Paramodulation

Summary

- Paramodulation [Chang-Lee 8.3]
- Linear Paramodulation [Chang-Lee 8.6]

Why a new inference rule

Need of a new inference rule

- Equality Relation: reflexivity, simmetry, transitivity, subs of equals
- Need extra equality axioms K to represent those properties
- Applying standard resolution to $S \cup K$ is inefficient
- Need a specific inference rule to handle equality: Paramodulation

Paramodulation: Example

Example (unit clause)

Consider the following clauses:

- $C_1 : P(a)$
- $C_2 : a = b$

By Exploiting the equality axioms, we can infer $C_3 : P(b)$.

Equality substitution

Definition (Equality substitution)

If a clause C contains a term t ($C[t]$) and if a unit clause is $t = s$ then we can infer a new clause by substituting s for a single occurrence of t (denoted by $C[s]$).

Paramodulation for Ground clauses

Ground Paramodulation

- $C_1 : L[t] \vee C_1'$
- $C_2 : t = s \vee C_2'$

We can infer the paramodulant

$$L[s] \vee C_1' \vee C_2'$$

Paramodulation: Example

Example (Ground)

- $C_1 : P(a) \vee Q(b)$
- $C_2 : a = b \vee R(b)$

We can infer the paramodulant: $P(b) \vee Q(b) \vee R(b)$

Paramodulation for General clauses

General Clauses

- Paramodulation can be applied to general clauses
- Instantiate before paramodulation

Example

- $C_1 : P(x) \vee Q(b)$
- $C_2 : a = b \vee R(b)$
- $\sigma = \{a/x\}$ and $C_1' : C_1\sigma = P(a) \vee Q(b)$
- $C_3' : P(b) \vee Q(b) \vee R(b)$ paramodulant of C_1 and C_2

Paramodulation

Definition (Paramodulation)

- $C_1 : L[t] \vee C_1'$
- $C_2 : r = s \vee C_2'$
- C_1 and C_2 have no variables in common, and σ MGU for t and r.

We can infer the binary paramodulant

$$L\sigma[s\sigma] \vee C_1'\sigma \vee C_2'\sigma$$

Where $L\sigma[s\sigma]$ is obtained by replacing a single occurrence of $t\sigma$ in $L\sigma$ by $s\sigma$.

- The inferred clause is a binary paramodulant of C_1 and C_2
- L and $r = s$ are the literals paramodulated upon
- paramodulation from C_2 into C_1

Paramodulation: Example

Example (Paramodulation)

- $C_1 : P(g(f(x))) \vee Q(x)$
- $C_2 : f(g(b)) = a \vee R(g(c))$

Applying paramodulation

- $t : f(x)$, $L[t] : P(g(\underline{f(x)}))$
- $r : f(g(b))$, $r = s : f(g(b)) = a$
- $\sigma = \{g(b)/x\}$
- $P(g(a)) \vee Q(g(b)) \vee R(g(c))$

Paramodulation: Example II

Example (Paramodulation)

- $C_1 : P(f(x, a), y) \lor R(y)$
- $C_2 : f(c, a) = g(b) \lor R(g(b))$

Paramodulants

- $P(g(b), y) \lor R(y) \lor R(g(b))$
- $P(f(x, a), g(b)) \lor R(f(c, a)) \lor R(g(b))$
- $P(f(x, a), f(c, a)) \lor R(g(b))$

Paramodulant

Definition (Paramodulant)

A paramodulant of (parent) clauses C_1 and C_2 is one of the following binary paramodulant:

1. A binary paramodulant of C_1 and C_2;
2. A binary paramodulant of a factor of C_1 and C_2;
3. A binary paramodulant of C_1 and a factor of C_2;
4. A binary paramodulant of a factor of C_1 and a factor of C_2;

Paramodulation: characteristics

characteristics

- Paramodulation combined with resolution is refutationally complete for E-Satisfiablity
- Using paramodulation and resolution we can always generate $\square$ from an E-unsatisfiable set of clauses
- Many refinements are still complete: e.g., linear paramodulation

Linear Paramodulation

Definition (Linear paramodulation)

- S: set of clauses, C_0: clause in S
- A linear deduction of C_n from S with a top clause C_0 by resolution and paramodulation is a sequence of clauses $R_1, \cdots, R_n$ where $R_n = C_n$ and:
  - $R_0 = C_0$
  - for $i = 0, \cdots, n-1$ R_{i+1} is a resolvent or a paramodulant of C_i (called center clause) and B_i (called side clause)
  - each B_i is either a clause in S or is a C_j for $j < i$
- A linear refutation by resolution and paramodulation is a linear deduction of $\square$ by resolution and paramodulation.

Linear Paramodulation: Example I

Example (Linear Paramodulation I)

- $S = \{\neg Q(d), Q(c) \vee c \neq d, c = d\}$
- Find a linear refutation from S with top clause $c = d$

Functionaly Reflexive axioms

Definition

- S: set of clauses
- $f(x_1, \cdots, x_n)$ n-place function symbol occurring in S
- Set F of functionally reflexive axioms for S is defined as

$$F \triangleq \{f(x_1, \cdots, x_n) = f(x_1, \cdots, x_n)\}$$

Linear Paramodulation: Example II

Example (Example: Linear Paramodulation II)

- $S = \{\neg Q(c) \vee c = d, \neg Q(c) \vee f(c) \neq f(d), Q(c) \vee a = b, Q(c) \vee f(a) \neq f(b), f(x) = f(x)\}$
- Find a linear refutation of S with top clause $\neg Q(c) \vee c = d$

Note

Can we derive $\Box$ with $S \setminus \{f(x) = f(x)\}$

Completeness Ground

Theorem (ground completeness)

- *S: E-unsatisfiable set of ground clauses and C clause in S*
- *S contains all ground instances of* $x = x$
- *If* $S - \{C\}$ *is E-satisfiable, then S has a linear refutation by resolution and paramodulation with top clause C*

Completeness

Theorem (completeness)

- *S: E-unsatisfiable set of clauses and C clause in S*
- *S contains $x = x$ and the set of functionally reflexive axioms for S*
- *If $S - \{C\}$ is E-satisfiable, then S has a linear refutation by resolution and paramodulation with top clause C*

discusison

- Very similar to linear ordered resolution
- But here we have to store center clauses, less efficient
- Above completeness results does not hold for ordered linear paramodulation

Exercise I

Exercise I

Find all paramodulant of C_1 and C_2

- $C_1 : P(f(x, g(x))) \vee Q(x)$
- $C_2 : a = b \vee g(a) = a \vee f(a, g(a)) = b$

Exercise II

Exercise II

Given

$$S = \{R(a) \vee R(b), \neg D(y) \vee L(a,y), , \neg R(x) \vee \neg Q(y) \vee \neg L(x,y), , D(a) \vee \neg Q(a), Q(b) \vee \neg R(b), a = b\}$$

Find a linear refutation from S with top clause $a = b$.

Exercise III

Exercise III

Given

$$S = \{P(b) \vee Q(a), P(a) \vee \neg Q(b), \neg P(a) \vee Q(b), \neg P(b) \vee \neg Q(a), a = b\}$$

Find a linear refutation from S with top clause $a = b$.

Further exercise

Try with $S' = S \setminus \{a = b\} \cup \{b = a\}$ and top clause $b = a$