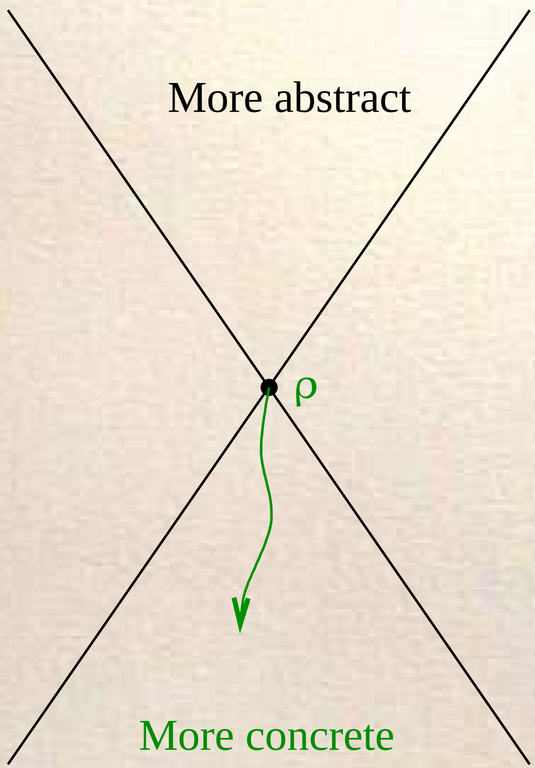

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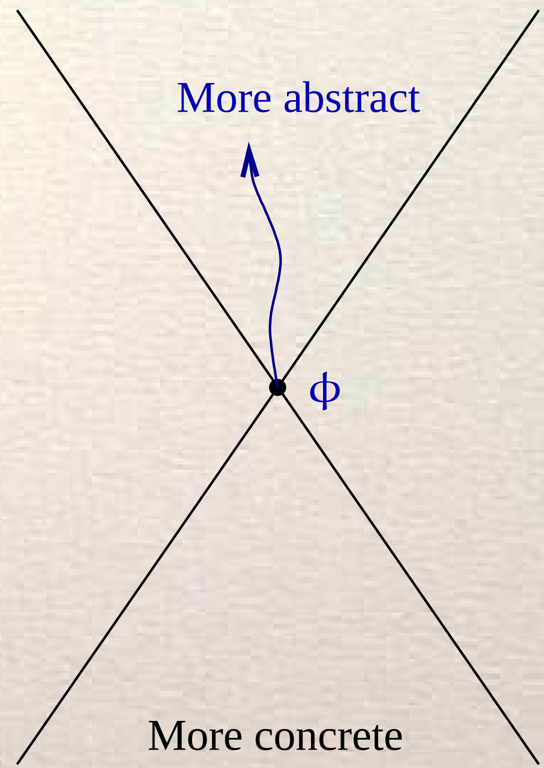

$uco(\wp(\mathbb{V}^{\mathsf{H}}))$

ANI AS COMPLETENESS

Let $\rho \in uco(\wp(\mathbb{V}^{\mathsf{L}})) \Rightarrow \mathcal{H}_\rho(X) \stackrel{\text{def}}{=} \langle \top^{\mathsf{H}}, \rho(X^{\mathsf{L}}) \rangle \in uco(\wp(\mathbb{V}))$

- *Narrow abstract non-interference:* $\mathcal{H}_\rho \circ [\![P]\!] \circ \mathcal{H}_\eta = \mathcal{H}_\rho \circ [\![P]\!]$;
- *Abstract non-interference:* $\mathcal{H}_\rho \circ [\![P]\!]^{\eta,\phi} \circ \mathcal{H}_\eta = \mathcal{H}_\rho \circ [\![P]\!]^{\eta,\phi}$

ANI AS COMPLETENESS

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$$\Downarrow$$

- PUBLIC OBSERVER AS COMPLETENESS CORE:
  $(\eta)P(\phi \rightsquigarrow\!\!\rrbracket \mathcal{C}^{\mathcal{H}_\eta}_{[\![P]\!]^{\eta,\phi}}(\mathcal{H}))$

ANI AS COMPLETENESS

Let $\rho \in uco(\wp(\mathbb{V}^{\mathsf{L}})) \Rightarrow \mathcal{H}_\rho(X) \stackrel{\mathsf{def}}{=} \langle \top^{\mathsf{H}}, \rho(X^{\mathsf{L}}) \rangle \in uco(\wp(\mathbb{V}))$

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$$\Downarrow$$

- PUBLIC OBSERVER AS FORWARD COMPLETENESS CORE:
  $(\eta)P(\phi \rightsquigarrow\!\!\!]\, \mathcal{C}^{\mathcal{H}_\eta}_{[\![P]\!]^{\eta,\phi}}(\mathcal{H}))$

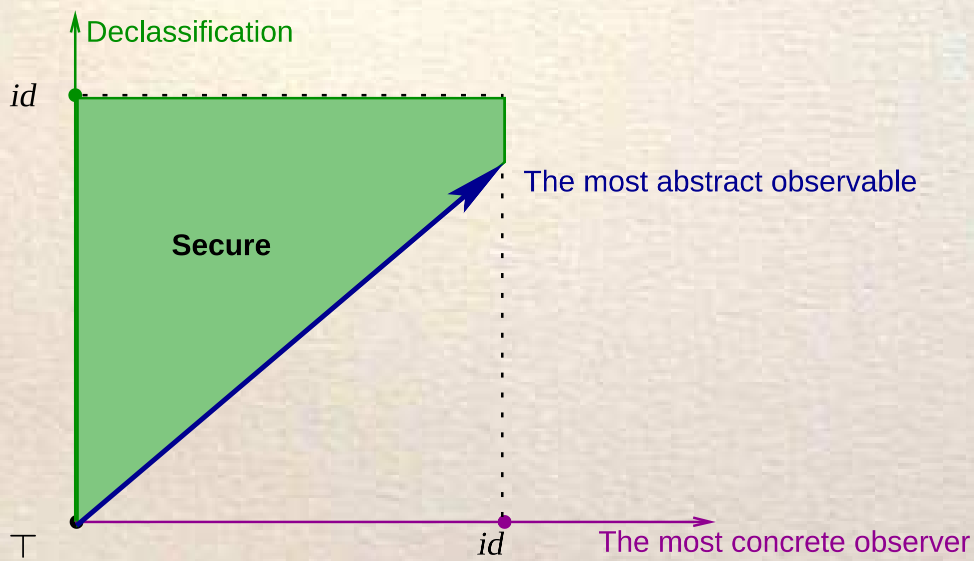
Strongest harmless attacker

- PRIVATE OBSERVABLE AS FORWARD COMPLETENESS SHELL:
  $(\eta)P(\mathcal{R}^{\mathcal{H}_\rho}_{[\![P]\!]^{\eta,id}}(\mathcal{H}_\eta) \Rightarrow \rho)$

Maximal information released

ANI AS COMPLETENESS

- ADJOINING ATTACKERS AND DECLASSIFICATION BY COMPLETENESS



DECLASSIFICATION

[Banerjee, Giacobazzi and Mastroeni ’07]

- By exploiting the strong relation between completeness and non-iterference we can obtain the following results:
  - Model declassification as a forward completeness problem for the weakest precondition semantics;
  - Derive counterexamples to a given declassification policy;
  - Refine a given declassification policy (Shell);

DNI: A COMPLETENESS PROBLEM

Let $\mathcal{H}^{\phi}$ the abstract domain declassifying the property ϕ of the private *input*:

$$\boxed{\mathcal{H}\circ[\![P]\!]\circ\mathcal{H}^{\phi} = \mathcal{H}\circ[\![P]\!] \ \Leftrightarrow\ \mathcal{H}^{\phi}\circ Wlp_P\circ\mathcal{H} = Wlp_P\circ\mathcal{H}}$$

$$\Downarrow$$

To release ϕ *means* to distinguish between elements in ϕ!

