Artificial Intelligence: Written Exam

29 September 2015

1 Exercise 1 (Points 25)

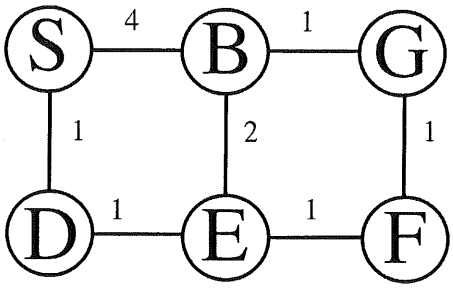


Figure 1: Mobility graph

Consider the mobility graph in Figure 1 where S and G are the start and goal positions respectively and labels on edges represent the moving cost between the nodes. Consider the problem of finding a minimum cost path between nodes S and G on this mobility graph and assume we want to solve this problem using search techniques. Answer the following questions:

- State whether a Breadth First Search would return the minimum cost path for this problem. Motivate your answer.
- Compute the maximum number of nodes that a BFS approach must store in memory for this problem in the worst case.
- Show the execution trace of A* (**do not** avoid repeated states on the same branch) and compute the maximum number of nodes that must be stored in memory in the worst case.

2 Exercise 2 (Points 30)

Give an instance of a graph coloring problem and an order for variable expansion such that an approach to find all solutions that employs backtracking plus forward checking is expanding less nodes than backtracking.

3 Exercise 3 (Points 25)

Consider the following **binary** cost network: Variables, $X = \{x_1, x_2, x_3, x_4\}$. Constraints $C_h = \emptyset$ and $C_s = \{F_{12}(x_1, x_2), F_{13}(x_1, x_3), F_{14}(x_1, x_4), F_{23}(x_2, x_3), F_{34}(x_3, x_4)\}$ and $D_1 = D_2 = D_3 = D_4 = \{0, 1\}$. Consider the Bucket Elimination algorithm and the variable ordering $o = \{x_2, x_1, x_4, x_3\}$. Answer the following questions:

- Compute the number of entries for the biggest table generated by the bucket elimination algorithm when using order o.
- is it possible to find a better order for the variables ? Motivate your answer.

4 Exercise 4 (Points 20)

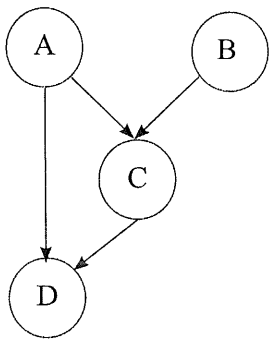


Figure 2: Bayesian Network.

Consider the Bayesian Network in Figure 4, Answer the following questions:

1. State whether the equation $P(D|A, C) = P(D|A, B, C)$ holds. Motivate your answer.
2. Write down the equation to compute $P(B|C = true)$;

ESAME 29 SEPT. 2015

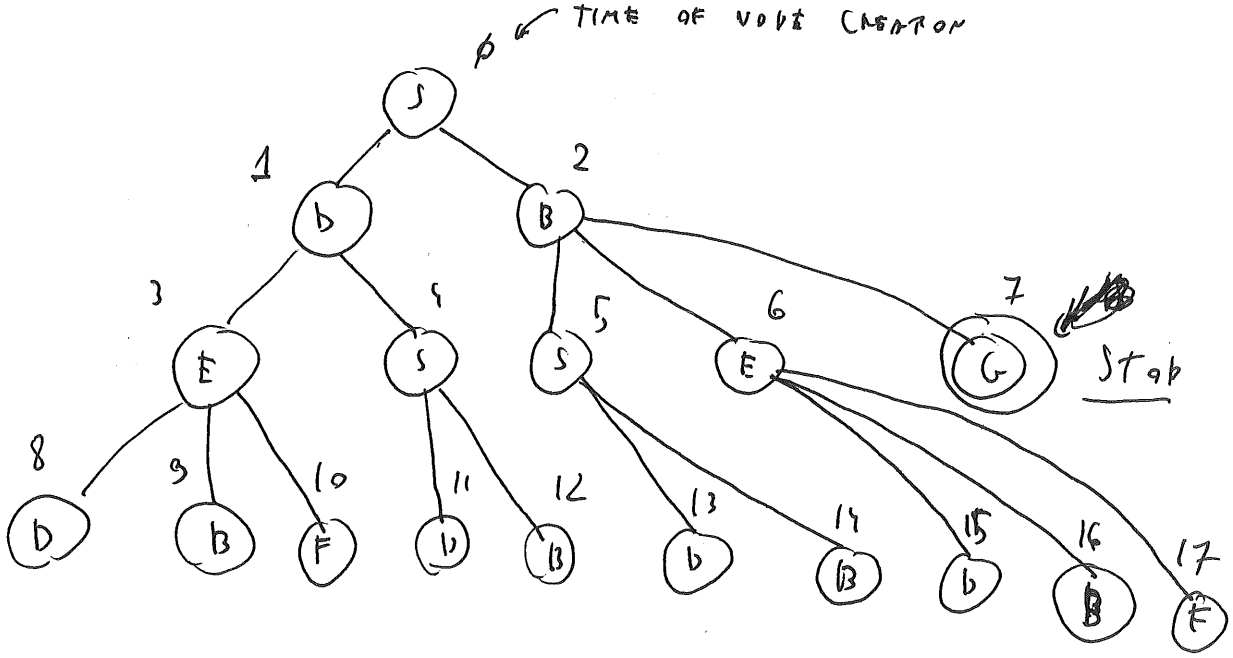
Ex 1 [SEARCH]

