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Diffusion Geometry in Shape Analysis

SOME APPLICATIONS

Alex Bronstein, Umberto Castellani, Michael Bronstein

Overview

- Diffusion geometry in neuroscience
  - HKS-based descriptors
  - Learning best scales
- Robust large-scale shape retrieval benchmark
  - Benchmark definition
  - Different variations of local shape descriptors: FEM-HKS, SI-HKS, VHKS
  - Global shape descriptor by Bag of Features approach

Diffusion geometry in neuroscience

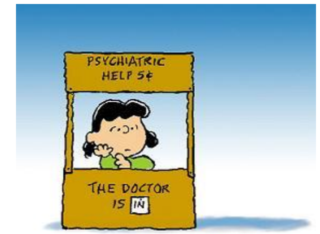
- 2-class classification problem
  - Characterizing healthy (*controls*) and pathological subjects (*patients*) based on the observation of morphological properties of the brain
- Challenging problem
  - Currently not diagnosed from MRI images
- Encouraged by medical studies

Find possible connections between brain morphological abnormalities and the disease

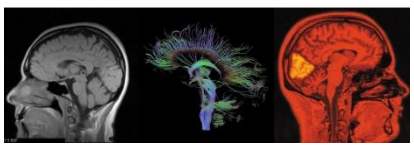
Overall scheme

Clinical method

Schizophrenia → Interview



Research

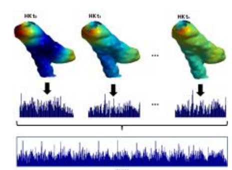


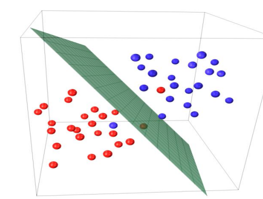
sMRI DTI fMRI

Imaging

[VBM: J. Ashburner 00]
[DBM: Gaser 00]
[SBM: Styner 06]

Brain Morphometry



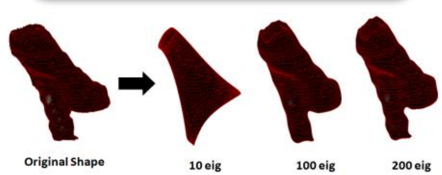


[Shape DNA: Reuter 09]

Spectral shape Analysis → Diffusion shape description → SVM classification

[Castellani 11]

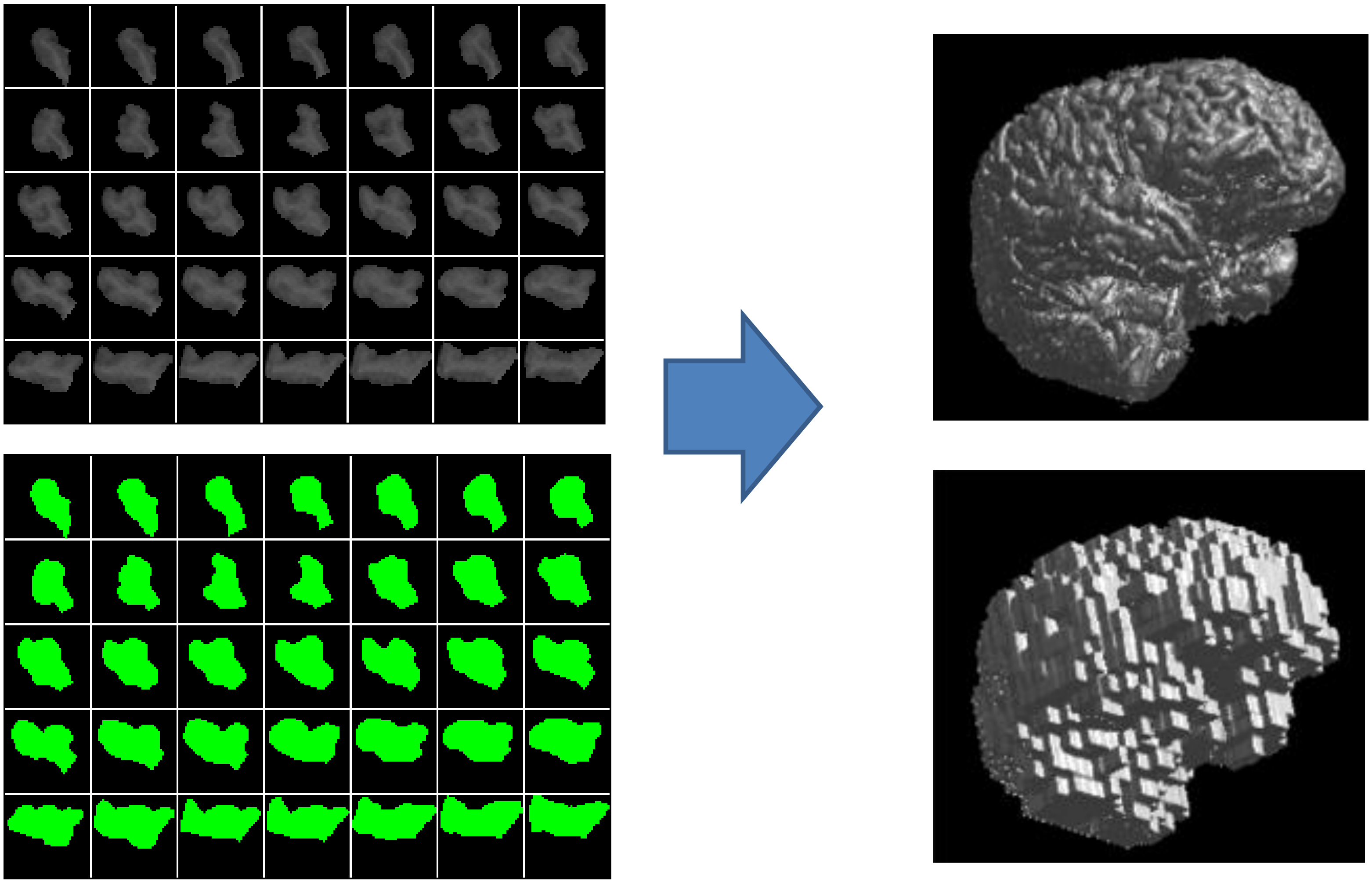
[Vapnik 95]