DNI: A COMPLETENESS PROBLEM

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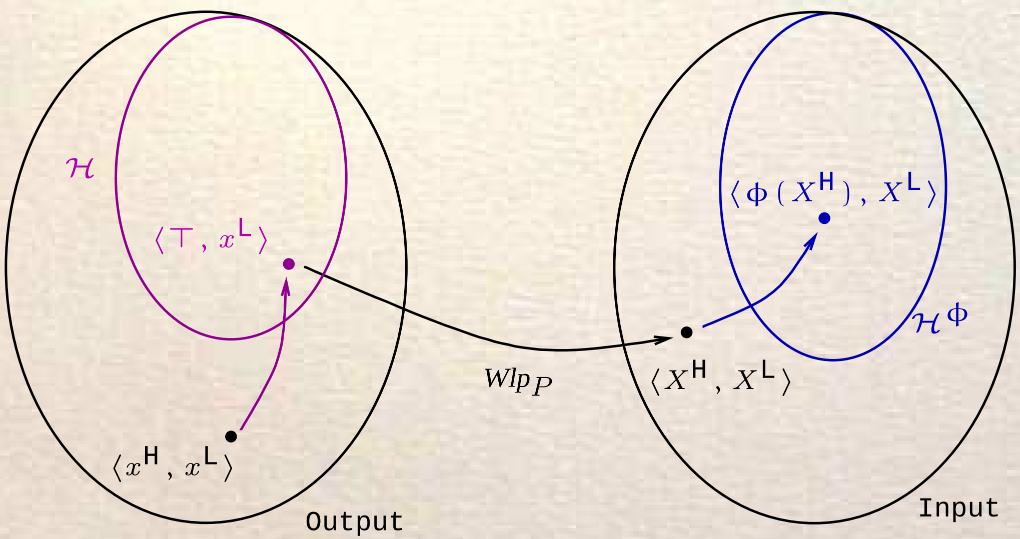

DNI: A completeness problem

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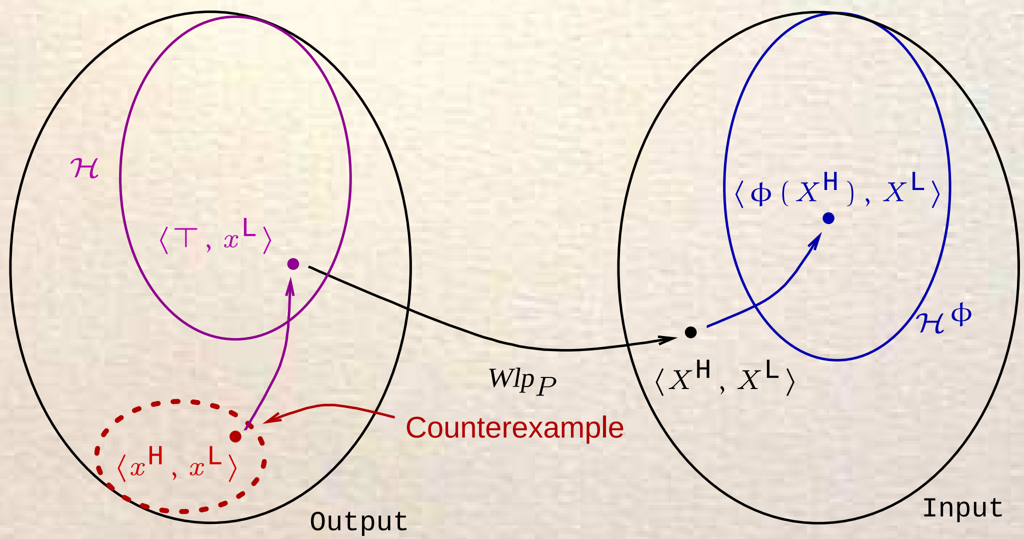

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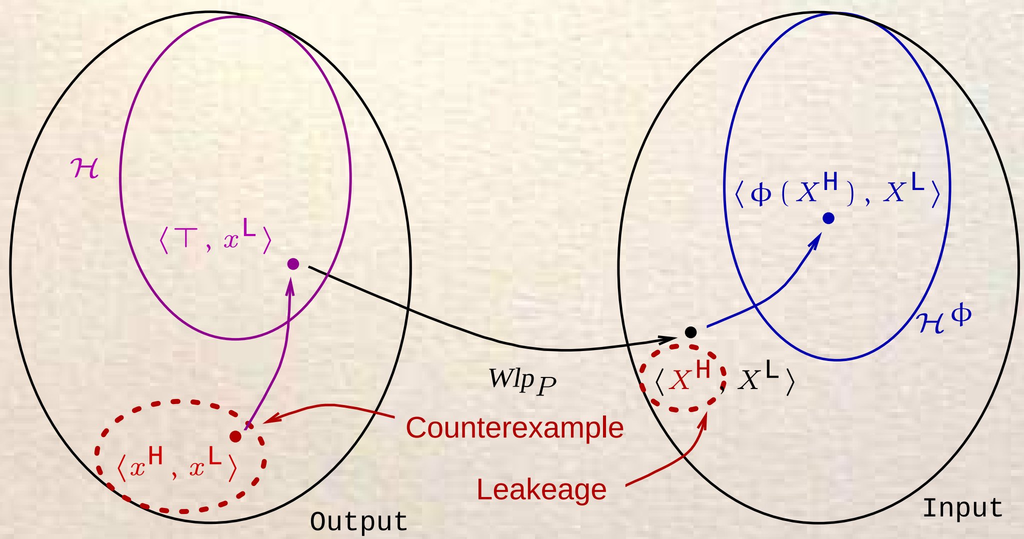

DNI: A COMPLETENESS PROBLEM

Let $\mathcal{H}^{\phi}$ the abstract domain declassifying the property ϕ of the private *input*:

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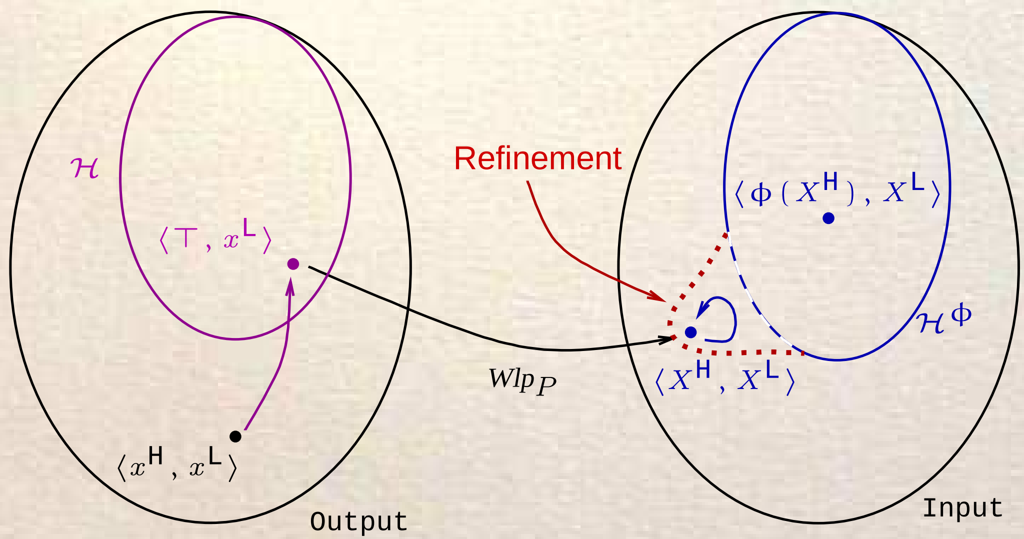

