Exercises

Bucket Elimination

02 May 2012, Exercise 2 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{X_1, X_2, X_3, x_4\}$, Domains, $D_1 = D_2 = D_4 = \{R, B\}$, $D_3 = \{G, B\}$, Constraints $C_h = \{R_{12}, R_{13}, R_{23}, R_{24}\}$ and $C_s = \{F_1(x_1), F_2(x_2), F_3(x_3), F_4(x_4)\}$. Where each R_{ij} is an inequality constraint (i.e., $R_{ij} = \{< R, B >< B, R >\}$) and $F_i(x_i)$ is of the following form:

$$F_i(x_i) = \begin{cases} 1 & \text{if } x_i = B \\ 0 & \text{otherwise} \end{cases}$$

Provide a solution for this cost network using Bucket Elimination. Use the ordering $o = \{x_4, x_2, x_1, x_3\}$.

29 Sept 2015, Exercise 3 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{x_1, x_2, x_3, x_4\}$. Constraints $C_h = \{\}$ and $C_s = \{F_{12}(x_1, x_2), F_{13}(x_1, x_3), F_{14}(x_1, x_4), F_{23}(x_2, x_3), F_{34}(x_3, x_4)\}$ and $D_1 = D_2 = D_3 = D_4 = \{0, 1\}$. Consider the Bucket Elimination algorithm and the variable ordering $o = \{x_2, x_1, x_4, x_3\}$. Answer the following questions:

- Compute the number of entries for the biggest table generated by the bucket elimination algorithm when using order o.
- is it possible to find a better order for the variables ? Motivate your answer.

Lookahead

02 May 2012, Exercise 3 (Points 25)

Consider the following Graph coloring problem: Variables $X = \{x_1, x_2, x_3, x_4, x_5\}$, Domains $D_1 = \{R, G\}$, $D_2 = D_3 = D_4 = D_5 = \{R, B\}$, Constraints $R = \{R_{12}, R_{13}, R_{23}, R_{24}, R_{35}, R_{45}\}$. Solve it with backtracking plus forward checking and with backtracking forcing arc consistency at each step. Use the following fixed ordering for variable expansion $o = \{x_1, x_4, x_5, x_2, x_3\}$ and always expand R first. Comment on whether AC is helping w.r.t. forward checking in this case (i.e., highlight the search space avoided by AC).

30 Apr 2014, Exercise 2 (Points 30)

Consider the following Graph coloring problem: Variables $X = \{x_1, x_2, x_3, x_4\}$, Domains $D_1 = \{G, B\}$, $D_2 = D_3 = D_4 = \{R, B\}$, Constraints $R = \{R_{12}, R_{13}, R_{23}, R_{34}\}$. Answer to the following questions:

- Find all solutions by using backtracking with the following fixed ordering for variable expansion $o = \{x_4, x_1, x_2, x_3\}$ always expand R before B and G before B.
- Find all solutions by using backtracking plus arc consistency, quantify the gain with respect to simple backtracking as number of nodes that are not expanded.
- Order variables by using the Minimum Remaining Values plus Degree Heuristic, break ties by always choosing the lowes Id. Show the search space for backtracking with this order and show the reduction when forcing Arc Consistency (in terms of number of not expanded nodes)

Agents and Search

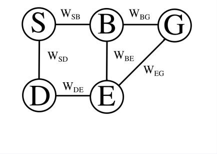
02 May 2012; Exercise 4 (Points 30)

Consider the labyrinth in the Figure, where S and G are the start and goal positions respectively. Consider the path planning problem associated to this labyrinth as a search problem. Assume: i) cycles are detected and states can not be repeated in a branch from root to leaves; ii) agents can move in four directions (N,S,E,W) if no obstacles; iii) cost of each move is 1. Answer to the following questions.

S	4	8	12
1	5	9	13
2	6	10	14
3	7	11	G

1. How many iterations would a IDS do ?
2. Given the following heuristics i) manhattan distance h_1, ii) manhattan distance with diagonal moves h_2, which one is preferable for doing an A* search (motivate your answer) ?
3. Show execution of A* using the heuristic of your choice

30 Apr 2014; Exercise 1 (Points 25)



Consider the mobility graph in the left figure where S and G are the start and goal positions respectively and $w_{i,j}$ represent the moving cost between the nodes i and j. Consider the problem of finding a minimum cost path between nodes S and G on this mobility graph and assume we want to solve this problem using search techniques. Consider as a heuristic function the minimum number of edges between the node and the goal. Answer to the following questions.