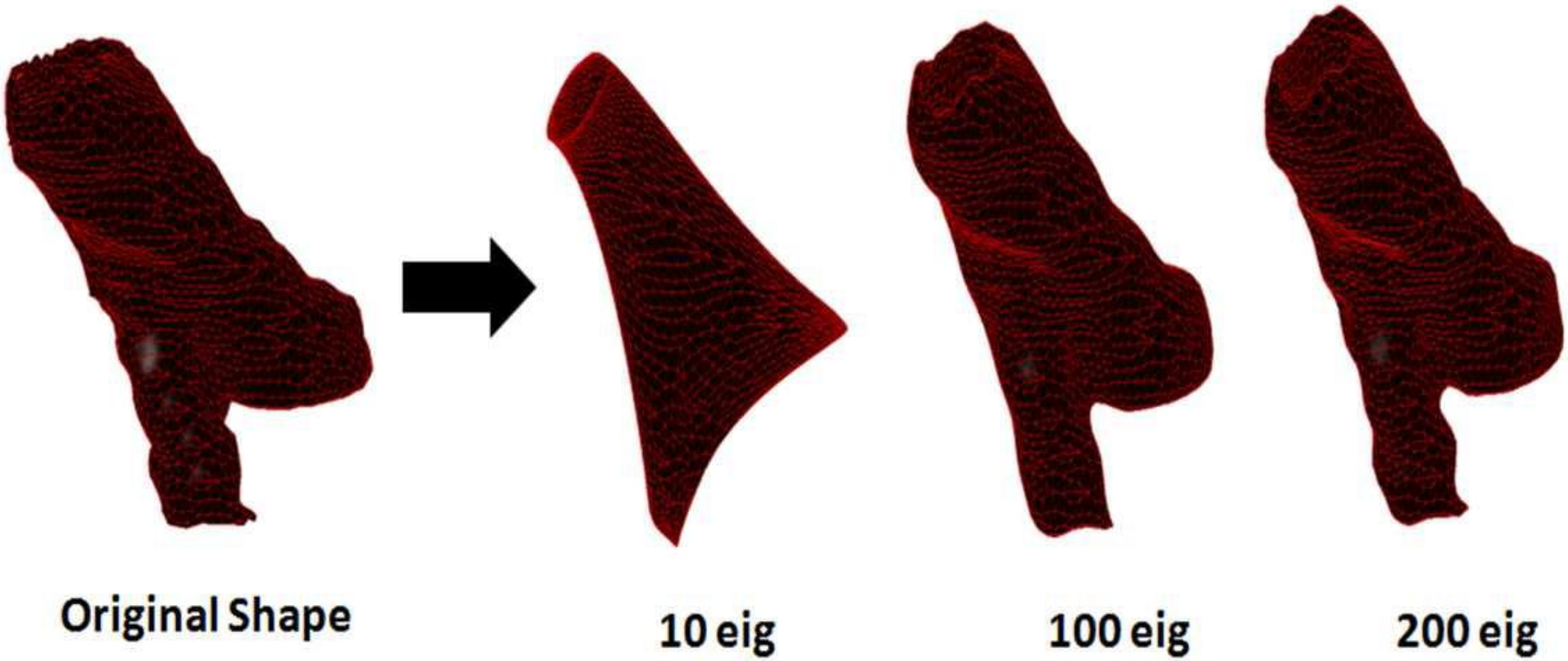
Main steps

1. From MRI slices to 3D surfaces or solid
2. Spectral shape analysis
3. Pointwise Heat diffusion process
4. Global shape descriptor (GHKS)
5. Classification

From MRI slices to 3D surface or solid



Spectral shape analysis



Thalamus

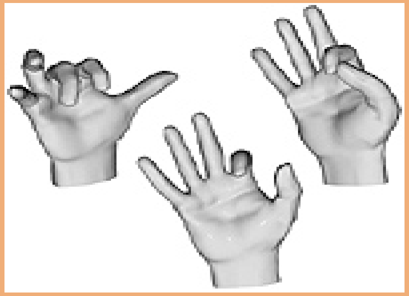
Spectral shape analysis

Surface-based

