Artificial Intelligence: Partial Written test

15 May 2013

1 Exercise 1 (Points 30)

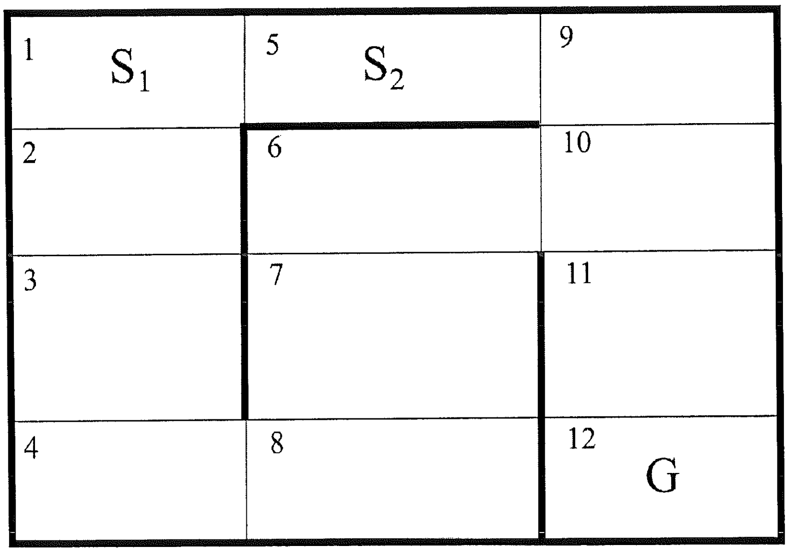


Figure 1: Mobility graph, S_1 and S_2 are the starting positions, thick lines are obstacles and G is the goal position

Consider the labyrinth in Figure 1 where S_1 and S_2 are two starting positions, G is the goal position. Consider the problem of finding a minimum cost path for two agents that move in the labyrinth, starting from S_1 and S_2 and to reach G. Agents move at the same time and they can reach the four neighbours and they must move at each time step (i.e., they do not have a "do nothing" operation). Agents can not traverse walls and each agent can not move to a cell if it is occupied by the other agent (the check is done before moving). When an agent reaches the goal cell it disappears from the environment (i.e., the goal cell is never occupied and once an agent reaches the goal cell it will not move again). Assume we want to solve this problem using search. A state is the position of the two agents, hence the start state is $< 1, 2 >$ and the goal state is $< 12, 12 >$ and movement cost is one for each move and for each agent. Define an admissible heuristic for this problem and trace the execution of a greedy local search approach (avoid repeated state in the same search branch). State whether the solution obtained is optimal.

2 Exercise 2 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{x_1, x_2, x_3, x_4\}$, Domains, $D_1 = D_2 = D_3 = D_4 = \{R, B\}$, Constraints $C_h = \emptyset$ and $C_s = \{F_{12}(x_1, x_2), F_{14}(x_1, x_4), F_{24}(x_2, x_4), F_{34}(x_3, x_4)\}$. Where each F_{ij} has the following form

$$F_{ij}(x_i, x_j) = \begin{cases} RR & -1 \\ RB & 0 \\ BR & 0 \\ BB & -2 \end{cases}$$

Provide a solution for this cost network using Bucket Elimination. Use the ordering $o = \{x_4, x_3, x_2, x_1\}$.

3 Exercise 3 (Points 20)

Consider the following Graph coloring problem: Variables $X = \{x_1, x_2, x_3, x_4, x_5\}$, Domains $D_1 = D_4 = \{G, B\}$, $D_2 = D_3 = D_5 = \{R, B\}$, Constraints $R = \{R_{12}, R_{13}, R_{23}, R_{24}, R_{34}, R_{35}\}$. Solve it with backtracking plus forward checking and with backtracking forcing arc consistency at each step. Use the following fixed ordering for variable expansion $o = \{x_1, x_4, x_5, x_2, x_3\}$ and always expand R before B and G before B. Comment on whether AC is helping w.r.t. forward checking in this case (i.e., highlight the search space avoided by AC).

4 Exercise 4 (Points 25)

Consider the same Graph coloring problem defined in Exercise 3. Find one solution using the Join Tree Clustering approach. When building the max-cardinality order start from x_1 and break ties by selecting the variables with lowest ID.

