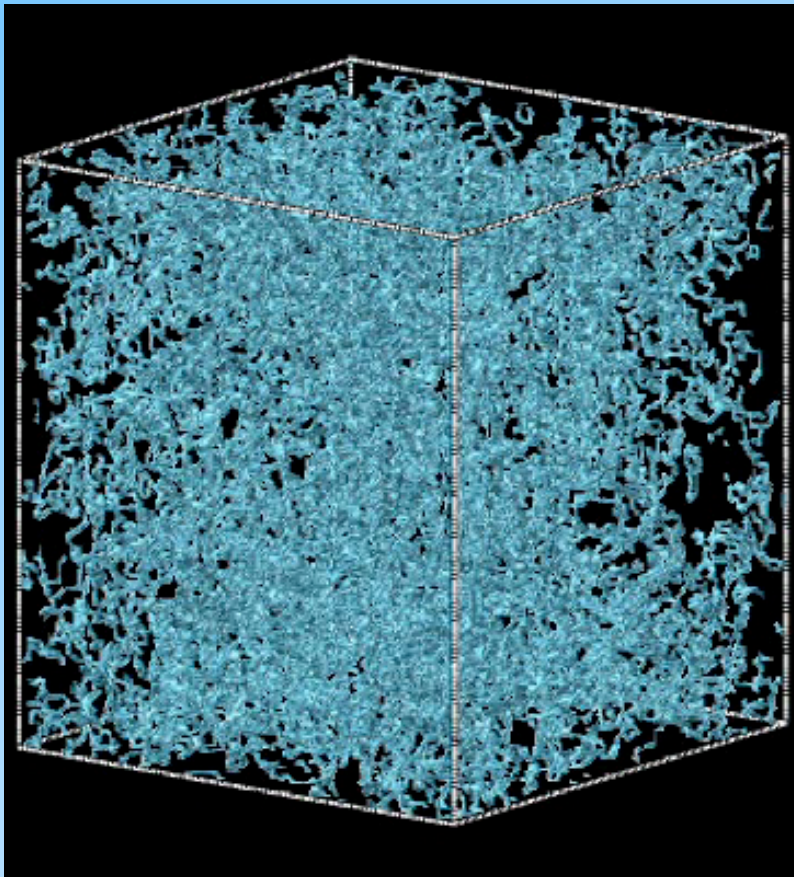


Quantum Turbulence

- from Superfluid Helium to Atomic Bose-Einstein Condensates -



Makoto TSUBOTA
Department of Physics,
Osaka City University, Japan

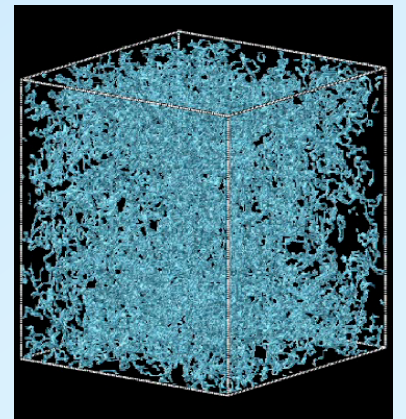
Thanks to the many collaborators

Review article

- M. Tsubota, J. Phys. Soc. Jpn. 77 (2008) 111006
- Progress in Low Temperature Physics Vol.16, eds. W. P. Halperin and M. Tsubota, Elsevier, 2008

My messages

1. Quantum turbulence (QT) has recently become one of the most important branches in low temperature physics.
2. QT has been studied thoroughly in superfluid ^4He and ^3He , and recently even in atomic Bose-Einstein condensates.
3. The research of QT can make a major breakthrough in the great mystery in nature, namely *turbulence*.





Leonardo Da Vinci
(1452-1519)



**Da Vinci observed turbulent flow
and found that turbulence consists
of many vortices.**

*Turbulence is not a simple disordered state
but having some structures with vortices.*

Certainly turbulence looks to have many vortices.

Turbulence behind a dragonfly

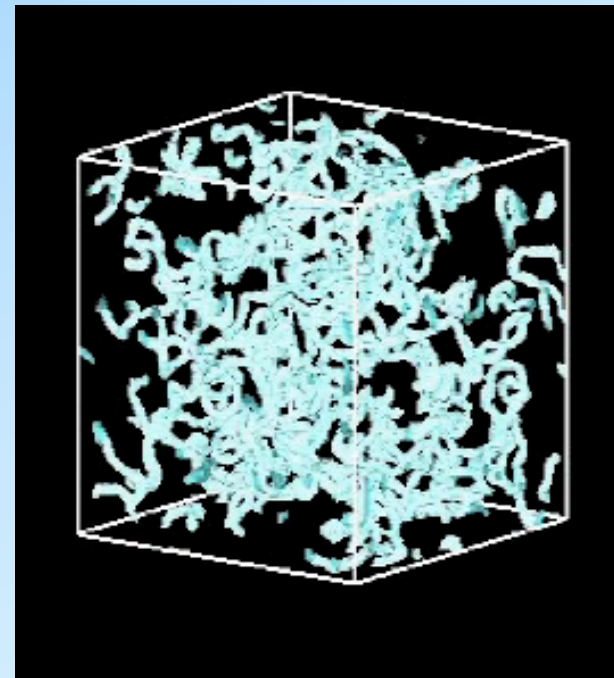


<http://www.nagare.or.jp/mm/2004/gallery/iida/dragonfly.html>

It is not so straightforward to confirm the Da Vinci message in classical turbulence.

Key concept of my talk

The Da Vinci message is actually realized in quantum turbulence comprised of quantized vortices.



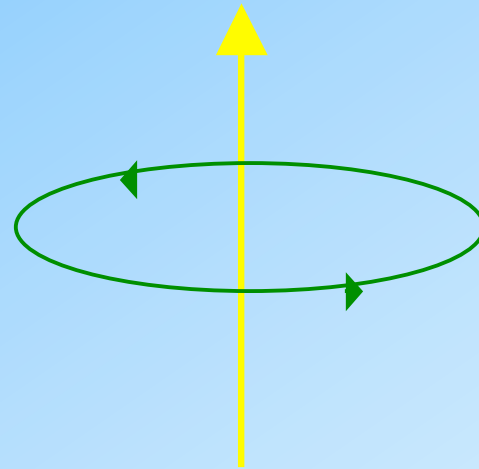
**A quantized vortex is a vortex of superflow in a BEC.
Any rotational motion in superfluid is sustained by
quantized vortices.**

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \quad (n = 0, 1, 2, \dots)$$
$$\kappa = h / m$$

A vortex with $n \geq 2$ is unstable.