- only the boundary of the shape is considedered
- Surface is considered as Riemannian manifold
- It is invariant to surface isometries

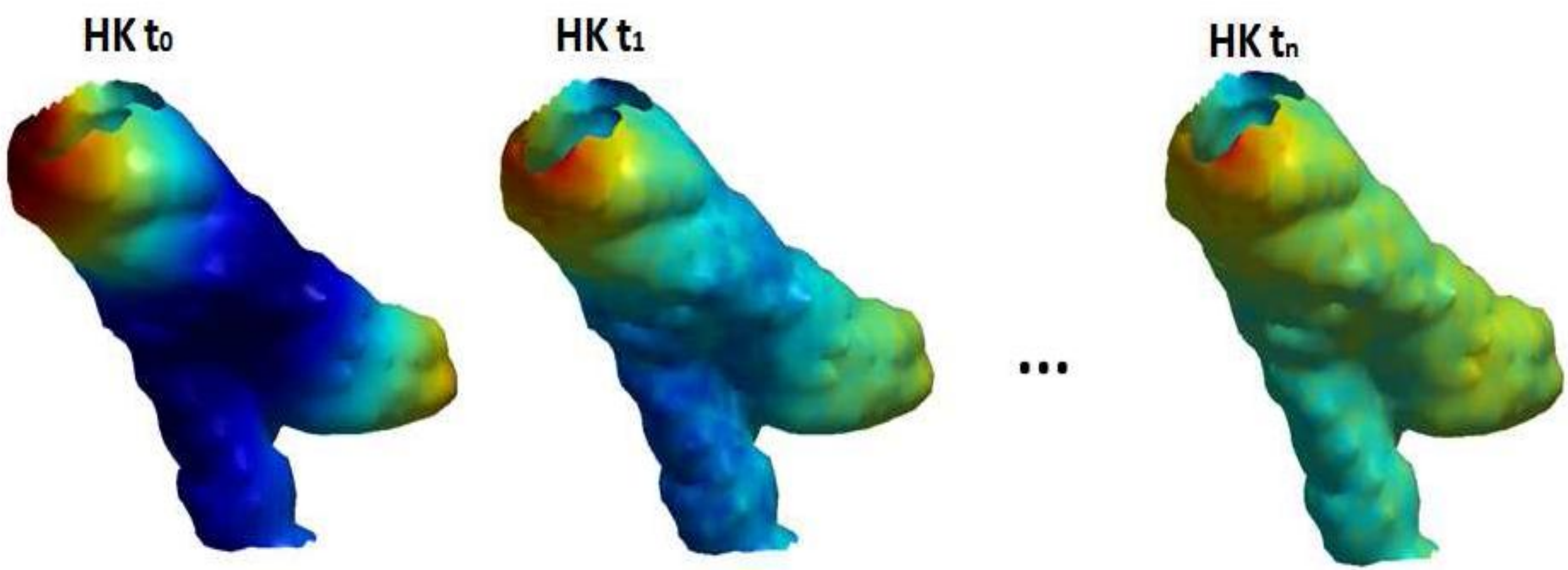
Volume-based

- Also the internal part of the shape (i.e., voxels) is considered
- Voxels are on a regular grid
- It is invariat to volume isometries (i.e., isometries preserving volume).