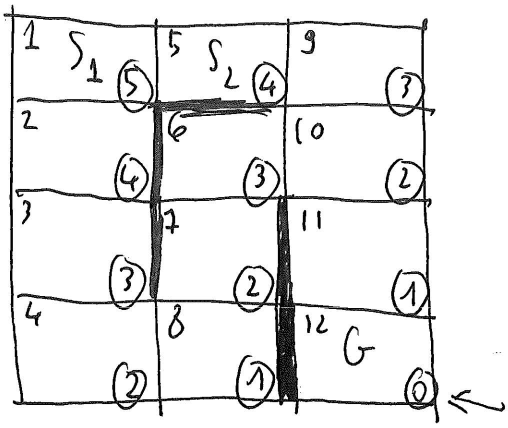
EX 1 [SEARCH]

STATE ⇒ < POS_AGENTE_1, POS_AGENTE_2 >

$\overline{X} = \langle x_1, x_2 \rangle$

$x_i = \langle 1, \dots, 12 \rangle$

STATE INIT. ⇒ < 1, 5 >



MANHATTAN DISTANCE

OP → N S W E

NO MOVE IF WALL, NO MOVE IF OTHER AGENT (UNLESS TARGET POSITION IS GOAL), MOVE AT THE SAME TIME, CAN NOT STAY IN SAME POS. (UNLESS GOAL STATE)

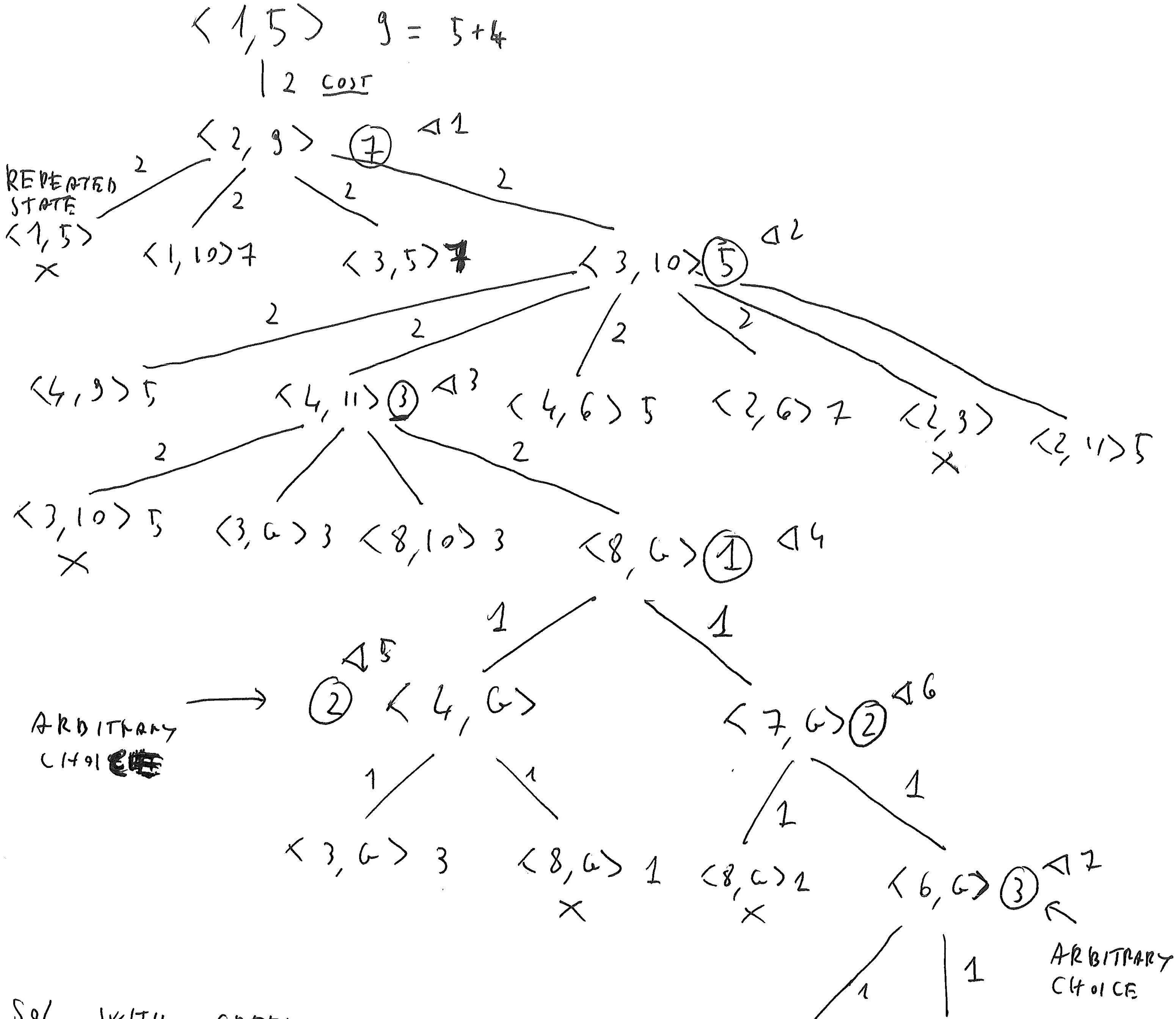
1) ADMISSIBLE HEURISTIC SUM OF MANHATTAN DISTANCE FOR AGENT 1 AND AGENT 2. EACH AGENT WILL NEED TO PERFORM AT LEAST THE NUMBER OF MOVES PROVIDED BY THE HEURISTIC. THE SYSTEM MUST PERFORM AT LEAST THE SUM OF SUCH MOVES.

