Local search algorithms

AIMA sections 4.1,4.2

Summary

- Hill-climbing
- Simulated annealing
- Genetic algorithms (briefly)
- Local search in continuous spaces (very briefly)

Iterative improvement algorithms

♢ In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

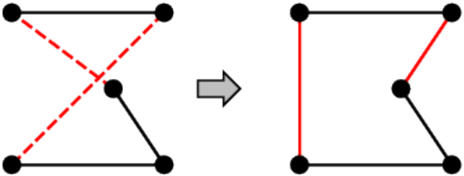
♢ Then state space = set of "complete" configurations;
find **optimal** configuration, e.g., TSP, etc.
or, find configuration satisfying constraints, e.g., n-Queens

♢ In such cases, can use iterative improvement algorithms;
keep a single "current" state, try to improve it

♢ Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

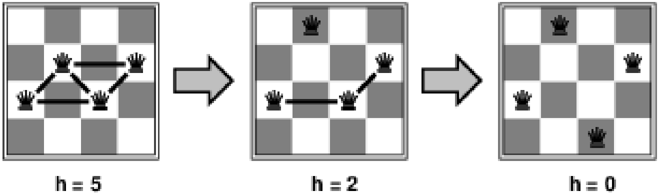


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n-queens

◇ Put n queens on an $n \times n$ board with no two queens on the same, row, column, or diagonal

◇ Move a queen to reduce number of conflicts



Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1 million$

Hill-climbing (or gradient ascent/descent)

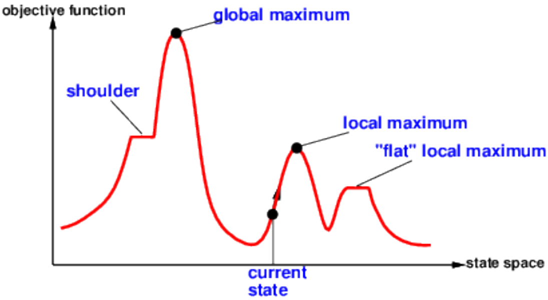
"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing( problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← Make-Node(problem.Initial-State)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbour.Value ≤ current.Value then return
            current.State
        end if
        current ← neighbor
    end
```

Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves 😊 escape from shoulders ☹ loop on flat maxima

Simulated Annealing

- Inspired by statistical mechanics
- Idea: escape local maxima by allowing some “bad” moves, **but gradually decrease their frequency**
- Allow more random moves at the beginning
  - we can reach zones with better solutions
- Diminish probability of having a random move towards the end
  - refine search around a good solution

Simulated annealing (pseudo-code)

```
function Simulated-Annealing( problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← Make-Node(problem.Initial-State)
    for t ← 1 to ∞ do
        T ← schedule(t)
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← next.Value – current.Value
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(Δ E/T)
```

Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state x^*
because $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T
Is this necessarily an interesting guarantee??
$\diamondsuit$ Devised by Metropolis et al., 1953, for physical process modelling
$\diamondsuit$ Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!

Searches that find good states recruit other searches to join them

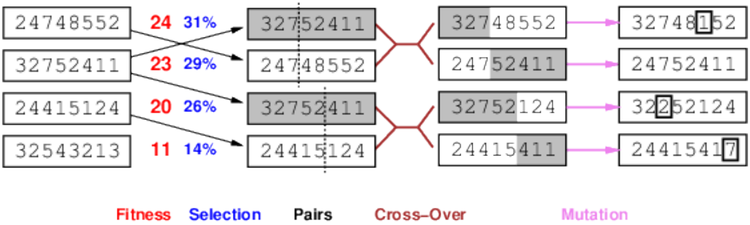
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

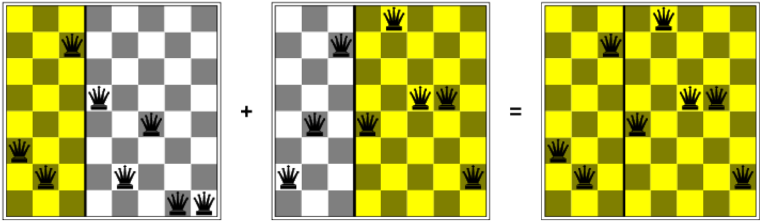
Genetic algorithms

= stochastic local beam search + generate successors from **pairs** of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)
Crossover helps **iff substrings are meaningful components**



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three airports in Romania:
– 6-D state space defined by (x_1, y_1), (x_2, y_2), (x_3, y_3)
– objective function $f(x_1, y_1, x_2, y_2, x_3, y_3) =$
sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space,
e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).

Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x}) \nabla f(\mathbf{x})$ to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$

Exercise: Local Search for the 4-Queens problem

Consider the 4-Queens problem. Assume the evaluation function is the number of pairs of queens that attack each other. Assume initial state is (1234)

- What is the current score for the initial state
- Write down the values of all successor states for this initial state
- Implement a simple program that computes the next best state(s) for your hill-climbing approach
- Trace a possible execution of a (deterministic) hill-climbing approach
- Comment on optimality of final state
- sol: eserc1LocalSearch.m

Exercise: Local Beam Search for the 4-Queens problem

Consider the 4-Queens problem and the deterministic hill climbing approach described above. Assume $k = 3$ and initial states are: $(1234), (2222), (3333)$.

- Trace execution of a **parallel** search
  sol: eserc2Parallel.m
- Trace the execution of a **beam** search
  sol: eserc2BeamSearch.m