Pointwise diffusion process

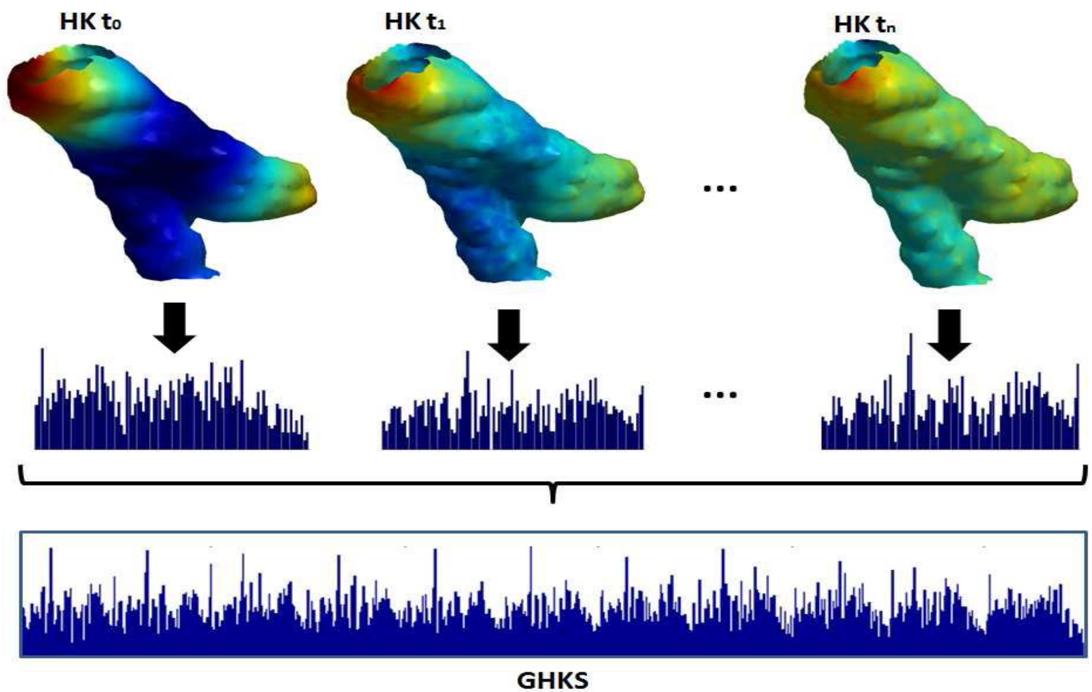
- Heat kernel signature:
  - $HKS(x) = [k_{t_0}(x, x), \cdots, k_{t_n}(x, x)]$.



Global shape descriptor

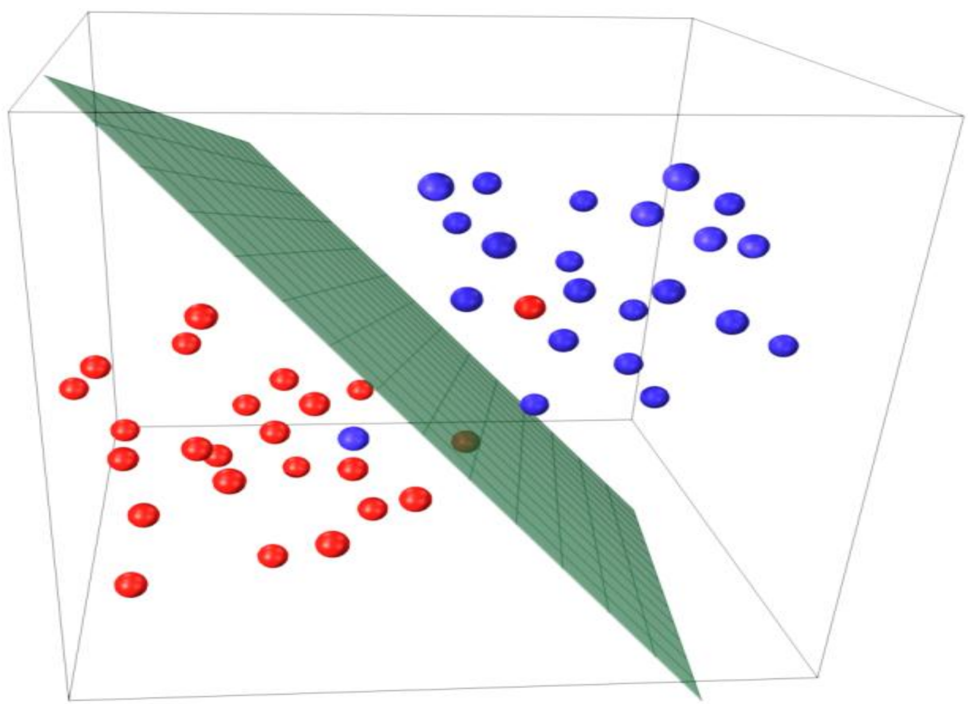
- From point description to whole shape descriptor:
  - $GHKS(M) = [hist(Kt_0(M)), \cdots, hist(Kt_n(M))]$,