SHELL: THE MAXIMAL RELEASED INFORMATION

Consider $\rho =$Parity$\stackrel{\text{def}}{=}\{\top, \mathit{Even}, \mathit{Odd}, \varnothing\}$, as the information observed by the attacker.

$$P = \Big[\qquad l := l * h^2;$$

Shell: The maximal released information

Consider $\rho = \text{Parity} \stackrel{\text{def}}{=} \{\top, \mathit{Even}, \mathit{Odd}, \varnothing\}$, as the information observed by the attacker.

$$\begin{array}{ll} & (l \in \mathit{Even} \;\vee\; (l \in \mathit{Odd},\; h \in \mathit{Even})) \\ l := l * h^2; & \\ & (l \in \mathit{Even}) \end{array} \qquad \text{OR} \qquad \begin{array}{ll} & (l \in \mathit{Odd} \;\wedge\; h \in \mathit{Odd}) \\ l := l * h^2; & \\ & (l \in \mathit{Odd}) \end{array}$$

Let $l = 3$, $h = 2 \in \mathit{Even}$:

$$\mathcal{H}_{Par}[\![P]\!](\langle 2, 3\rangle) = \langle \top, \mathit{Even}\rangle \neq \langle \top, \top\rangle = \mathcal{H}_{Par}[\![P]\!](\langle \top, 3\rangle) = \mathcal{H}_{Par}[\![P]\!](\mathcal{H}(\langle 2, 3\rangle))$$

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$$
\begin{array}{lll}
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l := l * h^2; & \text{OR} & l := l * h^2; \\
\quad (l \in \mathit{Even}) & & \quad (l \in \mathit{Odd})
\end{array}
$$

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WE RELEASE SOMETHING ABOUT THE PRIVATE INPUT!

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Let us compute the shell of the input domain $\mathcal{H}$:

$$\mathcal{H}' \stackrel{\text{def}}{=} \mathcal{R}^{\mathcal{H}_{Par}}_{[\![P]\!]}(\mathcal{H}) = \mathcal{H} \sqcap (\langle \top, \mathit{Even} \rangle \cup \langle \mathit{Even}, \mathit{Odd} \rangle, \langle \mathit{Odd}, \mathit{Odd} \rangle, \langle \mathit{Odd}, \mathit{Even} \rangle)$$

Shell: The maximal released information

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Hence (NB: By reduced product in $\mathcal{H}'$ we have the elements $\langle \mathit{Even}, l \rangle$)

Let $l = 3$, $h = 2 \in \mathit{Even}$:

$$\mathcal{H}_{Par}[\![P]\!](\langle 2, 3 \rangle) = \langle \top, \mathit{Even} \rangle = \mathcal{H}_{Par}[\![P]\!](\langle \mathit{Even}, 3 \rangle) = \mathcal{H}_{Par}[\![P]\!](\mathcal{H}'(\langle 2, 3 \rangle))$$

CORE: THE MOST POWERFUL ATTACKER

$$P = \Big[\quad \textbf{while } (h \neq 0) \textbf{ do } (h := 0; l := 2l) \textbf{ endw}$$

CORE: THE MOST POWERFUL ATTACKER

$((l \in Even, h = 0) \ \vee \ (h \neq 0))$

while $(h \neq 0)$ **do** $(h := 0; l := 2l)$ **endw**;

$(l \in Even)$

OR

$(h = 0)$

while $(h \neq 0)$ **do** $(h := 0; l := 2l)$ **endw**

$(l \in Odd)$

Let $l = 5$, $h = 3$:

$$\mathcal{H}[\![P]\!](\langle 3, 5\rangle) = \langle \top, 10\rangle \neq \langle \top, \top\rangle = \mathcal{H}[\![P]\!](\langle \top, 5\rangle) = \mathcal{H}[\![P]\!](\mathcal{H}(\langle 3, 5\rangle))$$

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$((l \in \mathit{Even}, h = 0) \ \vee \ (h \neq 0))$
while $(h \neq 0)$ **do** $(h := 0; l := 2l)$ **endw**;
$(l \in \mathit{Even})$

OR

$(h = 0)$
while $(h \neq 0)$ **do** $(h := 0; l := 2l)$ **endw**
$(l \in \mathit{Odd})$

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Let us compute the core of the output domain $\mathcal{H}$:

$$\mathcal{H}' \stackrel{\text{def}}{=} \mathcal{C}^{\mathcal{H}}_{[\![P]\!]}(\mathcal{H}) = \left\{ \langle \top, L \rangle \;\middle|\; \forall l \in \top.\, l \in L \Leftrightarrow 2l \in L \right\} = \curlyvee \left(\left\{ n\{2\}^{\mathbb{N}} \;\middle|\; n \in \mathit{Odd} \right\} \right)$$

Core: The most powerful attacker

$$\begin{array}{ll} ((l \in Even, h = 0) \ \vee \ (h \neq 0)) & (h = 0) \\ \textbf{while } (h \neq 0) \textbf{ do } (h := 0; l := 2l) \textbf{ endw}; \quad \text{OR} & \textbf{while } (h \neq 0) \textbf{ do } (h := 0; l := 2l) \textbf{ endw} \\ (l \in Even) & (l \in Odd) \end{array}$$

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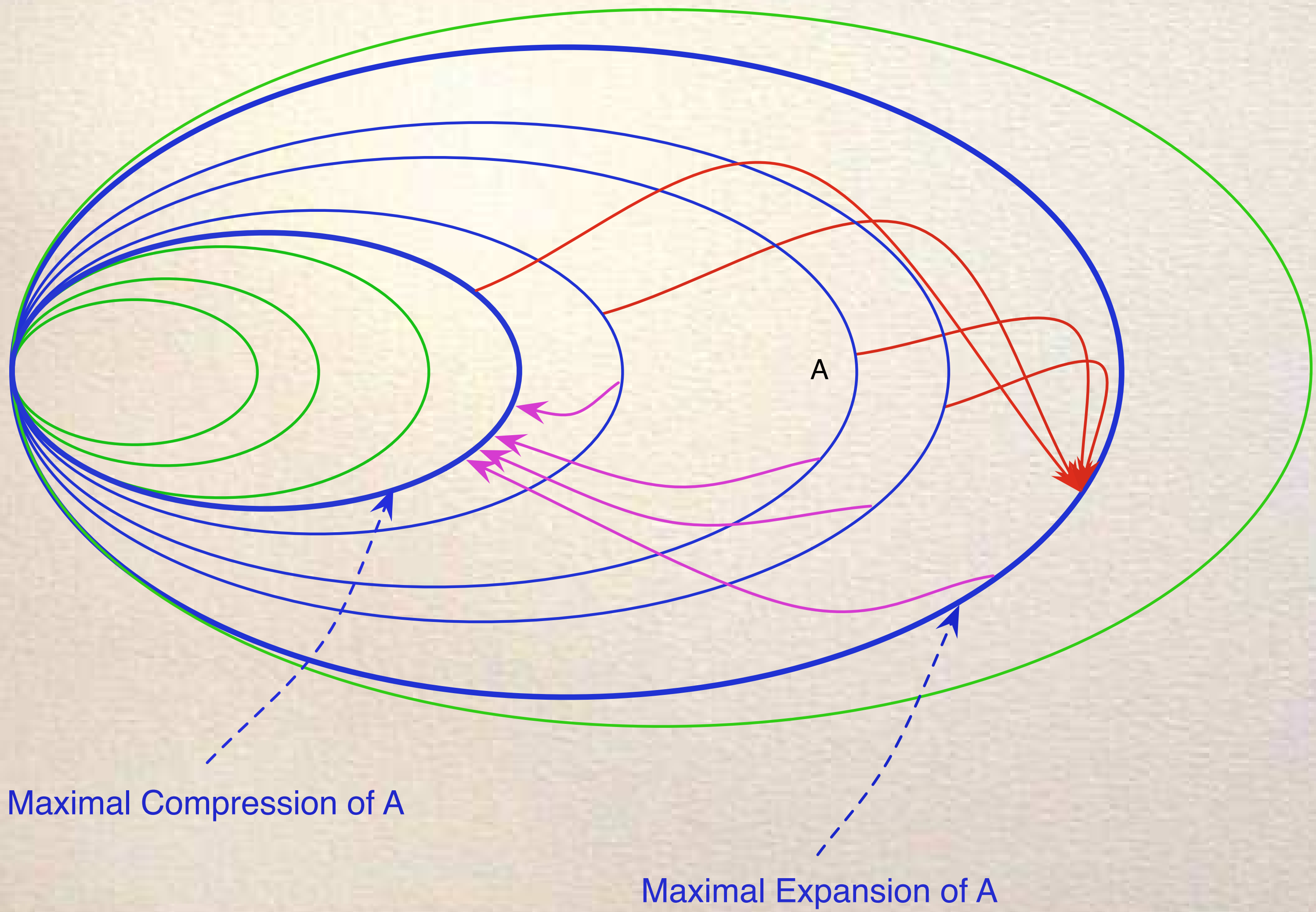
Hence