⇒ **Every vortex has the same circulation.**

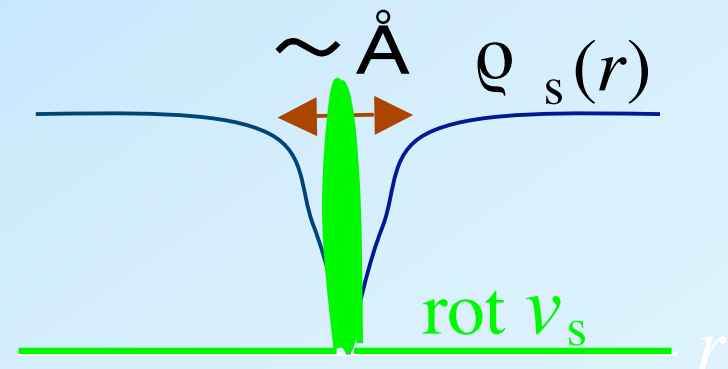


(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

⇒ **The vortex is stable.**

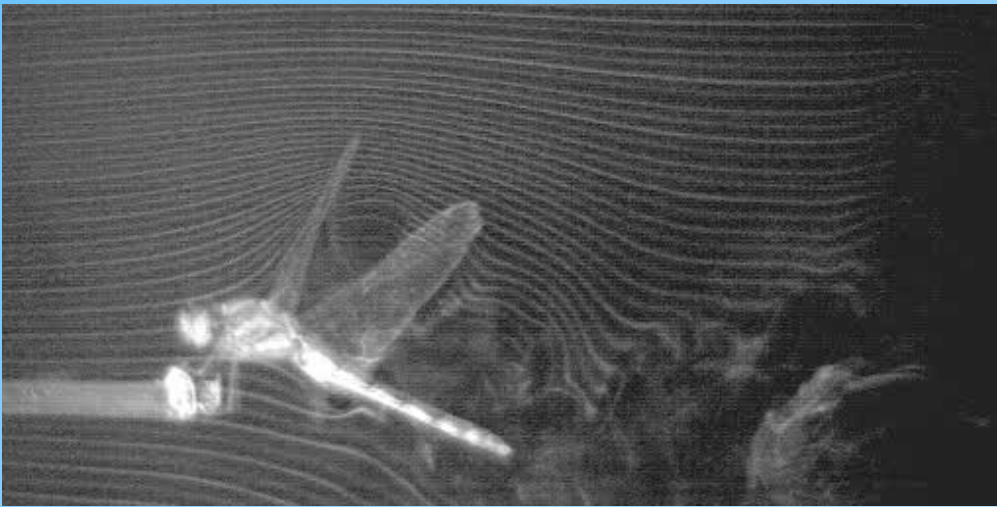
(iii) The core size is very small.

⇒ **The order of the coherence length.**



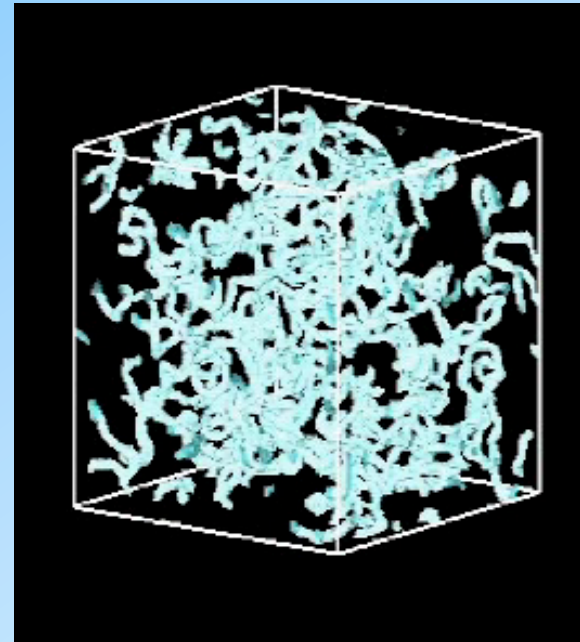
Classical Turbulence (CT) vs. Quantum Turbulence (QT)

Classical turbulence



QT is much simpler than CT, because each element of turbulence is definite.

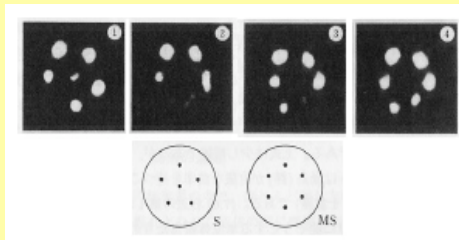
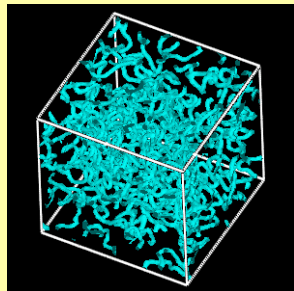
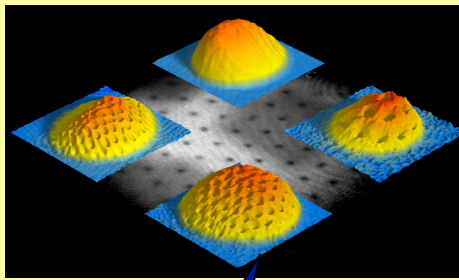
Quantum turbulence



Motion of
vortex
cores

- The quantized vortices are stable topological defects.
- Every vortex has the same circulation.
- Circulation is conserved.