1.1 BFS NOT OPTIMAL $S \xrightarrow{4} B \xrightarrow{1} G \Rightarrow COST = 5$

$S \xrightarrow{1} D \xrightarrow{1} E \xrightarrow{1} F \xrightarrow{1} G \Rightarrow COST = 4$

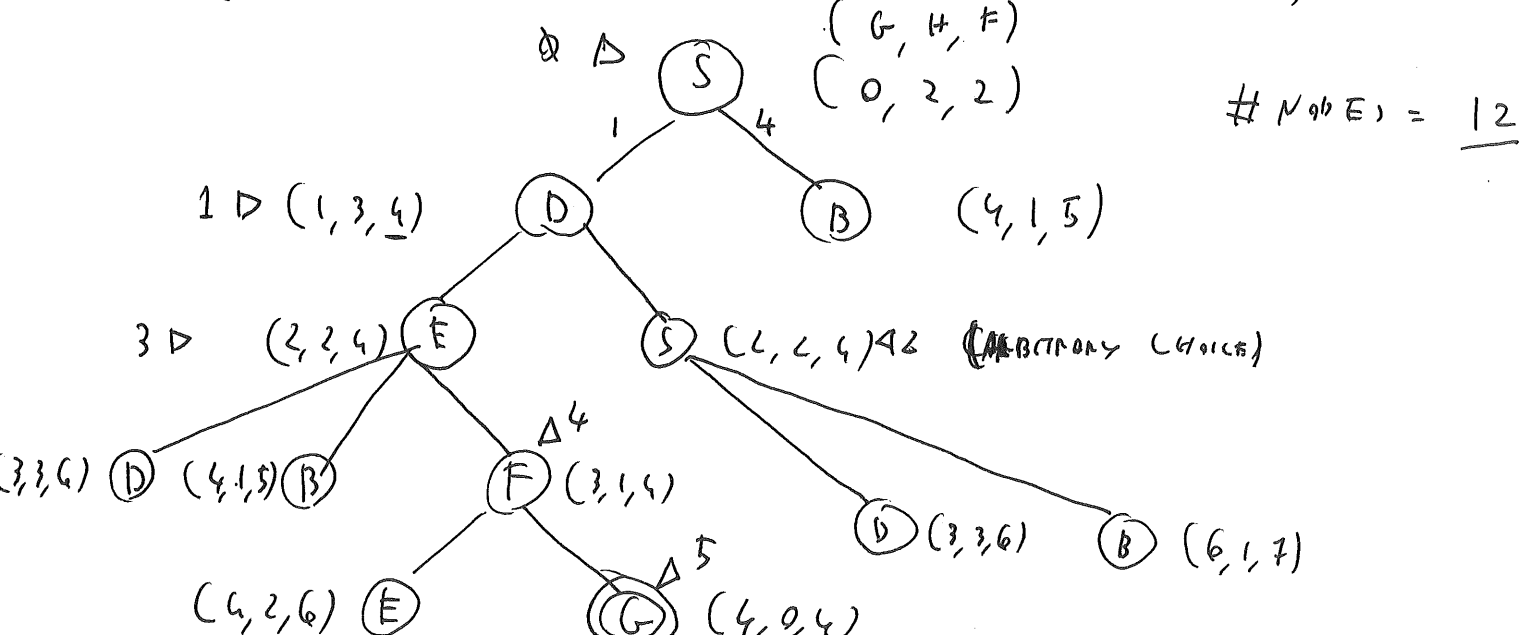
(SEE THE TREE BELOW)

1.2 WORST CASE = GOAL NODE IS THE LAST EXPANDED NODE. IN THIS CASE WE HAVE ~~18~~ NODES IN MEMORY. (SEE THE TREE BELOW)



1.3 EXECUTION OF A* (NOT AVOIDING REPEATED STATES)

(USING NUMBER OF HOPS TO DEST. AS HEURISTIC)



EX 2 [BACKTRACKING]

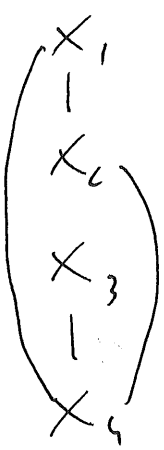
GRAPH COLORING PROBLEM $X = \{X_1, X_2, X_3, X_4\}$