Let $l = 5$, $h = 3 \in Even$:

$$\mathcal{H}'[\![P]\!](\langle 3, 5 \rangle) = \mathcal{H}'(\langle \top, 10 \rangle) = \langle \top, 5\{2\}^{\mathbb{N}} \rangle = \mathcal{H}'(\{5, 10\}) = \mathcal{H}'[\![P]\!](\langle \top, 5 \rangle) = \mathcal{H}'[\![P]\!](\mathcal{H}(\langle 3, 5 \rangle))$$

CAN WE EXPAND AND COMPRESS DOMAINS?

THE GEOMETRY OF DOMAIN TRANSFORMERS



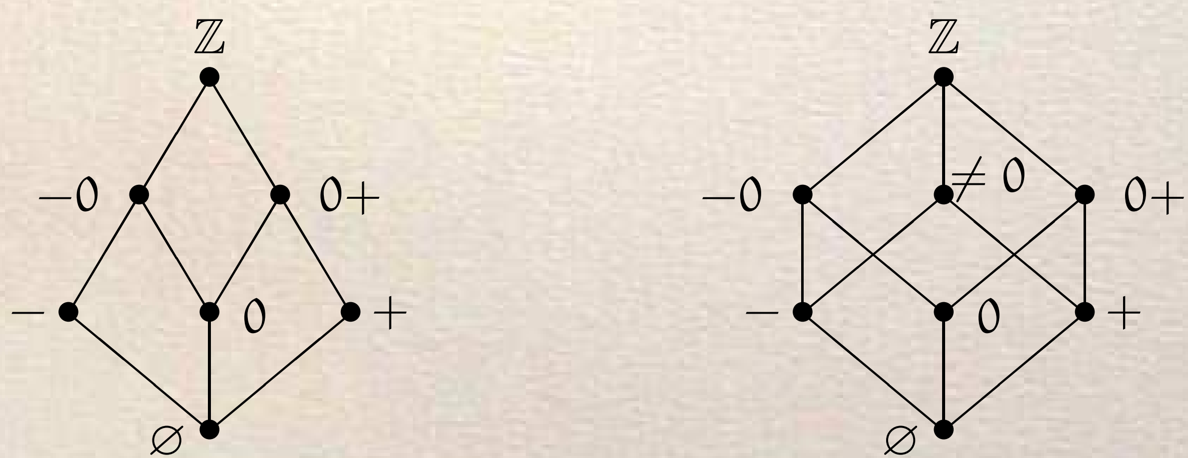
THE GEOMETRY OF DOMAIN TRANSFORMERS

DISJUNCTIVE COMPLETION

- Refinement: Forward Completeness for disjunction

$$\mathcal{R}(X) = \left\{ \bigvee Y \;\middle|\; Y \subseteq X \right\} \qquad \text{one step}$$

The least $X = \curlyvee(A)$: $X = A \sqcap \mathcal{R}(X)$ Disjunctive Completion