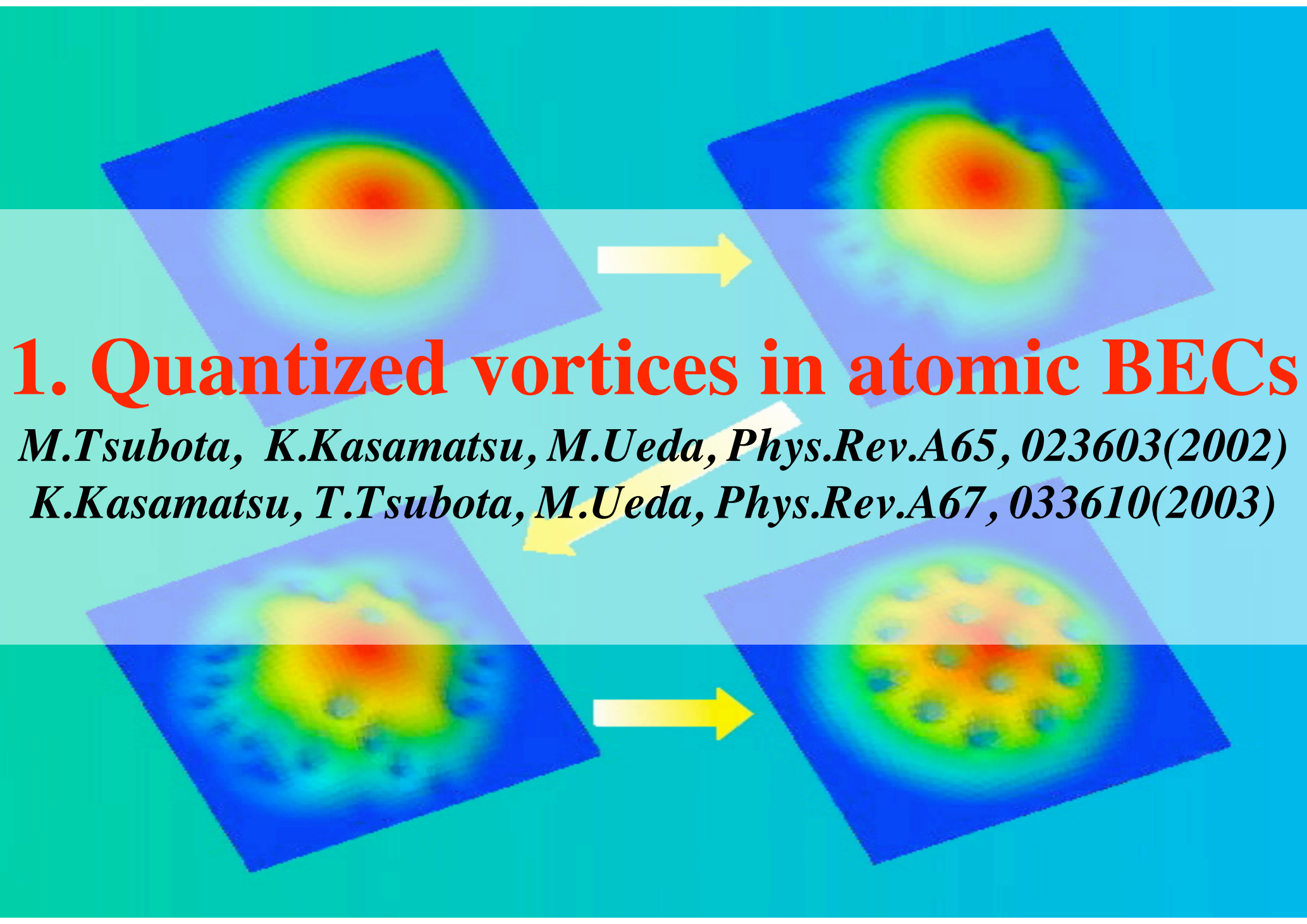
There are two main cooperative phenomena of quantized vortices; **Vortex array under rotation** and **Vortex tangle (Quantum turbulence)**.

	Vortex array	Vortex tangle
Superfluid He		
Atomic BEC		None

1.

2.

3.

The background of the slide features four heatmaps arranged in a 2x2 grid, each tilted at an angle. Each heatmap shows a central bright region (yellow/red) surrounded by a blue ring, representing the density distribution of an atomic Bose-Einstein condensate. Yellow arrows indicate a clockwise sequence from the top-left to the top-right, then to the bottom-right, and finally to the bottom-left. The heatmaps show the formation and evolution of quantized vortices, with the bottom-right image showing a more complex, multi-lobed structure.

1. Quantized vortices in atomic BECs

M.Tsubota, K.Kasamatsu, M.Ueda, Phys.Rev.A65, 023603(2002)

K.Kasamatsu, T.Tsubota, M.Ueda, Phys.Rev.A67, 033610(2003)

Advantages of atomic BECs as a physical system

Theoretical

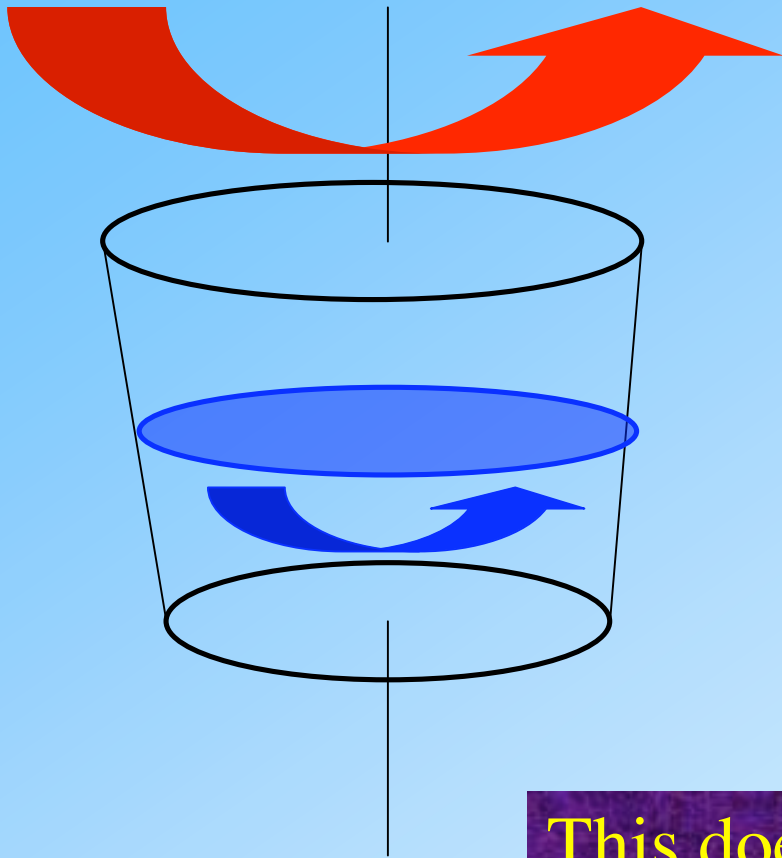
□ At low temperatures the model of dilute Bose gas and the mean field approximation theory (Gross-Pitaevskii (GP) model) are quantitatively correct.

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + g |\Psi(\mathbf{r}, t)|^2 \right] \Psi(\mathbf{r}, t)$$

Experimental

- We can directly control and visualize the condensate.
- We can change the atomic interaction by Feshbach resonance.

What happens if we rotate a vessel having a usual viscous classical fluid inside?



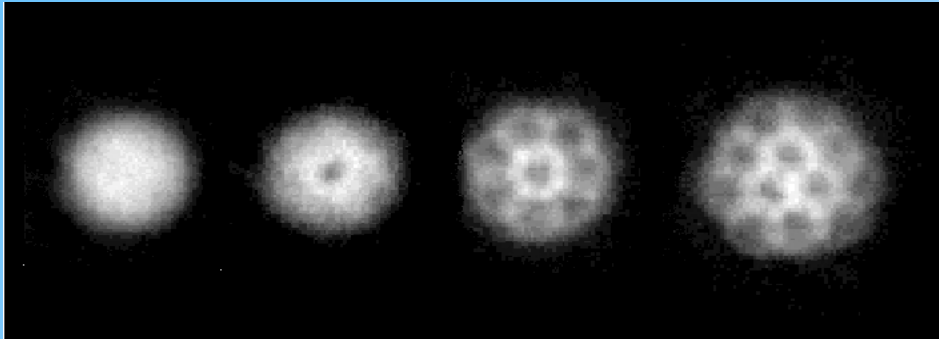
The fluid rotates with the same angular velocity with the vessel.