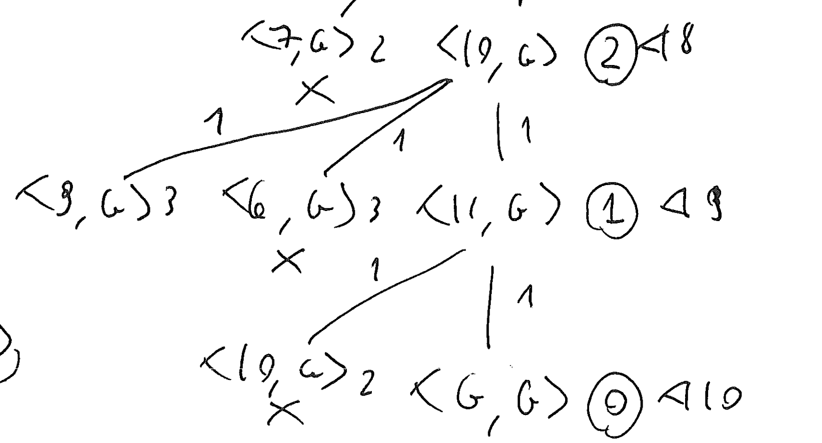
2) TRACE EXECUTION FOR GREEDY SEARCH

[SEE NEXT PAGE]

3) NOT OPTIMAL TOTAL COST OF GREEDY SEARCH EXECUTION IS 13. THE FOLLOWING SEQUENCE OF MOVES REACHES THE GOAL AND HAS A TOTAL COST OF 11

$$\langle 1,5\rangle \xrightarrow{2} \langle 2,9\rangle \xrightarrow{2} \langle 1,10\rangle \xrightarrow{2} \langle 5,11\rangle \xrightarrow{2} \langle 9,G\rangle \xrightarrow{1} \langle 10,G\rangle \xrightarrow{1} \langle 11,G\rangle \xrightarrow{1} \langle G,G\rangle$$





SOL WITH GREEDY SEARCH:
(AND THESE ARBITRARY CHOICES)

$\langle 1,5\rangle \xrightarrow{2} \langle 2,9\rangle \xrightarrow{2} \langle 3,10\rangle \xrightarrow{2} \langle 4,11\rangle \xrightarrow{2} \langle 8,G\rangle \xrightarrow{1} \langle 7,G\rangle \xrightarrow{1} \langle 6,G\rangle \xrightarrow{1} \langle 10,G\rangle \xrightarrow{1} \langle 11,G\rangle \xrightarrow{1} \langle G,G\rangle$

TOTAL COST = 13

EX 2 [BUCKET ELIMINATION]