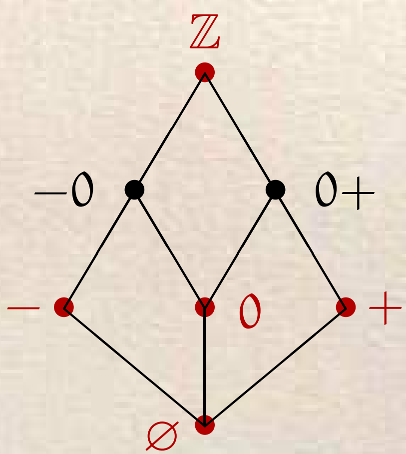


Sign $\qquad\qquad \curlyvee(\mathit{Sign})$

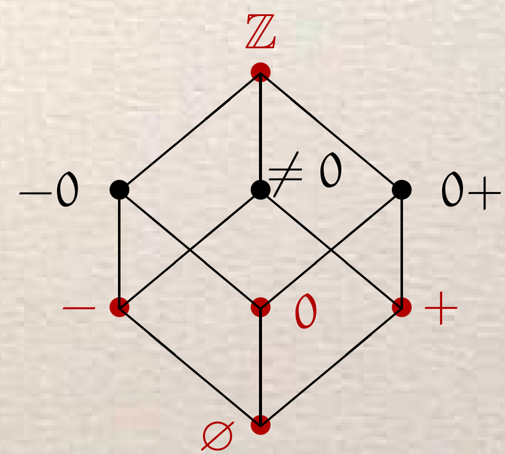
THE GEOMETRY OF DOMAIN TRANSFORMERS

DISJUNCTIVE COMPLETION

- Compressor: The domain of Join-Irreducible elements of X

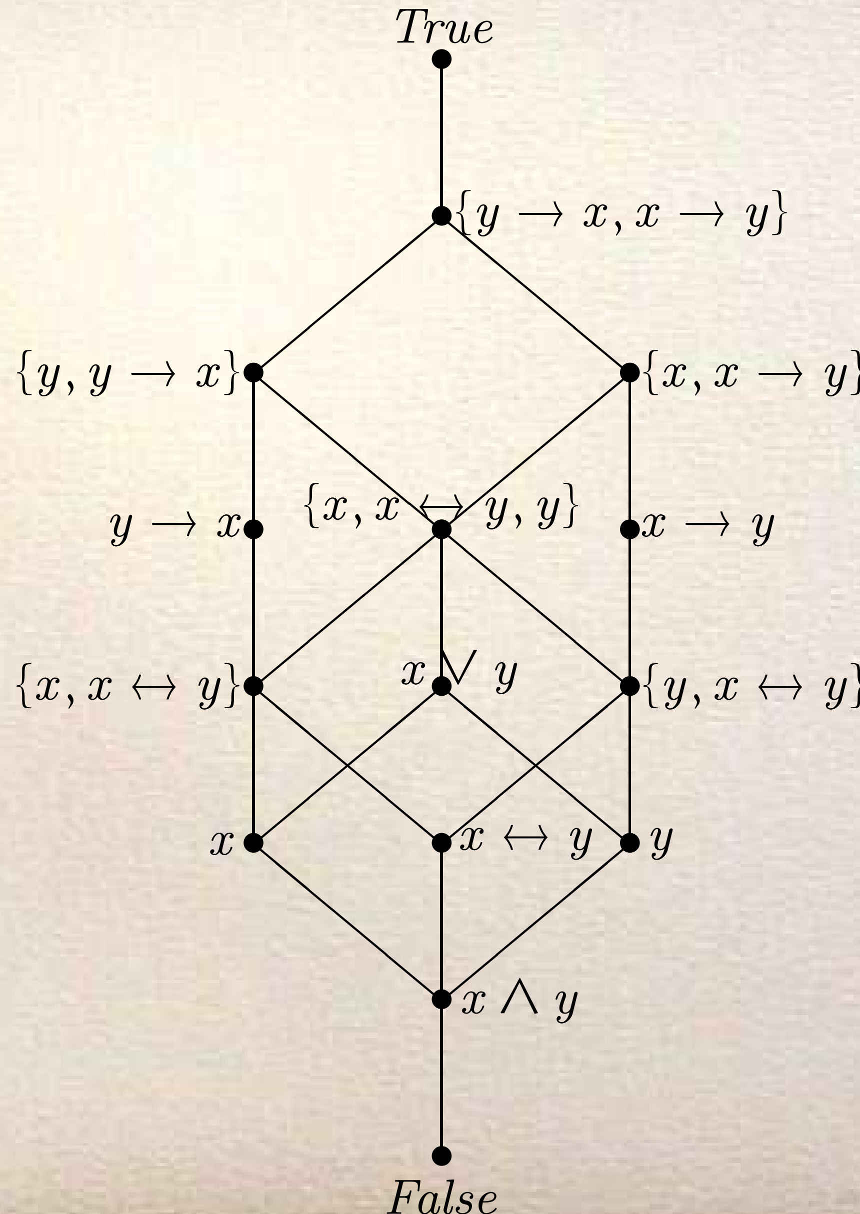


Sign

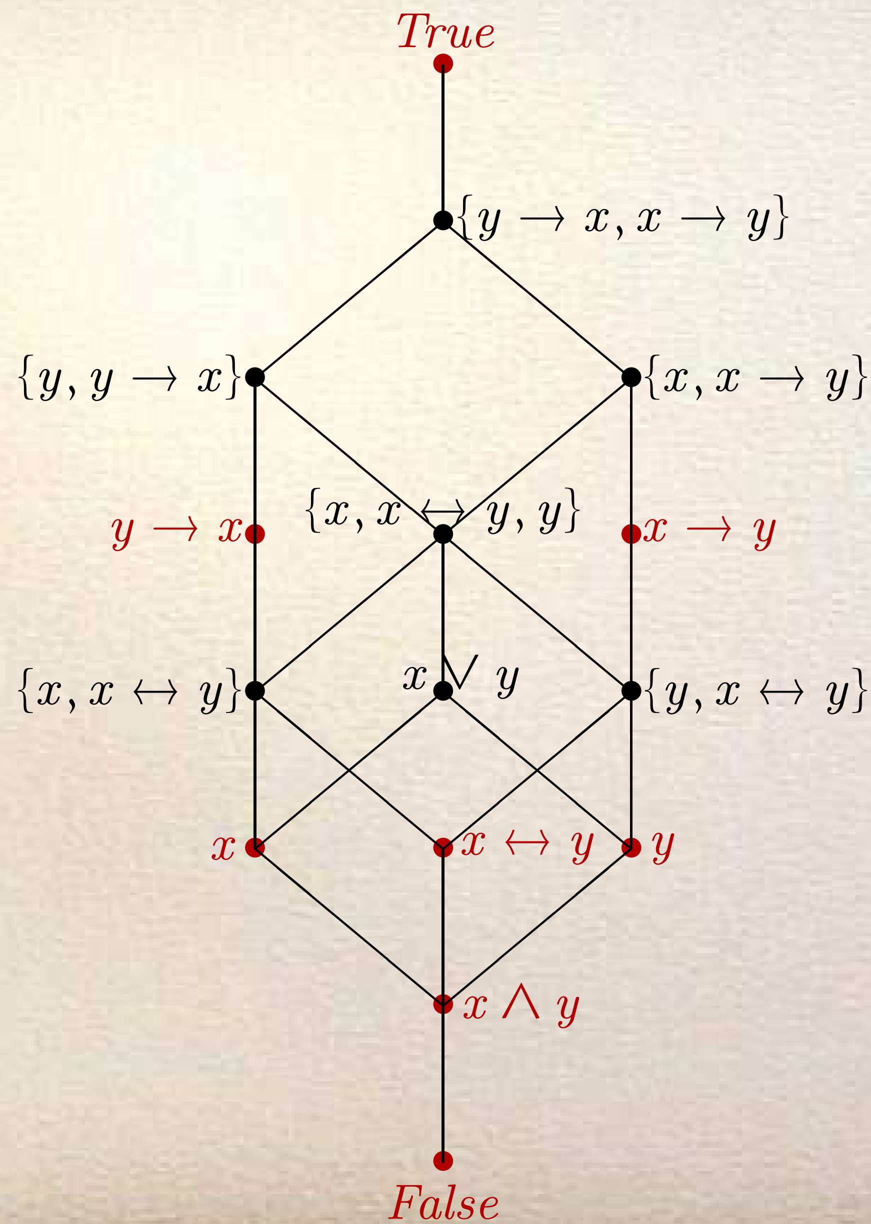


$\curlyvee(Sign)$

THE GEOMETRY OF DOMAIN TRANSFORMERS



THE GEOMETRY OF DOMAIN TRANSFORMERS



THE EXPANDER IN ANI

Let $l := 2h$ and $\mathcal{E}(X) = \bigsqcap \left\{ \, Y \, \middle| \, \mathcal{C}(X) = \mathcal{C}(Y) \, \right\}$.

- The most powerful harmless attacker for P is: $\mathcal{H}' = \curlyvee(\{Even, \{1\}, \{3\}, \ldots\})$

- Suppose the initial observer is $\rho = \{\top, Even \smallsetminus \{0\}, \{0\}, Odd, \varnothing\}$, then the most powerful harmless attacker more abstract than ρ is $Par = \mathcal{H}' \sqcup \rho$.

- The expander provides the most powerful attacker such that the <u>harmless</u> simplification is Par: $\curlyvee(\{Odd, \{0\}, \{2\}, \{4\}, \ldots\})$;

- WE OBTAIN THE MOST POWERFUL <u>MALICIOUS</u> ATTACKER, I.E., THE ONE THAT IS ABLE TO EXPLOIT AS MUCH AS POSSIBLE THE FAILURE OF NON-INTERFERENCE!

- Any more abstract (less powerful) attacker has to confuse some even inputs, for instance if it confuses $l = 0$ with $l = 2$ then it can not distinguish when $h = 0$ and $h = 1$.

THE COMPRESSOR IN ANI

Let **if** $h = 0$ **then** $l := 0$ **else** $l := |l|(h/|h|)$ and $\mathcal{E}(X) = \bigsqcup\left\{ Y \;\middle|\; \mathcal{R}(X) = \mathcal{R}(Y) \right\}$.

- Suppose we let to flow $\phi = \{\top, \geq 0, < 0, \varnothing\}$;
- The maximal information released by P, is the shell of ϕ:
  $\phi' = \{\top, \geq 0, \neq 0, \leq 0, < 0, > 0, 0, \varnothing\}$
- THE COMPRESSOR PROVIDES THE MOST ABSTRACT DECLASSIFICATION POLICY WHICH CANNOT CAPTURE WHAT IS RELEASED BY AN ATTACKER
- The compressor is $\lambda X.\, \top$
- This means that each policy between ϕ' and $\lambda X.\, \top$ is not able to protect the program.

