Exercises, II part

Bucket Elimination

02 May 2012, Exercise 2 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{X_1, X_2, X_3, x_4\}$, Domains, $D_1 = D_2 = D_4 = \{R, B\}$, $D_3 = \{G, B\}$, Constraints $C_h = \{R_{12}, R_{13}, R_{23}, R_{24}\}$ and $C_s = \{F_1(x_1), F_2(x_2), F_3(x_3), F_4(x_4)\}$. Where each R_{ij} is an inequality constraint (i.e., $R_{ij} = \{< R, B >< B, R >\}$) and $F_i(x_i)$ is of the following form:

$$F_i(x_i) = \begin{cases} 1 & \text{if } x_i = B \\ 0 & \text{otherwise} \end{cases}$$

Provide a solution for this cost network using Bucket Elimination. Use the ordering $o = \{x_4, x_2, x_1, x_3\}$.

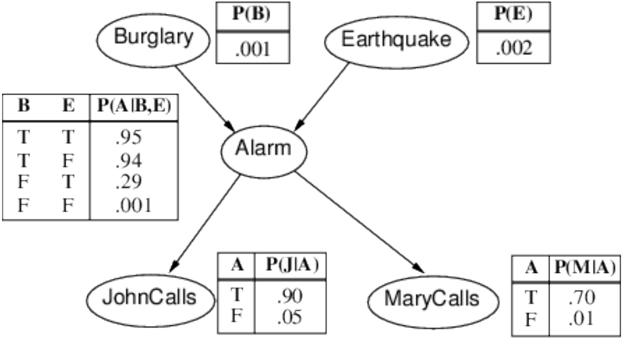
Bucket Elimination

29 Sept 2015, Exercise 3 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{x_1, x_2, x_3, x_4\}$. Constraints $C_h = \{\}$ and $C_s = \{F_{12}(x_1, x_2), F_{13}(x_1, x_3), F_{14}(x_1, x_4), F_{23}(x_2, x_3), F_{34}(x_3, x_4)\}$ and $D_1 = D_2 = D_3 = D_4 = \{0, 1\}$. Consider the Bucket Elimination algorithm and the variable ordering $o = \{x_2, x_1, x_4, x_3\}$. Answer the following questions:

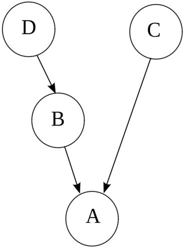
- Compute the number of entries for the biggest table generated by the bucket elimination algorithm when using order o.
- is it possible to find a better order for the variables ? Motivate your answer.

BN: Variable elimination



- Consider the Bayesian Network in the Figure, compute $P(B|j, m, e)$ using the variable elimination approach and the following variable order: $\{B, E, A, J, M\}$. Approximate numbers to the fourth decimal digit.
- Can you give an order for the variables that results in table(s) with more than four rows ?

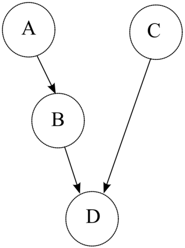
BN: 13 Feb. 2017



Consider the Bayesian Network in the Figure. Answer the following questions:

1. State whether P(A|B,C) = P(A|B,C,D). Motivate your answer.
2. State whether P(A|C) = P(A|B,C,D). Motivate your answer.
3. State how many numbers we need to represent the joint distribution for this network (variables are all boolean). Motivate your answer.

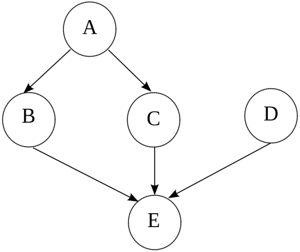
BN: 16 Giu. 2016



Consider the Bayesian Network in Figure, where every variable is binary. Answer the following questions:

1. Is it true that $P(B|A) = P(B|A,C)$? Motivate your answer.
2. Write down the equation to compute the query $P(D|A{=}true,C{=}true)$ using the CPT associated with the network.
3. How many independent numbers must be stored to answer all the possible queries for this Bayesian Network ?