1. Give a weight allocation such that a best first search algorithm would not be optimal in this setting
2. Considering the previous weight allocation, would Iterative Depening Search return the optimal solution ? (motivate your answer)
3. Show the execution of A* using the same heuristic function and the given weights (avoid repetition of states along the same branch).

30 Apr 2014; Exercise 3 (Points 20)

Consider the following **binary** cost network: Variables, $X = \{x_1, x_2, x_3, x_4, x_5\}$, Domains, $D_1 = \{G, B\}$,$D_2 = D_3 = D_4 = D_5\{R, B\}$, Constraints $C_h = \{\}$ and $C_s = \{F_{12}(x_1, x_2), F_{13}(x_1, x_3), F_{14}(x_1, x_4), F_{15}(x_1, x_5), F_{23}(x_2, x_2), F_{35}(x_3, x_5)\}$. Where each F_{ij} has the following form

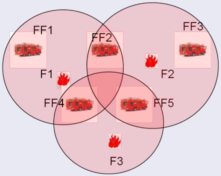
$$F_{ij}(x_i, x_j) = \begin{cases} -1 & \text{if values are the same} \\ 0 & \text{otherwise} \end{cases}$$

Answer the following questions:

- State whether this network is acyclic motivating your answer.
- Consider the bucket elimination algorithm. Give the dimension of the biggest table (in terms of values stored in the table) by using an order of your choice to process the bucket.

CSP formalization

02 May 2012; Exercise 1 (Points 20)



Consider a set of fire fighting units that must be assigned to a set of fires so to ensure that each fire has exactly two units allocated. Assume that: i) Each fire fighting unit can be assigned to just one fire; ii) A fire fighting unit can only be assigned to fires which are within a given distance from its initial position, i.e. for each fire F_i there is a set FFS_i that represents all the fire fighters that can be assigned to fire F_i (e.g., in the left figure $FFS_1 = FF_1, FF_2, FF_4$).

Formalize this task assignment problem as a CSP specifying (i) what the variables represent, (ii) the domain of the variables, and (iii) the constraints. State whether the constraint network associated to the situation in the left Figure is consistent, and if so provide a solution.

Backtracking with Cycle Cutset

Cycle cutset

Consider a 3-color graph coloring problem with variables x_1, x_2, x_3, x_4, x_5 and constraints $x_1 \neq x_2, x_1 \neq x_3, x_1 \neq x_4, x_1 \neq x_5, x_2 \neq x_3, x_3 \neq x_5, x_4 \neq x_5$ Povide all solutions using backtrack and

- Find a cycle cutset
- Find all solutions using backtrack only on the cycle cutset variable(s)

Bucket Elimination

Bucket Elimination

Given a combinatorial auction and the following set of bids:

- $B_1 = \langle\{1, 2, 3\}, 5\rangle$, $B_2 = \langle\{1, 4, 6\}, 4\rangle$, $B_3 = \langle\{4, 5\}, 1\rangle$, $B_4 = \langle\{2, 5\}, 3\rangle$

Answer to the following questions:

1. Provide a cost network formalisation of the winner determination problem, specifying the variables, their domains and the constraints (hard and soft).
2. Is the resulting cost network acyclic ? Motivate your answer
3. Solve the cost network with Bucket Elimination.

Based on Constraint Processing [Dechter] (Ex. 7 page 147)

Look ahead

Apply the following algorithm to find all solutions to the 4-Queens problem

- Backtracking
- Forward checking

Does Forward Checking help relative to Backtracking ? Justify the answer.

Agents and Search

Agents and Search

S	5	9	13
2	6	10	14
3	7	11	15
4	8	12	G

Consider the above maze where S is the start position and G is the goal. Assume: i) cycles are detected and states can not be repeated in a branch from root to leaves; ii) agents can move in four directions (N,S,E,W); iii) cost of each move is 1. Show an execution of the following search algorithms, comment on optimality of solution and number of states that are generated (break ties by natural order on state ids):

- Depth First; Iterative Depening
- Greedy search; A* (use manhattan distance as heuristic)

Based on Constraint Processing [Dechter] (Ex. 10 page 82)

arc and path consistency

Consider a Graph coloring problem with three colors and four nodes x_1, x_2, x_3, x_4 where every two nodes are connected.

- Is the problem arc-consistent ?
- Is the problem path consistent ?
- Is the problem 4-consistent ?

Arc Consistency and Path Consistency

local consistency

Consider the network $\mathcal{R}$: Variables $\{x, y, z, w\}$ Domains $D_i = \{1, 2, 3\}$ Constraints $x < y, x \leq z, z < y, y = w$

- Force Arc consistency