APPLY BUCKET ELIMINATION CONSIDERING GIVEN ORDER

X_1	B_1	① F_{12} ② F_{14}	$\rightarrow$ ③ $H^1(2,4)$		$X_1^* = R$ ↑
X_2	B_2	③ F_{24} ⑤ $H^1(2,4)$	$\rightarrow$ ⑥ $H^2(4)$		$X_2^* = R$ ↗
X_3	B_3	④ F_{34}	$\rightarrow$ ⑦ $H^3(4)$		$X_3^* = R$ ↑
X_4	B_4	⑥ $H^2(4)$ ⑦ $H^3(4)$	$\rightarrow$ ⑧ H^4		$X_4^* = B$

↑ ORDER

FUNCTIONS : $\{ \cancel{F_{12}}, \cancel{F_{14}}, \cancel{F_{24}}, \cancel{F_{34}} \}$ ① ② ③ ④

- CONSTRAINT PARTITIONING: CONSIDER BUCKETS FROM LAST TO FIRST AND PUT ALL FUNCTIONS DEPENDING ON CURRENT VARIABLE IN THE BUCKET

- PROCESS BUCKETS FROM LAST TO FIRST COMPUTING H-FUNCTIONS. PUT NEW FUNCTIONS IN APPROPRIATE BUCKET.

- FIND BEST VALUE FOR EACH BUCKET AND PROPAGATE UPWARD

$H^1(2,4)$	X_1	X_2	X_4	F_{12}	F_{14}	Σ
0	R	R	R	-1	-1	-2
	B	R	R	0	0	0
-1	$\boxed{R}$	R	B	-1	0	-1
	B	R	B	0	-2	-2
-1	R	B	R	0	-1	-1
	B	B	R	-2	0	-2
0	R	B	B	0	0	0
	[illegible]	B	[illegible]	-2	[illegible]	[illegible]

MAXIMISING ON $X_1 \Rightarrow$ REMOVE X_1 COLUMN AND PICK BEST VALUE.

SINCE $X_2^* = R$, $X_4^* = B$, $(X_3^* = R)$

$\Rightarrow X_1^* = R$

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EX2 [BUCKET ELIMINATION]