THE COMPRESSOR IN ANI

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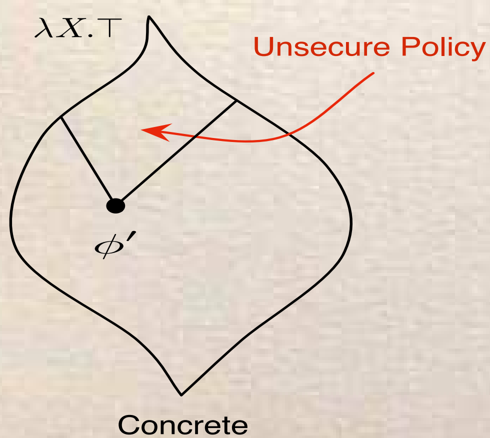


Can we make completeness by transforming semantics?

THE GEOMETRY OF SEMANTICS TRANSFORMERS

MAKING SEMANTICS COMPLETE (FROM ABOVE AND BELOW):

$$\mathbb{F}^{\uparrow}_{\eta,\rho}(f) = \bigsqcap\{h : C \longrightarrow C \mid f \sqsubseteq h,\ \rho \circ h \circ \eta = h \circ \eta\}$$
$$\mathbb{F}^{\downarrow}_{\eta,\rho}(f) = \bigsqcup\{h : C \longrightarrow C \mid f \sqsupseteq h,\ \rho \circ h \circ \eta = h \circ \eta\}$$

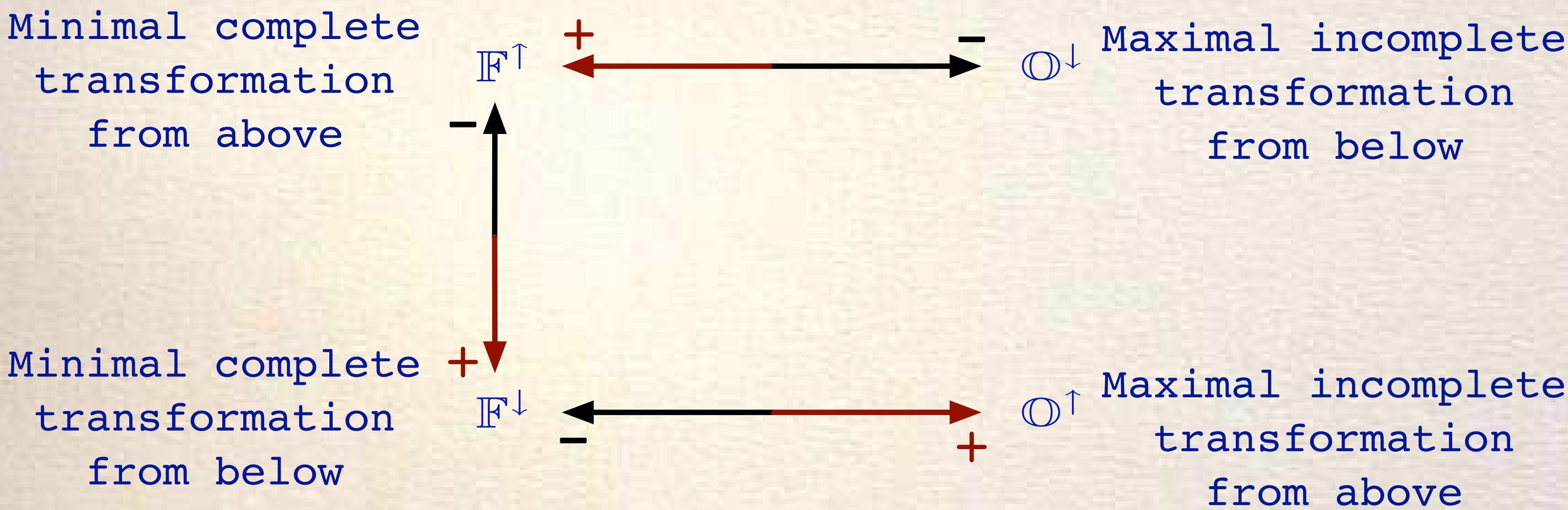
$\mathbb{F}^{\uparrow}_{\eta,\rho}(f)$ and $\mathbb{F}^{\downarrow}_{\eta,\rho}(f)$ are (Forward) complete

MAKING SEMANTICS MAXIMALLY IN-COMPLETE (FROM ABOVE AND BELOW):

$$\mathbb{O}^{\uparrow}_{\eta,\rho}(f) = \bigsqcup\{g : C \longrightarrow C \mid \mathbb{F}^{\downarrow}_{\eta,\rho}(g) = \mathbb{F}^{\downarrow}_{\eta,\rho}(f)\}$$
$$\mathbb{O}^{\downarrow}_{\eta,\rho}(f) = \bigsqcap\{g : C \longrightarrow C \mid \mathbb{F}^{\uparrow}_{\eta,\rho}(g) = \mathbb{F}^{\uparrow}_{\eta,\rho}(f)\}$$

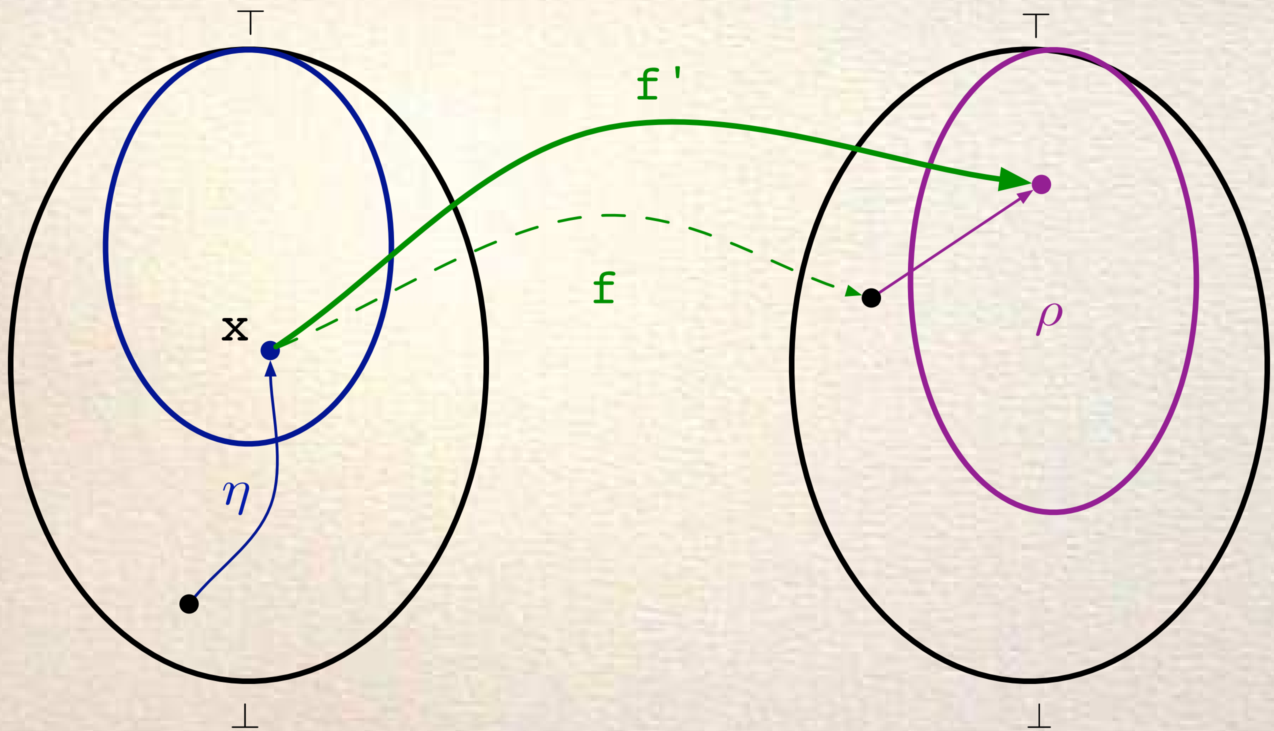
$\mathbb{O}^{\uparrow}_{\eta,\rho}(f)$ and $\mathbb{O}^{\downarrow}_{\eta,\rho}(f)$ are generally in-complete