This means that there appears one vortex in the vessel. The single vortex can make the solid-body rotation with any angular velocity.

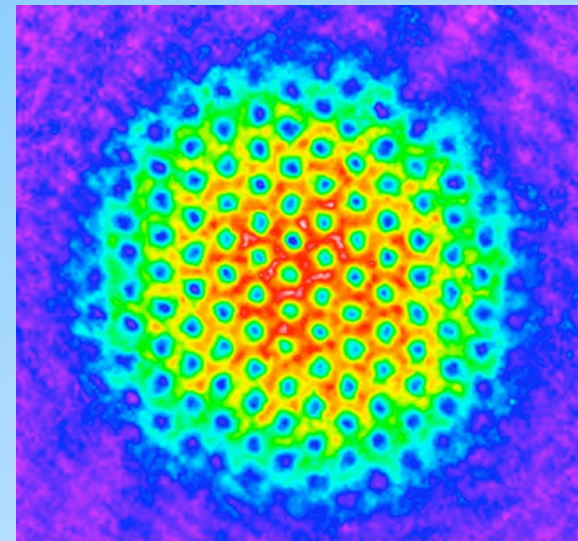
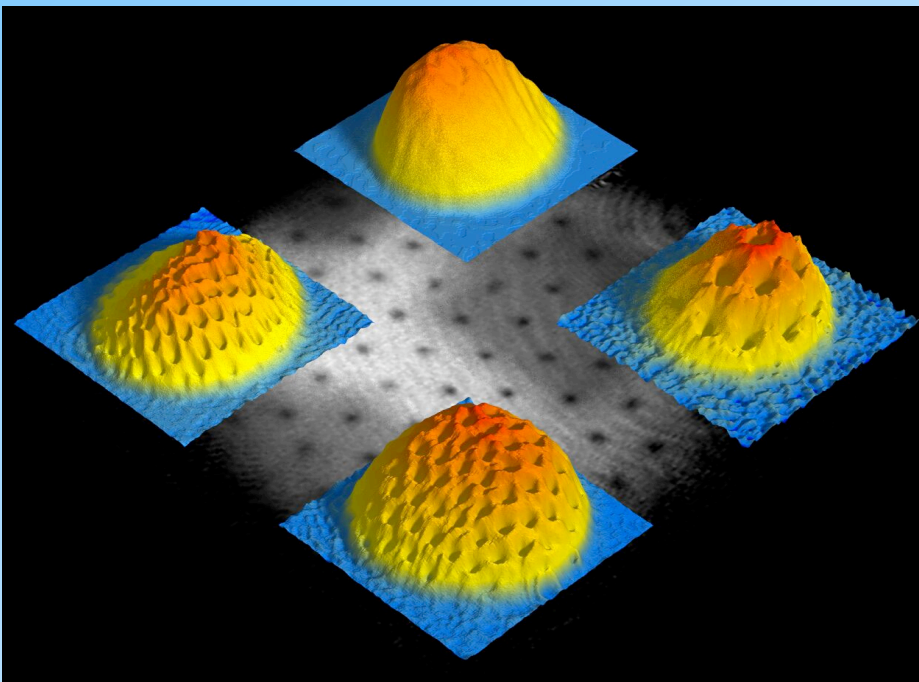
This does not occur in quantum fluids! Such experiments were done in atomic BECs.

Observation of quantized vortices in atomic BECs

ENS K.W.Madison, et.al PRL **84**, 806 (2000)



MIT J.R. Abo-Shaeer, et.al
Science **292**, 476 (2001)

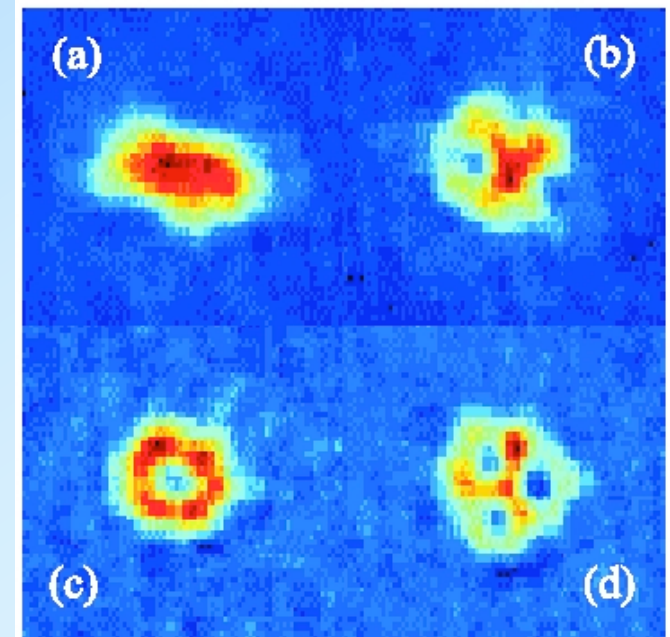


JILA

P. Engels, et.al
PRL **87**, 210403
(2001)

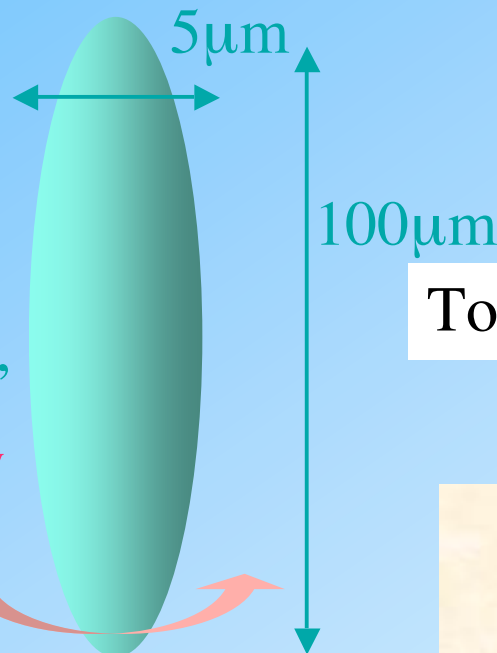
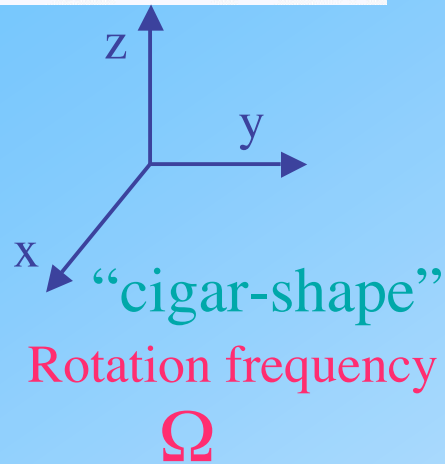
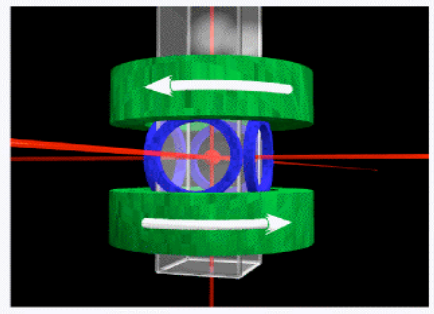
Oxford