    $Kt_i(M) = \{kt_i(x, x), \forall\, x \in M\}$



Classification

- A Support Vector Machine (SVM) can be used for classification



Results

Method	Linear-SVM	Polynomial-SVM	RBF-SVM
Surface GHKS	65.00%	66.67%	71.67%
Volumetric GHKS	81.67%	80.00%	**83.33%**
Surface ShapeDNA	50.00%	66.67%	70.00%
Volumetric ShapeDNA	50.00%	71.67%	73.33%

- 30 patients 30 controls
- LOO cross validation
- n=200 time values
- 100 bins per histogram

Learning best scales

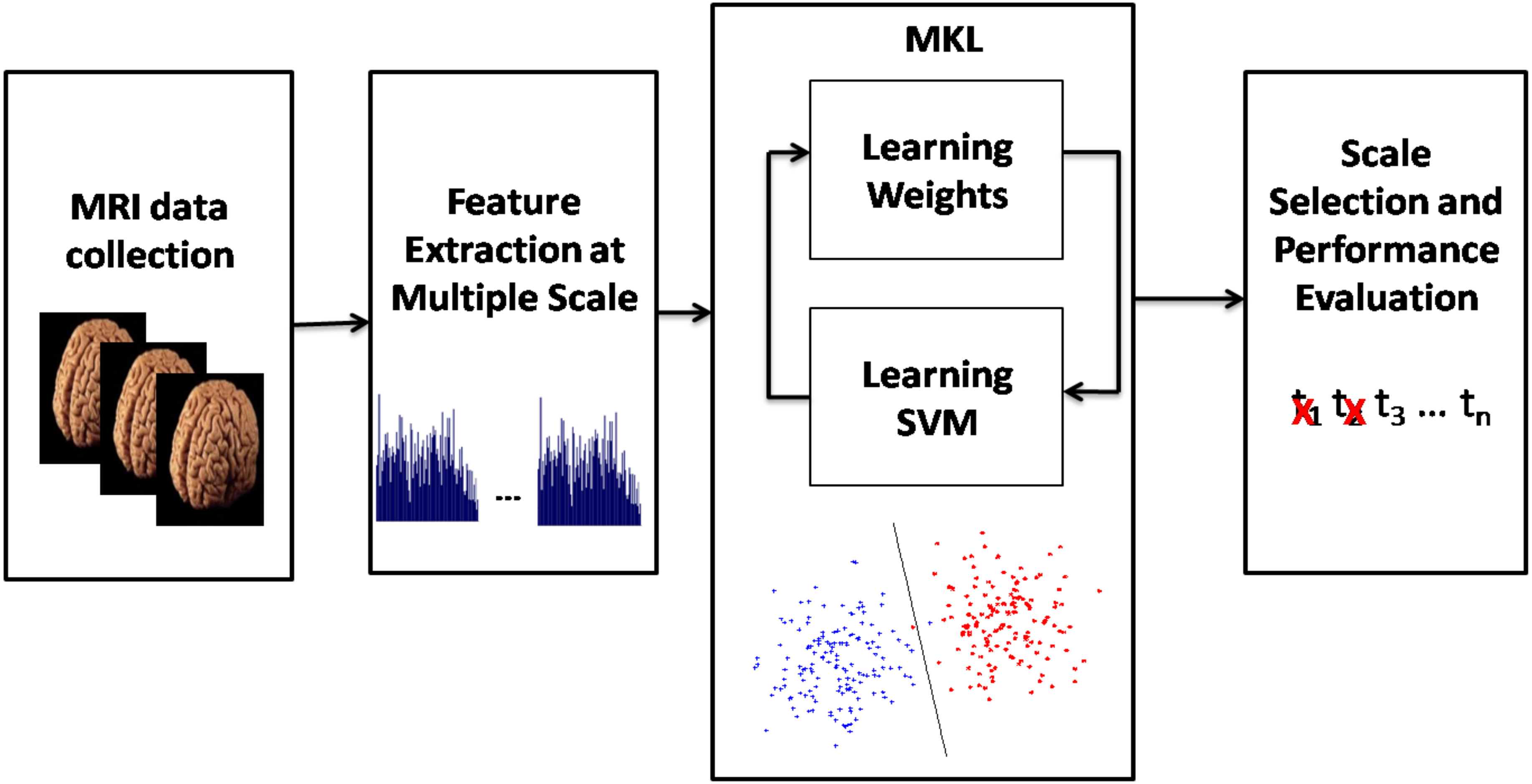
- Shape diffusion methods have proved to be very effective in providing useful descriptions for shape classification purposes:
  - They capture intrinsic properties of shape at different scales
  - They provide effective shape descriptors
  - They are very informative: *small* scales encode local properties, *large* scales encode global properties