TO PAGE 3

$H^2(4)$	x_2	x_4	F_{24}	$H^1(2,4)$	Σ
-1	R	R	-1	0	-1
	B	R	0	-1	-1
-1	R	B	0	-1	-1
	B	B	-2	0	-2

MAXIMISING ON x_2 ⟹ REMOVE x_2 COLUMN

SINCE $x_4^* = B$ ($x_3^* = R$) ⟹ $x_2^* = R$

$H^3(4)$	x_3	x_4	F_{34}
0	R	R	-1
	B	R	0
0	R	B	0
	B	B	-2

MAXIMISING ON x_3 ⟹ REMOVE x_3 COLUMN

SINCE $x_4^* = B$ ⟹ $x_3^* = R$

M	x_4	$H^2(4)$	$H^3(4)$	Σ
	R	-1	0	-1
	B	-1	0	-1

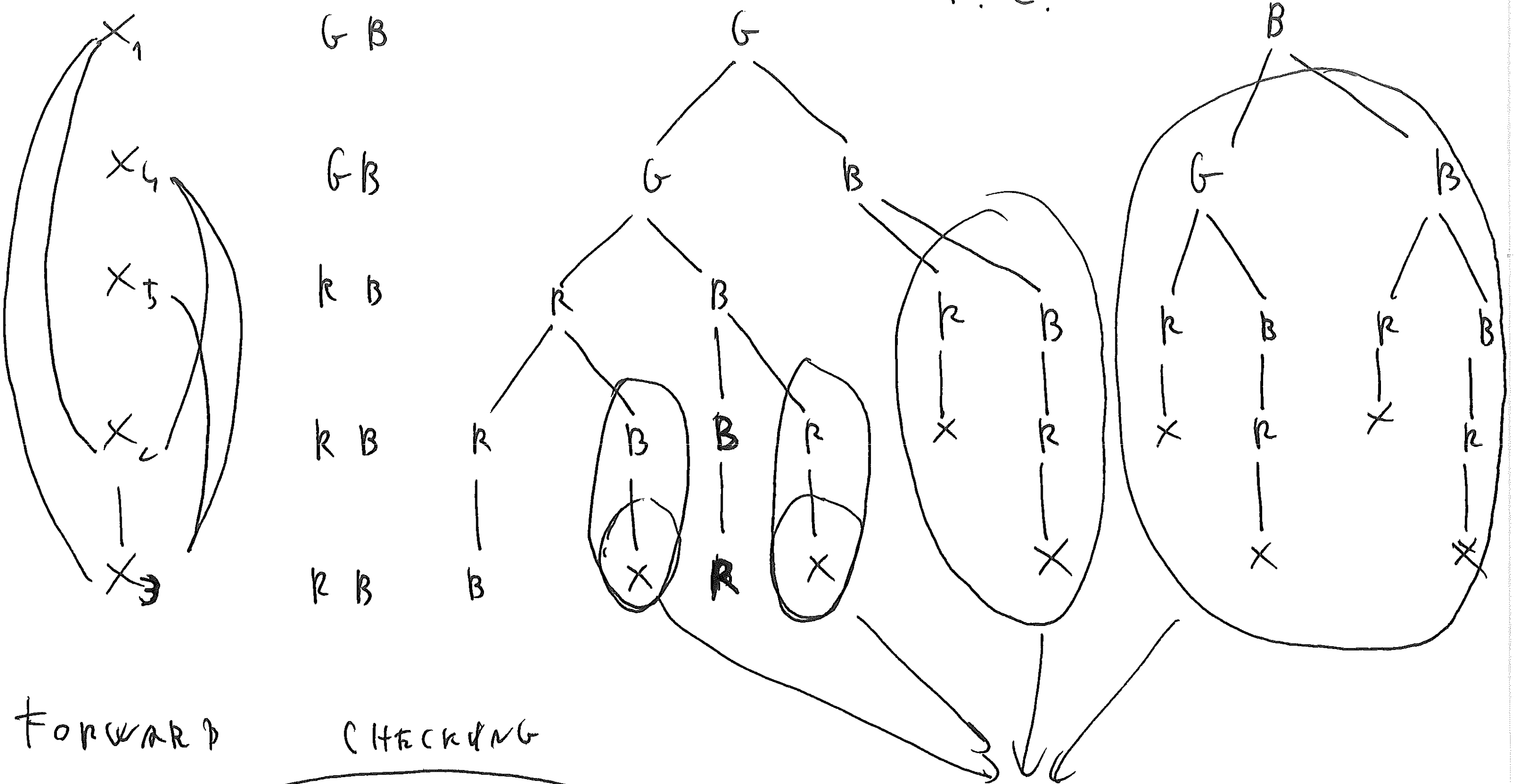
THE BEST WE CAN ACHIEVE IS -1

ARBITRARY CHOOSE

$x_4^* = B$ PROPAGATE UP

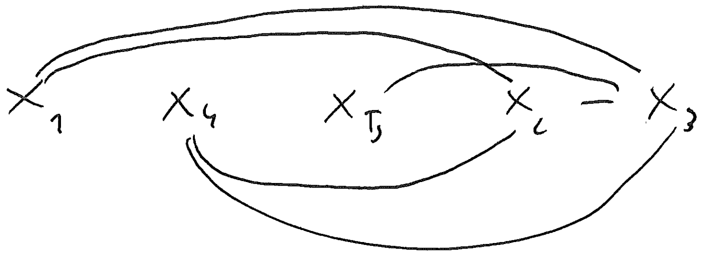
EX 3 [LOOK AHEAD SCHEMES]

BACKTRACKING WITH ~~NO LOOK AHEAD~~ F.C.



SEARCH SPACE AVOIDED BY A.C. W.R.T. F.C.