E. Hodby, et.al
PRL **88**, 010405
(2002)

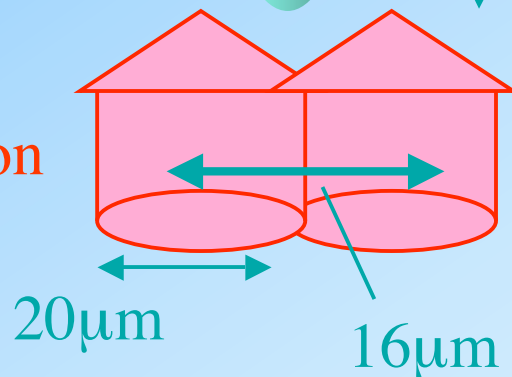


How can we rotate the trapped BEC ?

K.W.Madison et.al Phys.Rev Lett **84**, 806 (2000)



Optical spoon



Axisymmetric potential

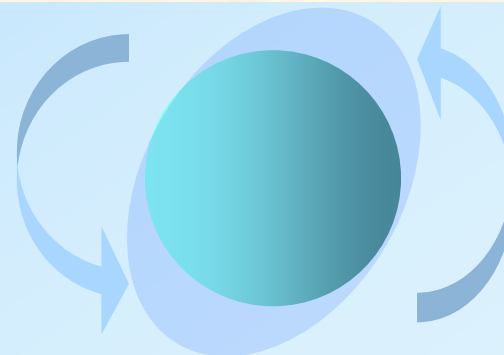
$$V_{\text{ext}}(\mathbf{R}) = V_{\text{trap}}(\mathbf{R}) + U_{\text{stir}}(\mathbf{R})$$

Total potential

Non-axisymmetric potential

$$U_{\text{stir}}(\mathbf{R}) = \frac{m}{2} \omega_{\perp}^2 (\epsilon_x X^2 + \epsilon_y Y^2)$$

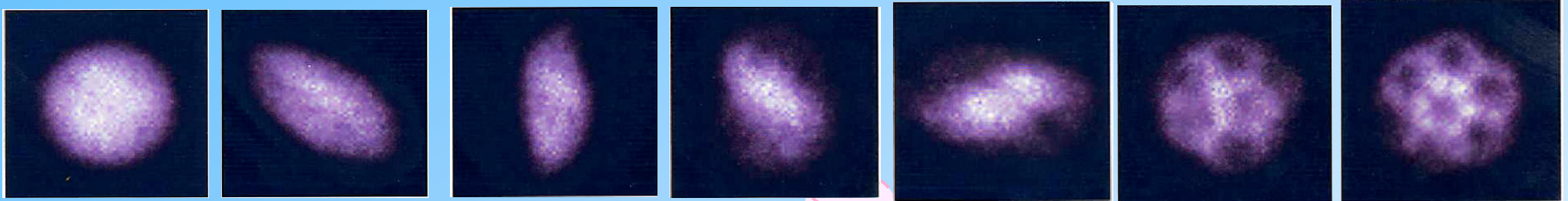
$$\epsilon_x \neq \epsilon_y$$



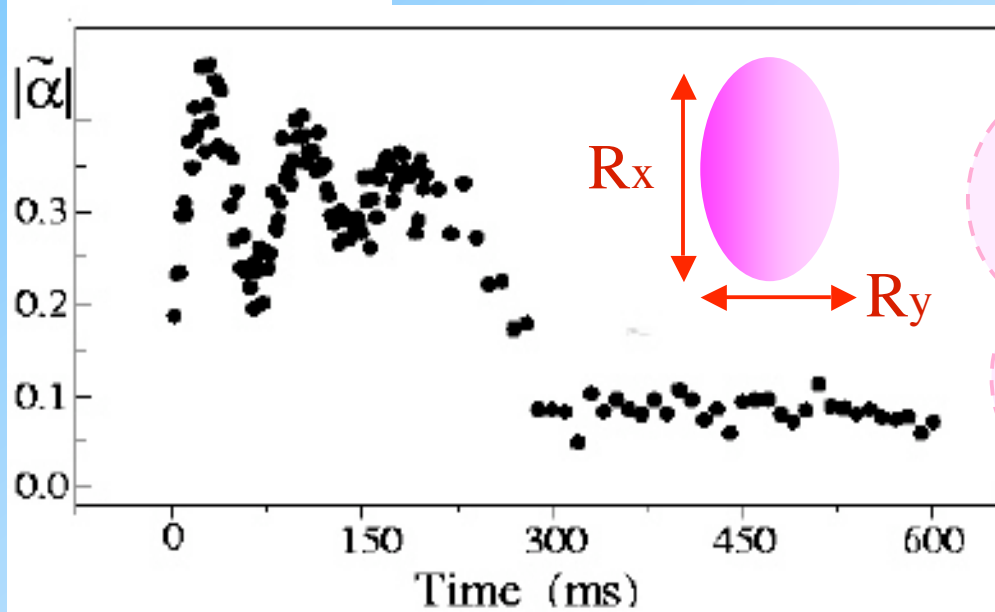
Direct observation of the vortex lattice formation

K.W. Madison *et al.* PRL **86**, 4443 (2001)

Snapshots of the BEC after turning on the rotation



$$\alpha = \Omega \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}$$



1. The BEC becomes elliptic, then oscillating.
2. The surface becomes unstable.
3. Vortices enter the BEC from the surface.
4. The BEC recovers the axisymmetry, the vortices forming a lattice.