Learning best scales

- The selection of the scales is very important:
  - for a particular shape, some scales may be highly discriminative, while some other scales should encode useless information

Scales can be selected by a learning procedure

General schema



Learning by MKL

- Learning can be addressed by Multiple Kernel Learning (MKL):

$$k_\eta(\boldsymbol{x}_i, \boldsymbol{x}_j; \boldsymbol{\eta}) = \sum_{m=1}^{P} \eta_m k_m(\boldsymbol{x}_i^m, \boldsymbol{x}_j^m)$$

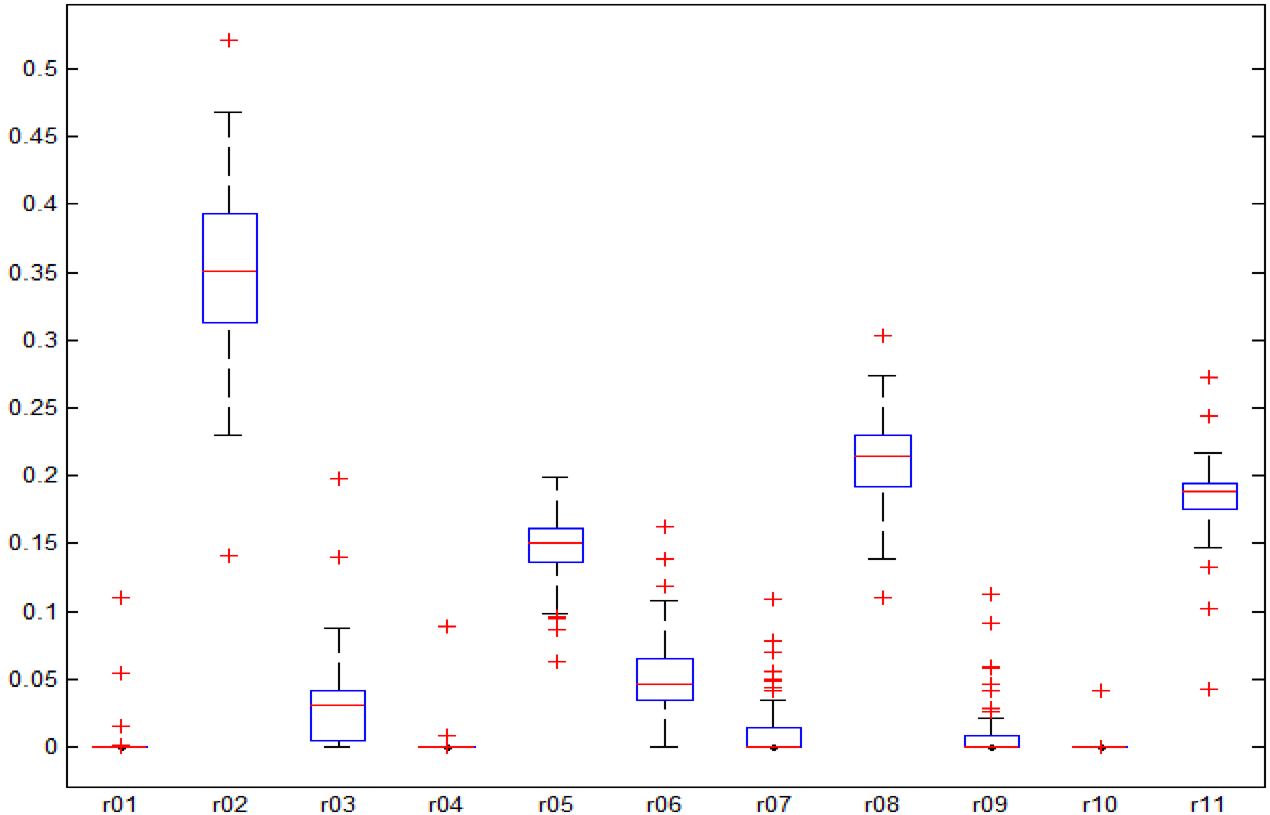
- In practice, each shape representation at scale $t=m$ is associated to a kernel k_m by leading to P kernels
- The final kernel is plugged into a Support Vector Machine for classification. According to MKL procedure both SVM parameters and kernel weights are estimated in the same learning procedure