THE GEOMETRY OF SEMANTICS TRANSFORMERS



$(\mathbb{F}^{\uparrow})^{+} = \mathbb{F}^{\downarrow}$ and $(\mathbb{F}^{\uparrow})^{-} = \mathbb{O}^{\downarrow}$

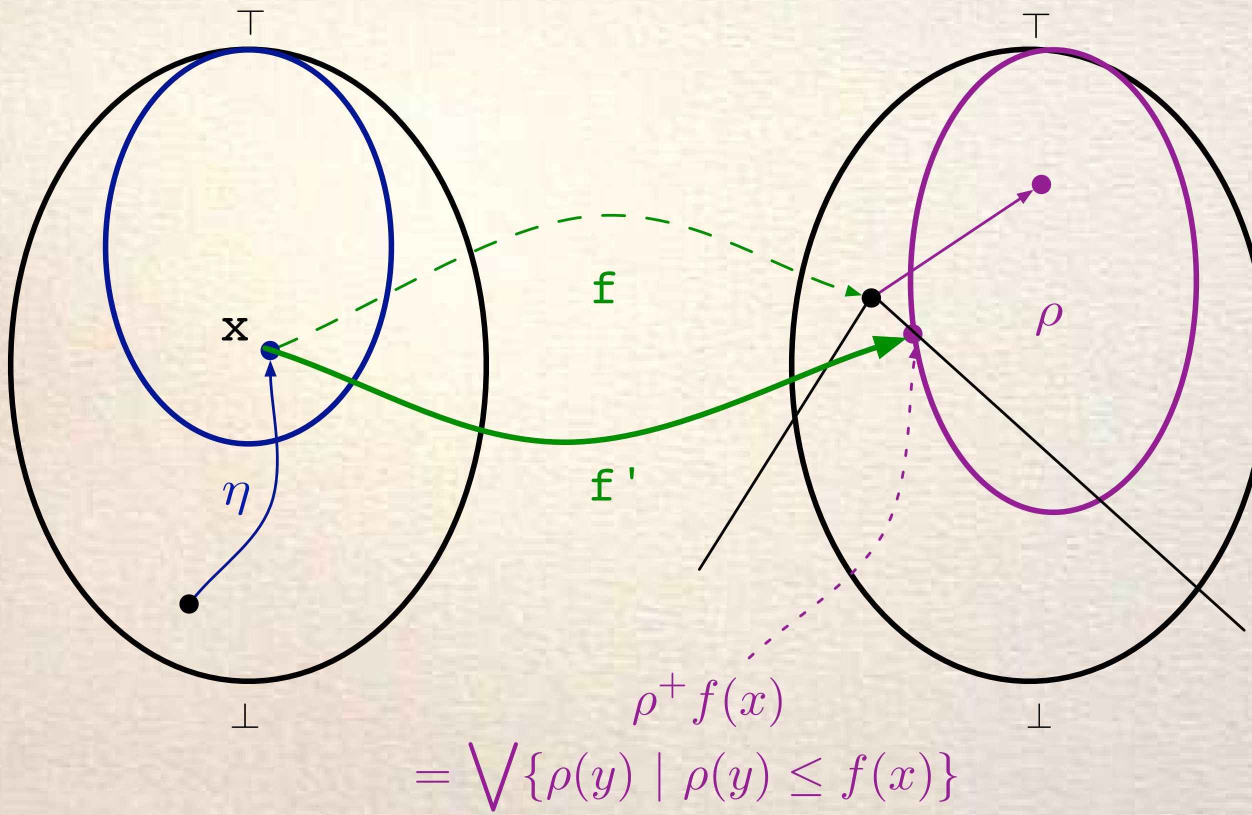
THE GEOMETRY OF SEMANTICS TRANSFORMERS



Making FORWARD COMPLETENESS: Transforming the semantics upwards

$$\mathbb{F}^{\uparrow}_{\eta,\rho} = \lambda f.\lambda x. \begin{cases} \rho \circ f(x) & \text{if } x \in \eta(C) \\ f(x) & \text{otherwise} \end{cases}$$

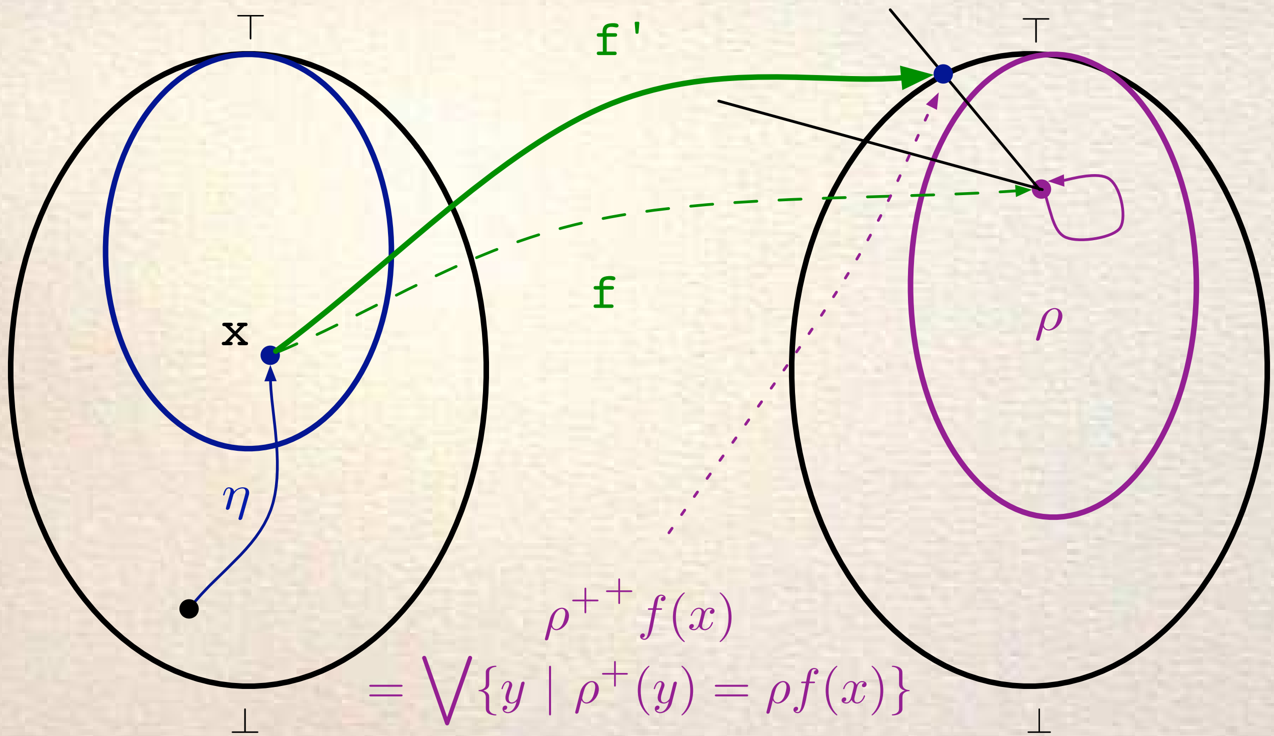
The Geometry of Semantics Transformers



Making FORWARD COMPLETENESS: Transforming the semantics downwards

$$\mathbb{F}^{\downarrow}_{\eta,\rho} = \lambda f.\lambda x. \begin{cases} \rho^{+} \circ f(x) & \text{if } x \in \eta(C) \\ f(x) & \text{otherwise} \end{cases}$$