The Gross-Pitaevskii (GP) equation in a rotating frame

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V_{\text{trap}} \Psi + g |\Psi|^2 \Psi$$

Wave function

$$\Psi(\mathbf{r}, t)$$

Interaction

$$g = \frac{4\pi\hbar^2 a_s}{m}$$

$$a_s$$

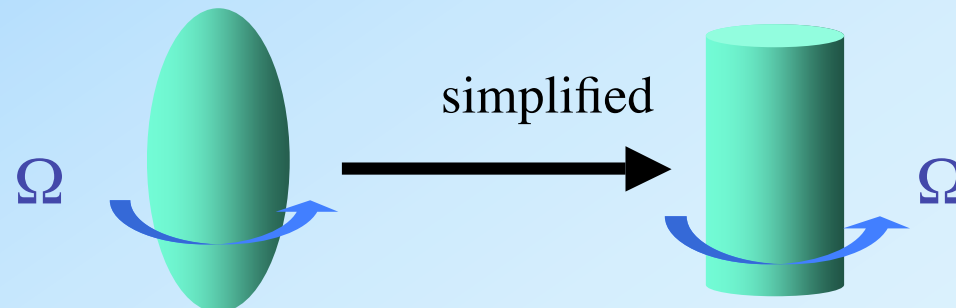
s-wave
scattering length

in a rotating frame

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + (V_{\text{trap}} + U_{\text{stir}}) \Psi + g |\Psi|^2 \Psi - \Omega L_z \Psi$$

Two-dimensional

$$U_{\text{stir}}(\mathbf{R}) = \frac{m}{2} \omega_{\perp}^2 (\varepsilon_x X^2 + \varepsilon_y Y^2)$$



The GP equation with a dissipative term

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + (V_{\text{trap}} + U_{\text{stir}}) + g|\Psi|^2 - \mu - \Omega L_z \right] \Psi$$

$$(i - \gamma)\hbar \frac{\partial \Psi}{\partial t} \quad \gamma = 0.03 : \text{dimensionless parameter}$$

S.Choi, et.al. PRA 57, 4057 (1998)

I.Aranson, et.al. PRB 54, 13072 (1996)

This dissipation comes microscopically from the interaction between the condensate and the noncondensate.

E.Zaremba, T. Nikuni, and A. Griffin, J. Low Temp. Phys. **116**, 277 (1999)

C.W. Gardiner, J.R. Anglin, and T.I.A. Fudge, J. Phys. B **35**, 1555 (2002)

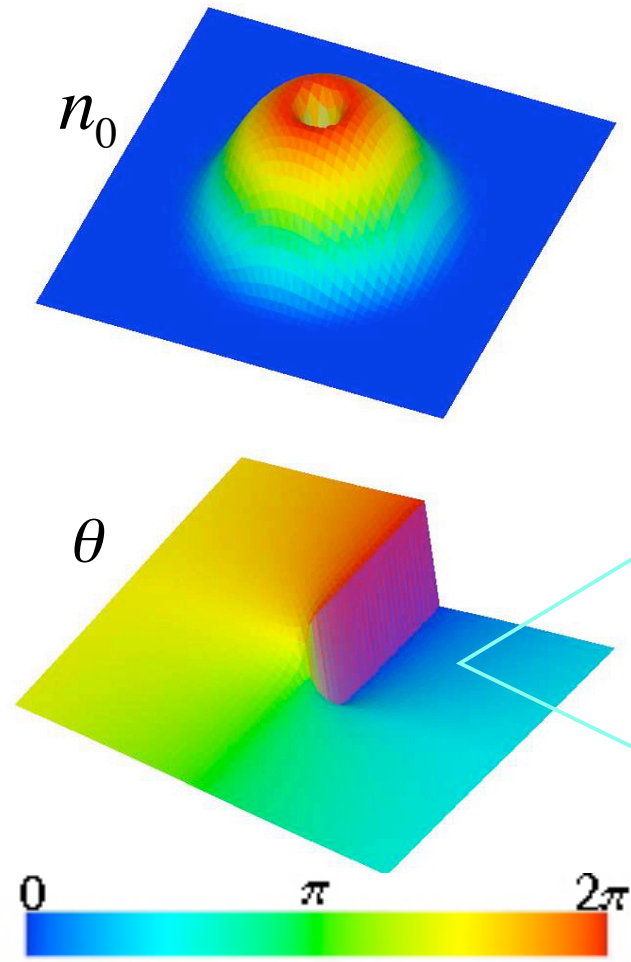
M. Kobayashi and M. Tsubota, PRL **97**, 145301 (2006)

Profile of a single quantized vortex

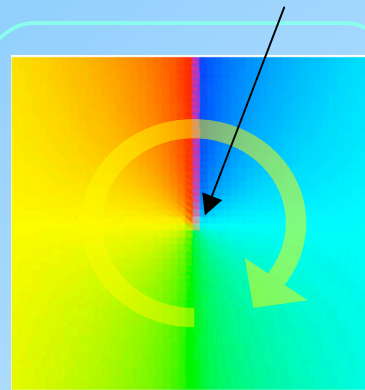
$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + V_{\text{trap}} \Psi + g|\Psi|^2 \Psi = \mu \Psi$$

$$\Psi(r) = \sqrt{n_0(r)} e^{i\theta(r)}$$

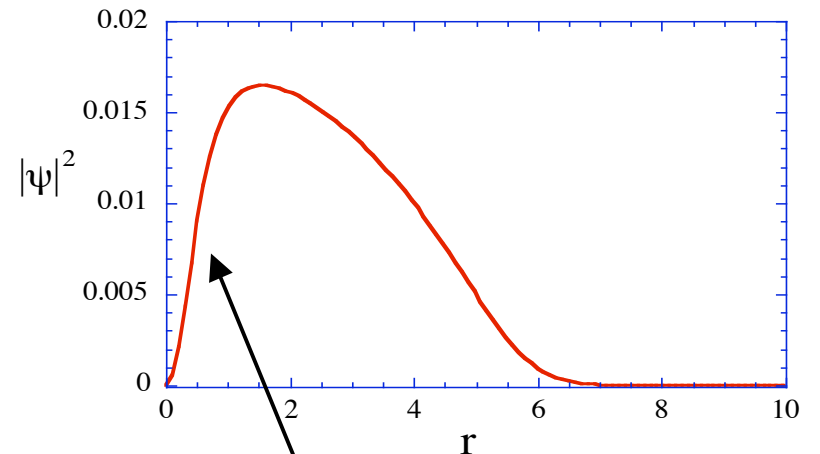
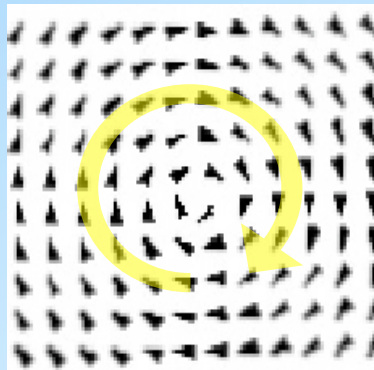
A quantized vortex