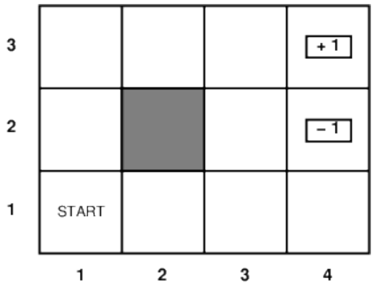
BN: 21 Sep. 2016

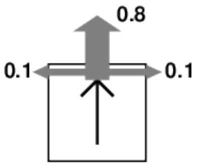


Consider the Bayesian Network in Figure, where every variable is binary. Answer the following questions:

1. State how many parameters must be provided to compute the joint probability table of this BN.
2. Assume the CPT that defines P(E|B,C,D) is specified with a noisy-or, state how many parameters must be provided to compute the joint probability table of this BN.
3. State whether P(D|C,E) = P(D|A,B,C,E). Motivate your answer.

MDP: probability of action sequence

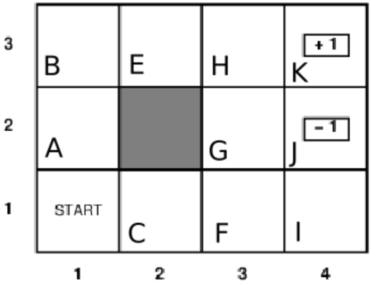


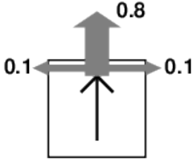


Consider the environment in the Figure. Answer the following questions:

1. Compute the probability that the sequence of actions $< U, U, R, R, R >$ ends in terminal state with reward $+1$;
2. Compute which states can be reached by the sequence of actions $< U, R >$ and with which probability.

MDP: probability of action sequence





Consider the environment in the Figure. Answer the following questions:

1. Assume that $\forall s, a, s' | s' \notin \{J, K\} \;\; R(s, a, s') = +2$, $\forall s, a \;\; R(s, a, K) = +1$ and $\forall s, a \;\; R(s, a, J) = -1$. State what is the optimal action for states F, G, H, I and motivate your answer.
2. Assume that $\forall s, a, s' | s' \notin \{J, K\} \;\; R(s, a, s') = -2$, , $\forall s, a \;\; R(s, a, K) = +1$ and $\forall s, a \;\; R(s, a, J) = -1$. State what is the optimal action for states F, G, H, I and motivate your answer.

MDP: value iteration, problem statement

Consider an undiscounted MDP having three states (1,2,3). State 3 is terminal. In state 1 and 2 there are two possible actions A and B. The transition and reward model is as follow:

- In state 1, action A moves the agent to state 2 with probability .8 and a reward of -2 while it leaves the agent in state 1 with probability .2 and a reward of -1. In state 1, action B moves the agent to state 3 with probability .1 and a reward of 0 while it leaves the agent in state 1 with probability .9 and a reward of -1.
- In state 2, action A moves the agent to state 1 with probability .8 and a reward of -1 while it leaves the agent in state 2 with probability .2 and a reward of -2. In state 2, action B moves the agent to state 3 with probability .1 and a reward of 0 while it leaves the agent in state 2 with probability .9 and a reward of -2.

MDP: value iteration, questions

1. Provide a transition diagram for the MDP described above.
2. Provide a qualitative discussion about the optimal policy for this MDP.
3. Show the value of $v(1)$ for the first two iterations of a Value Iteration algorithm. Assume $v(s) = 0 \ \ \forall s$.
4. Is it necessary to compute the value of $v(2)$ to answer to the previous question ?

Consider and environment with states $\{A, B, C, D, E\}$, actions $\{r, l, u, d\}$ where states $\{A, D\}$ are terminal. Consider the following sequence of learning episodes:

E1 $(B, r, C, -1)(C, r, D, +9)$

E2 $(B, r, C, -1)(C, r, D, +9)$

E3 $(E, u, C, -1)(C, r, D, +9)$

E4 $(E, u, C, -1)(C, r, A, -11)$

1. Build an estimate for $\hat{T}$ and $\hat{R}$
2. Compute $v(s)$ for all non-terminal states by using a direct evaluation approach
3. Compute $v(s)$ for all non-terminal states by using a sample-based evaluation approach (assume $\gamma = 1$ and $\alpha = 0.5$)

Inference: 12 Jul 2012

Consider the following Joint Probability Table for the three binary random variables A, B, C.

$P(A, B, C)$	A	B	C
0.108	T	T	T
0.012	T	T	F
0.072	T	F	T
0.008	T	F	F
0.016	F	T	T
0.064	F	T	F
0.144	F	F	T
0.576	F	T	F

Compute the following queries:

1. P(C|A=T,B=T)
2. P(C|A=T)