FORWARD CHECKING



G	G	R	R B	~~R~~ B	UP TO THIS ASSIGNMENT NO CHANGE
G	G	R	R	B	OK
G	G	R	B	$\emptyset$	STOP
G	G	B	R B	R ~~B~~	BACKTRACK TO $X_5 = B$
G	G	B	R	$\emptyset$	STOP
G	G	B	B	R	OK
G	B	R B	R ~~B~~	R ~~B~~	BACKTRACK TO $X_4 = B$
G	B	R	R	$\emptyset$	STOP
G	B	B	R	~~$\emptyset$~~	STOP
B	G B	R B	R ~~B~~	R ~~B~~	BACKTRACK TO $X_1 = B$
B	G	R B	R ~~B~~	R ~~B~~	
B	G	R	R ~~B~~	$\emptyset$	STOP
B	G	B	R ~~B~~	~~$\emptyset$~~	STOP

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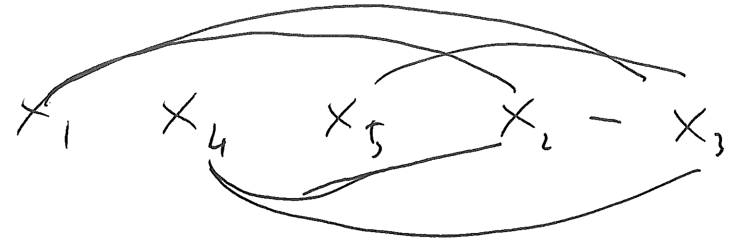
EX 3 [LOOKAHEAD SCHEMES]

↓ FROM PAGE 5

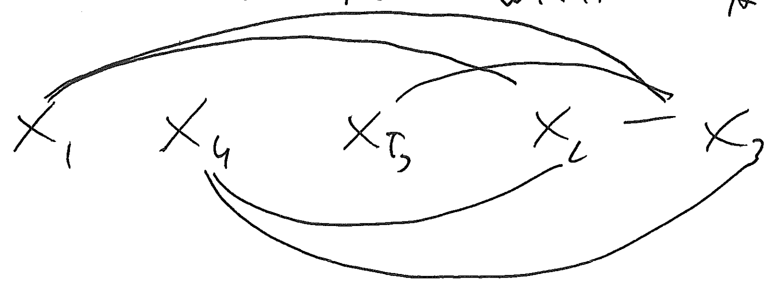


B B RB R~~B~~ R~~B~~ BACKTRACK TO $X_4 = B$

B B R R~~B~~ ~~R~~~~B~~ STOP

B B B R~~B~~ R~~B~~ STOP BACKTRACK TO $X_5 = B$

BACKTRACKING WITH A.C. LOOK AHEAD



GB GB RB RB RB A.C. AS IN EVERY GRAPH COLORING PROBLEM.

G GB RB RB RB A.C. [NO CHANGE TO X_2 AND X_3]

G G RB RB RB A.C. [NO CHANGE TO X_2 AND X_3]

REV((X_3), X_5)

G G R R~~B~~ ~~R~~B A.C. OK

REV((X_2) X_3)

G G B ~~R~~ B REV (X_2)(X_3) R~~B~~ A.C. OK.

REV((X_3) X_5)

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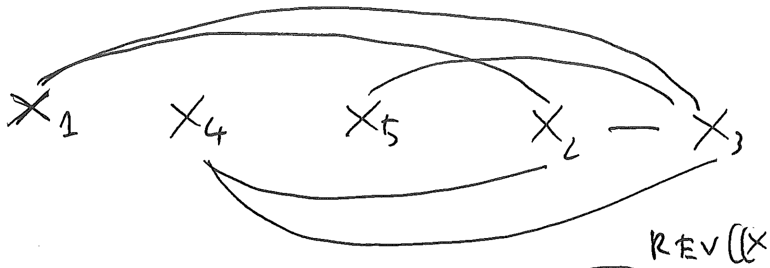
EX 3 [LOOK AHEAD SCHEMES]

↓ FROM PAGE 6

BACKTRACKING WITH A.C.