A vortex



Velocity field



Vortex core= healing length

$$\xi \approx \frac{\hbar}{\sqrt{2mgn_0}}$$

Dynamics of the vortex lattice formation (1)

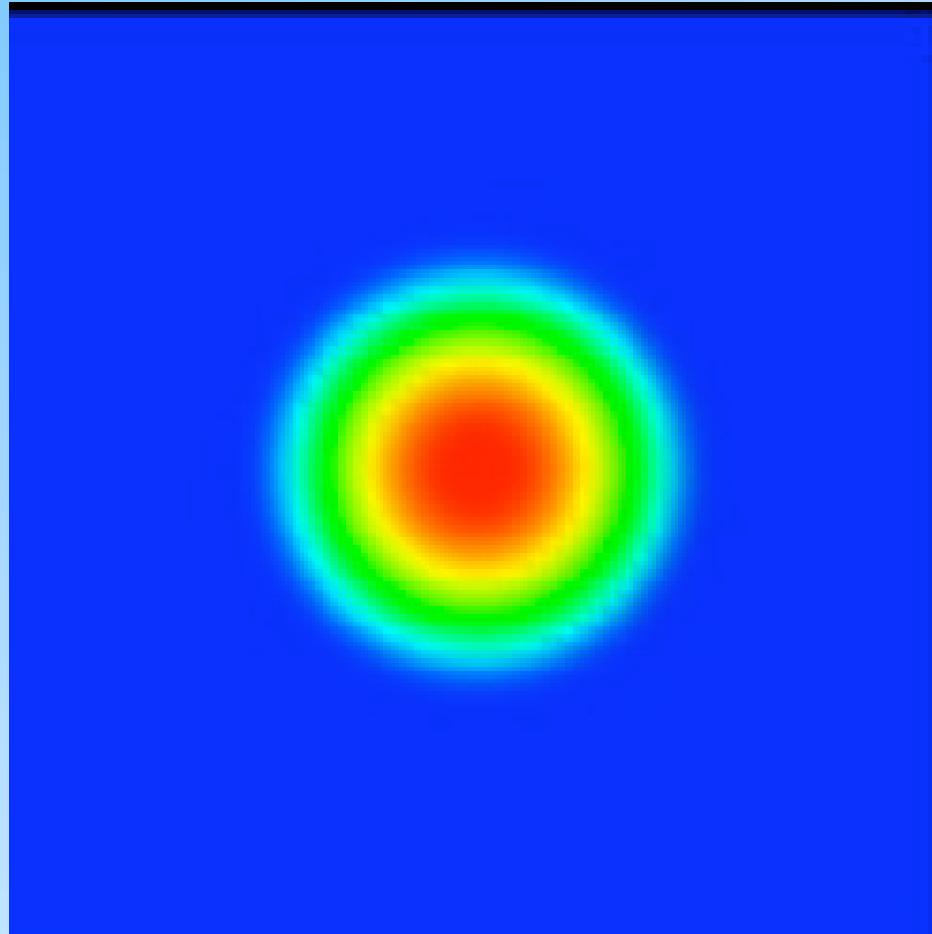
Time development of the condensate density n_0

$$\Omega = 0.7\omega_{\perp}$$

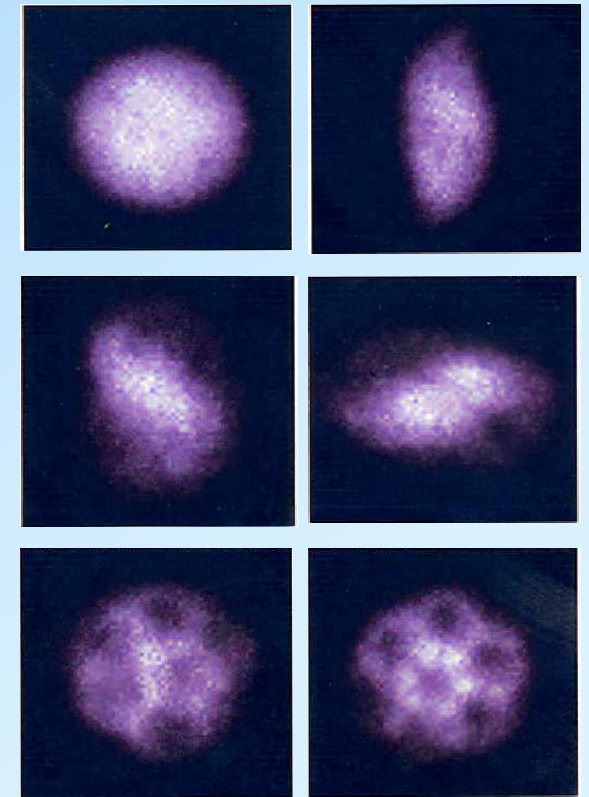
Tsubota *et al.*,
Phys. Rev. A **65**,
023603 (2002)

$$V_{\text{trap}}(r) = \frac{1}{2}m\omega_{\perp}^2 r^2$$

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$



Experiment

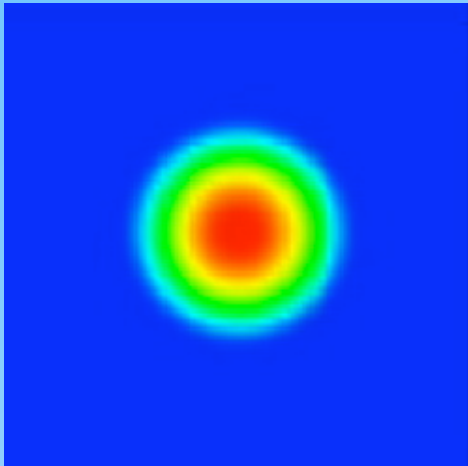


K.W.Madison *et al.* PRL **86**, 4443 (2001)

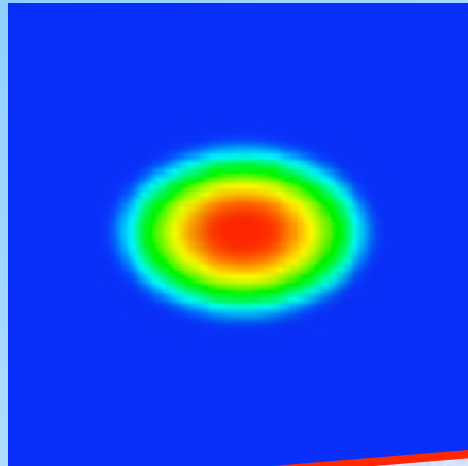
Dynamics of the vortex lattice formation (2)