Results

- 11 representation from 11 scales

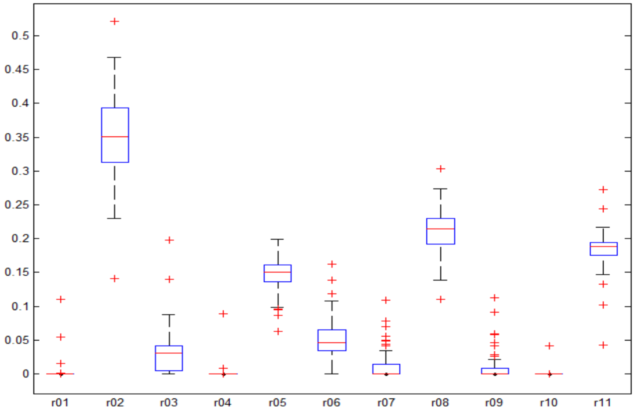
Single-best	SVM-con	RBMKL	SimpleMKL	GLMKL
78.33	83.33	81.67	**86.67**	85.00

Relevance of each scale



Take home message...

- Being driven by the training data, we are able to choose the scales of the heat kernel which are more suitable to describe our kind of shapes.



In this experiments both small and high scales are crucial

Robust large-scale shape retrieval benchmark

SHREC database

http://tosca.cs.technion.ac.il/book/shrec_robustness.html

- Retrieve shapes in large-scale dataset under a variety of transformations
- Test robustness to different types of transformations
- Test robustness to different strength of transformations