G   B   RB   R ~~B~~   ~~R~~ ~~B~~   STOP

REV((X_3) X_2)

REV((X_2) X_4)

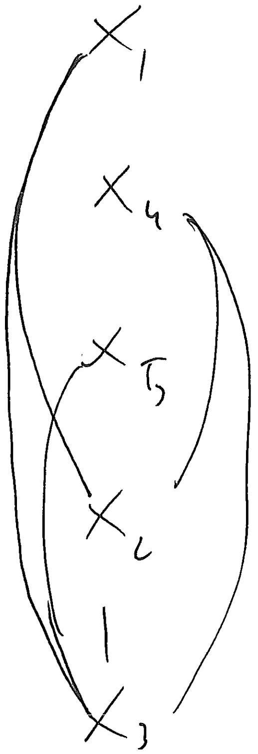
REV((X_3), X_1)

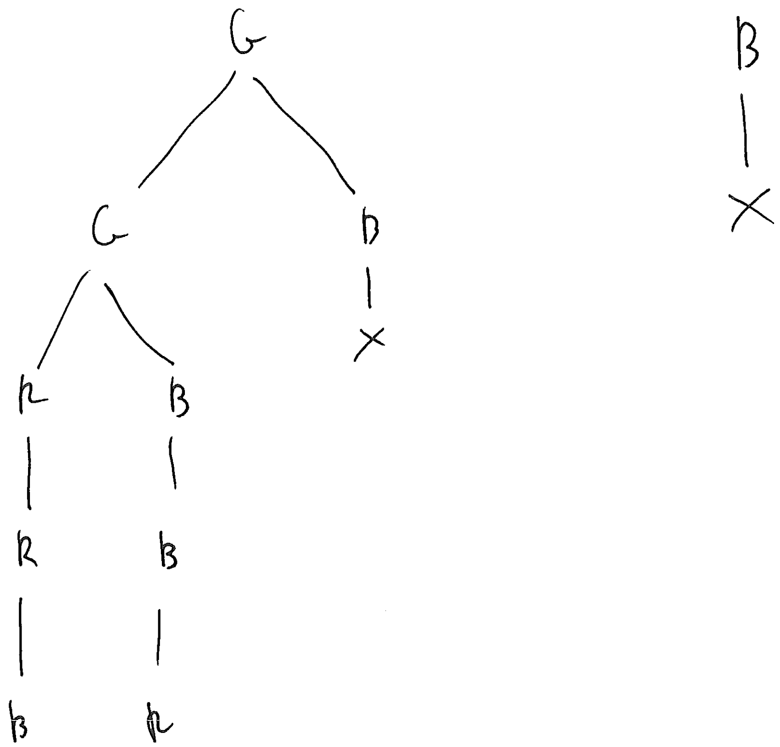
B   GB   R B   R ~~B~~   ~~R~~ ~~B~~   STOP.

REV((X_3), X_2)

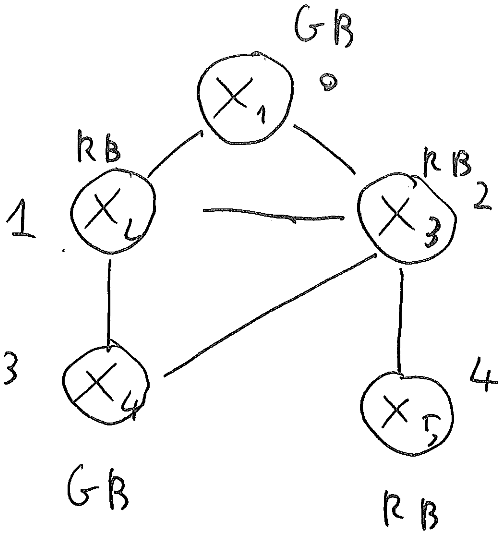
REV((X_2), X_1)

REV((X_3), X_1)





EX 4 [JOIN TREE CLUSTERING]



$C = \{R_{12}, R_{13}, R_{23}, R_{24}, R_{35}\}$

MAX CARD. ORDER FROM X_1 (BREAK TIES BASED ON LOWEST ID)