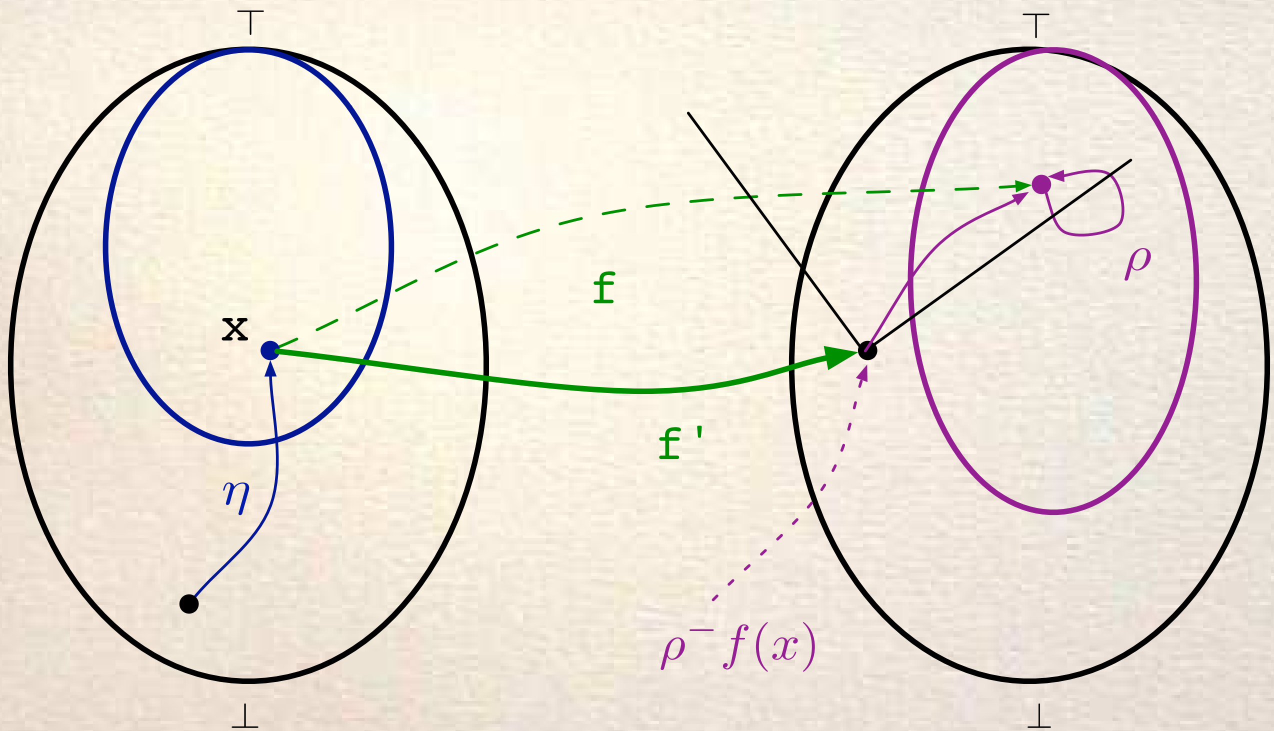
The Geometry of Semantics Transformers



Making FORWARD IN-COMPLETENESS: Transforming the semantics upwards

$$\mathbb{O}^{\uparrow}_{\eta,\rho}(f)(x) = \begin{cases} (\rho^{+})^{+}(f(x)) = \bigvee \left\{ y \;\middle|\; \rho^{+}(y) = \rho^{+}(f(x)) \right\} & \text{if } x \in \eta \\ f(x) & \text{otherwise} \end{cases}$$

The Geometry of Semantics Transformers



Making FORWARD IN-COMPLETENESS: Transforming the semantics downwards

$$\mathbb{O}^{\downarrow}_{\eta,\rho}(f)(x) = \begin{cases} \rho^{-}(f(x)) = \bigwedge \left\{ y \;\middle|\; \rho(y) = \rho(f(x)) \right\} & \text{if } x \in \eta \\ f(x) & \text{otherwise} \end{cases}$$

MAKING SEMANTICS COMPLETE: AN EXAMPLE

$$\textbf{while } (h > 0) \textbf{ do } (h := h - 1; l := h) \textbf{ endw}$$

Making semantics complete: An Example

$(h > 0) \;\vee\; (l = 0)$

while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**;

$(l = 0)$

Or

$(h = 0)$

while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**

$(l \neq 0)$

Let $l = 5$, $h_1 = 3$, $h_2 = 0$:

$\mathcal{H}[\![P]\!](\langle 3, 5\rangle) = \langle \top, 0\rangle \neq \langle \top, 5\rangle = \mathcal{H}[\![P]\!](\langle 0, 5\rangle)$

MAKING SEMANTICS COMPLETE: AN EXAMPLE

$$(h \geq 0)$$

while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**;

$$(l = 0)$$

OR

$$(h = 0)$$

while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**

$$(l \neq 0)$$

Let $l = 5$, $h_1 = 3$, $h_2 = 0$:

$$\mathcal{H}[\![P]\!](\langle 3, 5\rangle) = \langle \top, 0\rangle \neq \langle \top, 5\rangle = \mathcal{H}[\![P]\!](\langle 0, 5\rangle)$$

WE RELEASE SOMETHING (THE EQUALITY WITH 0) ABOUT THE PRIVATE INPUT!

MAKING SEMANTICS COMPLETE: AN EXAMPLE

$$(h \geq 0)$$
while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**;
$$(l = 0)$$

OR

$$(h = 0)$$
while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**
$$(l \neq 0)$$

The upward transformation inducing completeness of Wlp_P is:

$$\mathbb{F}^{\uparrow}(Wlp_P) : \{l = 0 \mapsto h \in \mathbb{Z} \text{ and } l \neq 0 \mapsto h \in \mathbb{Z}\}$$

MAKING SEMANTICS COMPLETE: AN EXAMPLE

$$(h \geq 0)$$

while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**;

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This is, for example, the semantics of the program

$$Q : \quad l_1 := l;\ P;\ l := l_1$$

DISCUSSION

- Encoding AI problems as completeness problems:
  - Systematic transformations for optimal models
  - Better understanding of the limits of abstractions
- Adequacy of the theory
  - Abstract interpretation is perfectly adequate to reason about itself
  - A calculational design of domain and code transformations can be done in abstract interpretation
  - Completeness is a driving force for understanding domain and code transformers
  - From semantics transformers to code transformations (and deformations) by AI [Cousot & Cousot '02]

Future directions

- Code obfuscation and sw watermarking
  - Completeness corresponds to maximal precision
  - Obfuscating P corresponds to make P maximally incomplete against a given attack ($\mathbb{O}$?)
  - Watermarks and fingerprints can be hidden in completeness holes
- Language-based security
  - $\mathbb{F}$ provides code protection against information release!
  - Can we design a monitor M such that $\mathbb{F}(\llbracket P \rrbracket) = \llbracket M; P \rrbracket$?
  - Models for active attackers as code transformations (code deformations)... and the corresponding completeness problem?
- Abstract Model Checking
  - Isolate temporal sub-logics which are complete for a given abstract system to analyse.

MANY THANKS!!