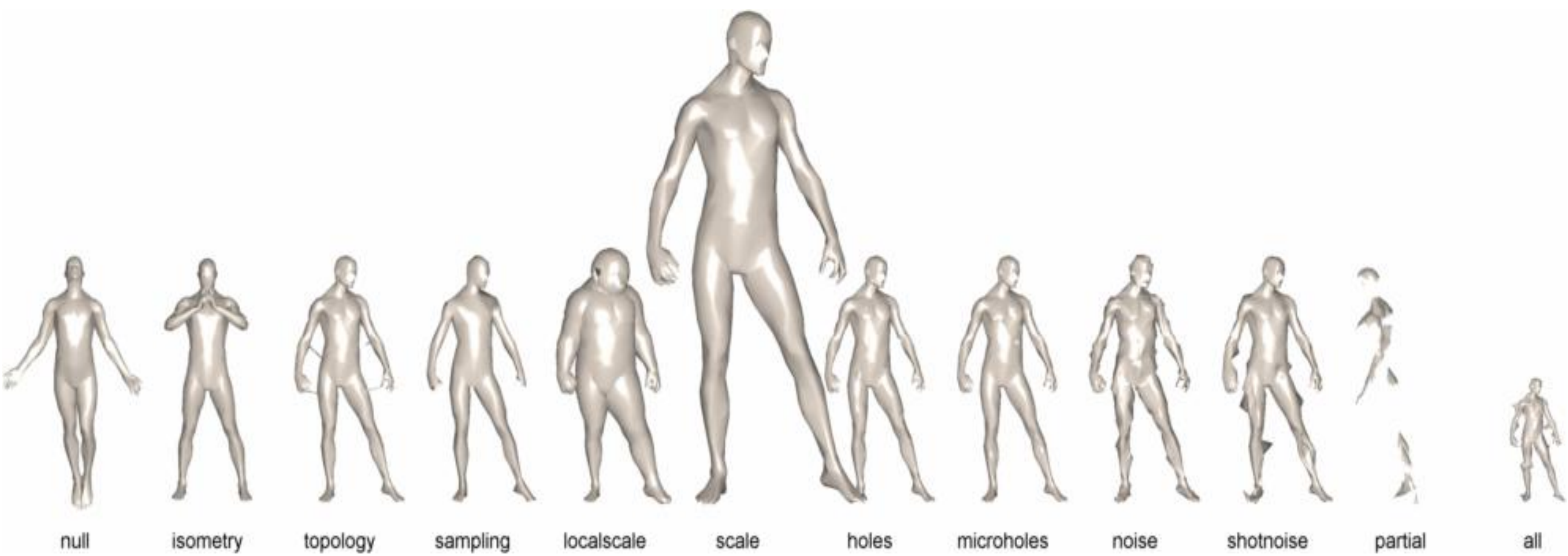
Dataset

- **Source:** shapes from TOSCA, Robert Sumner, and Princeton dataset
- **Positive:** 13 basic shapes (i.e., null shapes)
- **Negative**: 456 general shapes
- **Query set**: 13 shape classes X 11 transformation typesX5transformation strenghts $\Rightarrow$ 715 shapes

- **Total dataset size**: 1184 shapes

Positive and negative models

Query set



Null shape and 11 transformed shapes, the same transformations are applied to all 13 positive shapes, each transformation is applied at 5 different strengths

Evaluation

- **Goal**: retrieve transformed shapes from the query set in a database of null shapes (positive) and other general shapes (negative)
- **Retrieval performance**: mean average precision (mAP)
- Retrieval results broken down according to transformation **type** and **strength**

Diffusion methods

- ShapeGoogle with FEM heat kernel descriptors (SG-1:FEM-HKS)[1]
- ShapeGoogle with scale-invariant heat kernel descriptors (SG-2: SI-HKS)[2]
- ShapeGoogle with Volumetric heat kernel (SG-3:VHKS)[3]

1. **G. Patane, M. Spagnuolo, B. Falcidieno**
2. **M. M. Bronstein, I. Kokkinos**
3. **D. Raviv, A.M. Bronstein, M.M. Bronstein, R. Kimmel**

Global shape descriptor

- HKS-based descriptors encode local information
- In order to compare two different shapes a global signature i required

In ShapeGoogle methods global signature is defined by a bag of words approach

Results

- BoW signatures are compared by L1 or L2 norm (some ad hoc distance for histograms can be considered as well)

	Strength				
Transform.	1	≤2	≤3	≤4	≤5
Isometry	100.00	100.00	100.00	100.00	100.00
Topology	100.00	98.08	97.44	96.79	96.41
Holes	100.00	100.00	97.44	95.19	90.13
Micro holes	100.00	100.00	100.00	100.00	100.00
Scale	0.98	40.68	43.31	33.72	27.42
Local scale	100.00	100.00	98.72	89.38	80.22
Sampling	100.00	100.00	100.00	100.00	99.23
Noise	100.00	100.00	100.00	100.00	100.00
Shot noise	100.00	100.00	100.00	100.00	100.00
Partial	7.54	5.70	4.51	3.58	2.95
Mixed	53.13	55.86	47.77	37.54	30.34

(SG-1: FEM HKS)

	Strength				
Transform.	1	≤2	≤3	≤4	≤5
Isometry	100.00	100.00	100.00	100.00	100.00
Topology	96.15	96.15	94.87	93.27	92.69
Holes	100.00	100.00	100.00	94.71	89.97
Micro holes	100.00	100.00	100.00	100.00	100.00
Scale	91.03	95.51	97.01	97.76	98.21
Local scale	100.00	100.00	97.44	89.38	82.08
Sampling	100.00	100.00	100.00	100.00	97.69
Noise	100.00	100.00	100.00	100.00	100.00
Shot noise	100.00	100.00	100.00	100.00	100.00
Partial	17.43	10.31	9.57	8.06	6.61
Mixed	56.47	57.44	63.59	67.47	65.07

(SG-2: SI-HKS)

	Strength				
Transformation	1	≤2	≤3	≤4	≤5
Isometry	100.00	100.00	100.00	100.00	100.00
Topology	100.00	100.00	100.00	100.00	100.00
Holes	100.00	100.00	100.00	100.00	98.75
Micro holes	100.00	100.00	100.00	100.00	100.00
Scale	0.61	11.94	8.81	6.74	5.46
Local scale	100.00	93.35	81.86	69.04	60.81
Sampling	100.00	100.00	100.00	100.00	100.00
Noise	100.00	100.00	100.00	100.00	100.00
Shot noise	100.00	100.00	100.00	100.00	100.00
Mixed	100.00	61.14	41.65	31.47	25.29

(SG-3: VHKS)

Conclusions

- Diffusion geometry allows the definition of powerful shape descriptors for several applicative scenarios
- Performance of diffusion-geometry-based approaches are in general better than other state of the art methods
- Diffusion-geometry-based approaches perform well on challenging scenarios (i.e., medical domain)