0 X_1 $\quad ANC(X_1) = \emptyset$ ok

1 X_2 $\quad ANC(X_2) = (X_1)$ ok

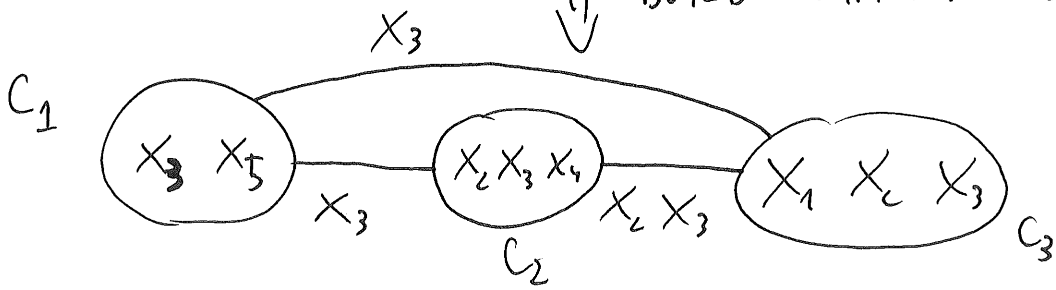
2 X_3 $\quad ANC(X_3) = (X_1\ X_2)$ ok

3 X_4 $\quad ANC(X_4) = (X_2\ X_3)$ ok

4 X_5 $\quad ANC(X_5) = (X_3)$ ok

→ GRAPH IS CHORDAL NO NEED TO ADD ANY LINK

⇓ BUILD MAXIMAL CLIQUES



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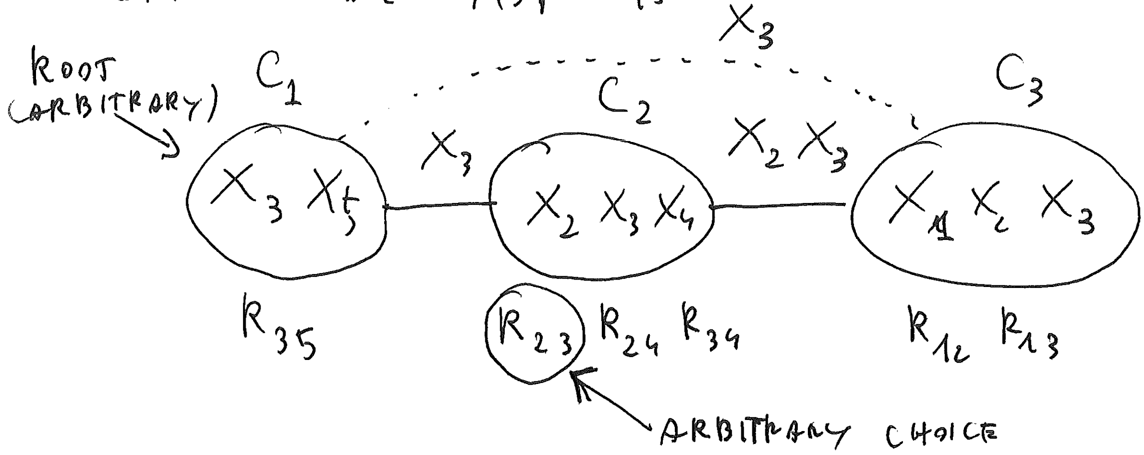
EX 4 [JOIN TREE CLUSTERING]

FROM PAGE 8

BUILD MAXIMUM SPANNING TREE

ONE POSSIBLE MST IS



ALLOCATE CONSTRAINTS AND BUILD SUBPROBLEM TO FORCE CONFORMITY OF NETWORK

C_1	X_3	X_5	
	R	B	0
	B	R	

C_2	X_2	X_3	X_4	
	R	B	G	
	B	R	G	1

C_3	X_1	X_2	X_3	
	G	R	R	
	G	R	B	
	G	B	R	2
	G	B	B	
	B	R	R	

USE TREE SOLVER TO CHECK CONSISTENCY AND GENERATE SOLUTION

EXTEND TO A SOLUTION

X_1	X_2	X_3	X_4	X_5
G	B	R	G	B
2	1	0	1	0

$$R'_{C_2} = \pi_{S_2}\left(R_{C_2} \bowtie R_{C_3}\right) = R_{C_2}$$

[ALL TUPLES HAVE A CORRESPONDING TUPLE IN C_3 THAT AGREE ON $X_2\ X_3$]

$$R'_{C_1} = \pi_{S_1}\left(R_{C_1} \bowtie R'_{C_2}\right) = R_{C_1}$$

[ALL TUPLES IN C_1 HAVE A CORRESPONDING TUPLE IN C_2 THAT AGREE ON X_3]

ANOTHER POSSIBLE SOLUTION

X_1	X_2	X_3	X_4	X_5
G	R	B	G	R