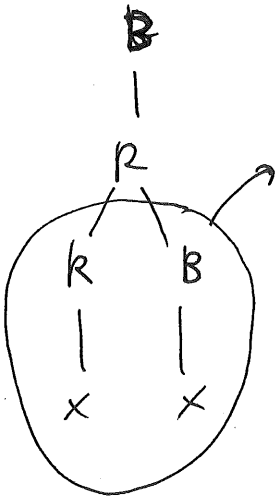
$D_1 = \{B, G\}$ $D_2 = D_3 = D_4 = \{R, B\}$ $C = \{R_{12}, R_{14}, R_{24}, R_{34}\}$

ORDER = $\{X_1, X_2, X_3, X_4\}$

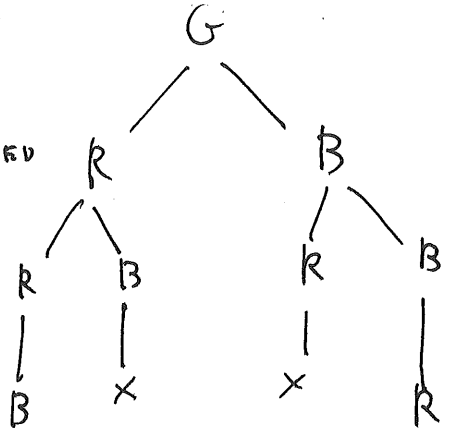
MOTIVATION :

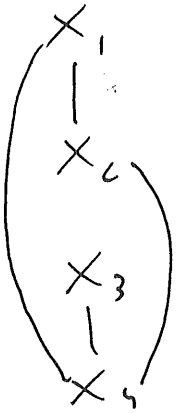
NOT F.C.

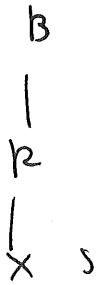




NOT EXPANDED BY F.C.



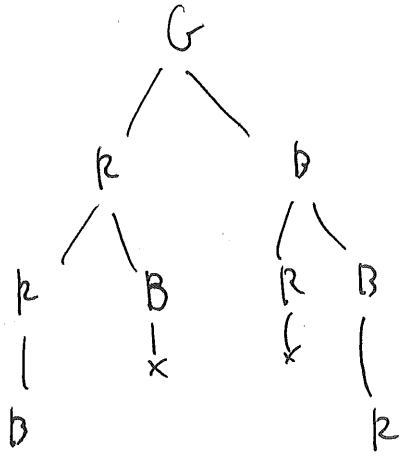




STOP

$X_1 = B$, $X_2 = R$

MAKE $X_4 = \emptyset$



EX 3 [BUCKET ELIMINATION]

X_3 F_{13} F_{34} F_{23} $\rightarrow$ H^3_{124} $\rightarrow$ $2^4 = 16$

X_4 F_{14} H^3_{124} $\rightarrow$ H^4_{12}

X_1 F_{12} H^4_{12} $\rightarrow$ H^1_2

X_2 H^1_2 $\rightarrow$ M

↑ 0

3.1 TO PROCESS BUCKET X_3 WE MUST GENERATE A TABLE WITH 4 COLUMNS, SINCE EACH VARIABLE HAS 2 POSSIBLE VALUES (I.E., THE NETWORK IS BINARY) AND SINCE WE DO NOT HAVE HARD CONS. THE TABLE WILL HAVE $2^4 = 16$ ENTRIES

3.2 YES IT IS POSSIBLE THE ORDER $\{X_1, X_2, X_3, X_4\}$ GIVES A LARGEST TABLE THAT IS $2^3 = 8$ ENTRIES.

SEE BELOW:

X_4 F_{14} F_{34} $\rightarrow$ H^4_{13} $\rightarrow$ $2^3 = 8$

X_3 F_{13} F_{23} H^4_{13} $\rightarrow$ H^3_{12} $\rightarrow$

X_2 F_{12} H^3_{12} $\rightarrow$ H^2_1

X_1 H^2_1 $\rightarrow$ M

Ex 4 [BAYESIAN NETWORK]

4.1 YES, BECAUSE THE MARKOV BLANKET FOR D IS $\{A, C\}$ AND THUS D IS CONDITIONALLY INDEPENDENT FROM ALL OTHER VARIABLES (I.E., B) GIVEN THE VARIABLES IN THE MARKOV BLANKET

4.2

$$P(B \mid C=T) = \alpha \sum_{A,D} P(D \mid A, C=T)\, P(C=T \mid A, B)\, P(A)\, P(B)$$

$$= \alpha\, P(B) \sum_{A} P(C=T \mid A, B)\, P(A) \sum_{D} P(D \mid A, C=T)$$

↑

$P(B)$ DOES NOT DEPEND ON A, D; $P(C=T \mid A, B)$ DOES NOT DEP. ON D; $P(A)$ DOES NOT DEP. ON D

$$= \alpha\, P(B) \sum_{A} P(C=T \mid A, B)\, P(A)$$

↑

$D \notin \text{ANCESTORS}(B, C) = \{A\}$

HENCE D IS IRRELEVANT.