Time-development of the condensate density n_0

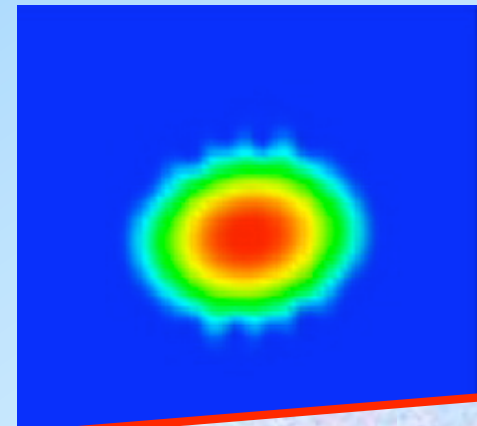
t=0



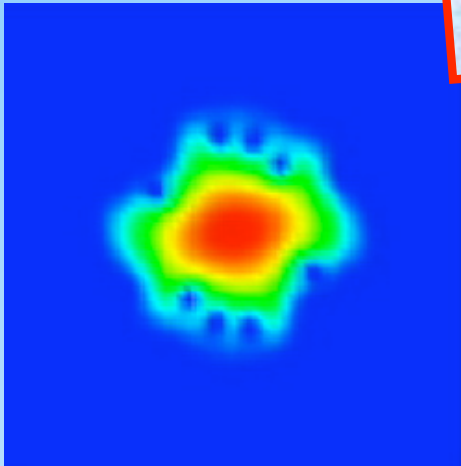
67ms



340ms

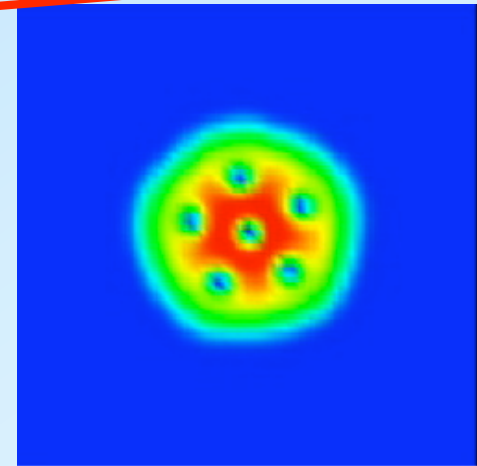


390ms



Are these holes actually quantized vortices?

450ms

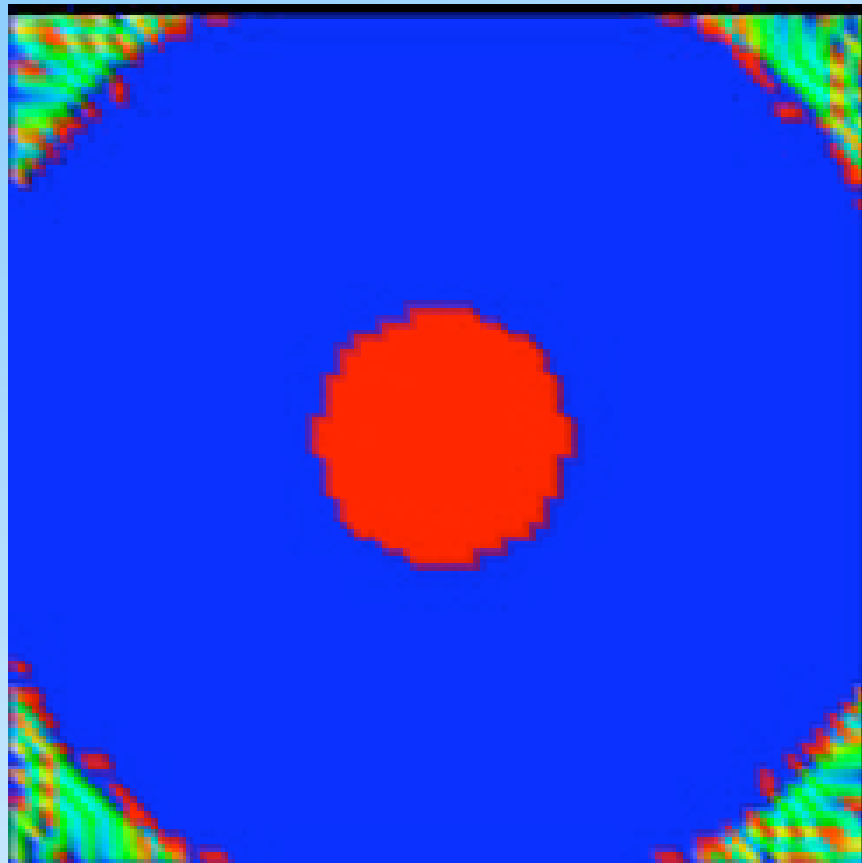


Dynamics of the vortex lattice formation (3)

Time-development of the phase θ

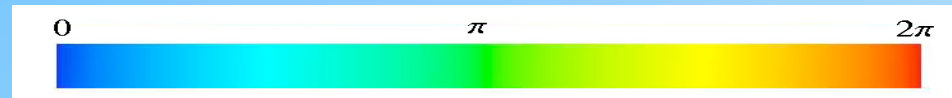


$$\Psi(r) = \sqrt{n_0(r)} e^{i\theta(r)}$$

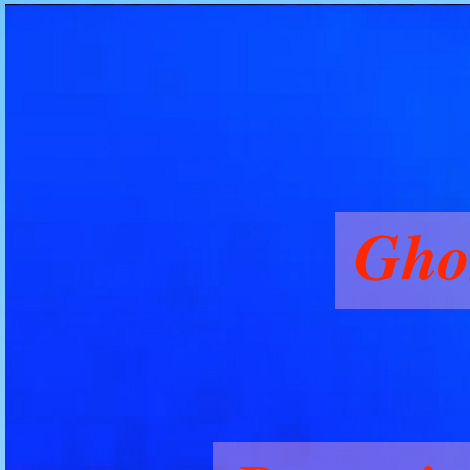


Dynamics of the vortex lattice formation (4)

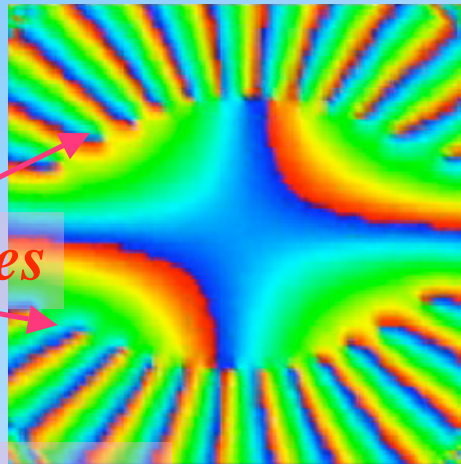
Time-development of the phase θ



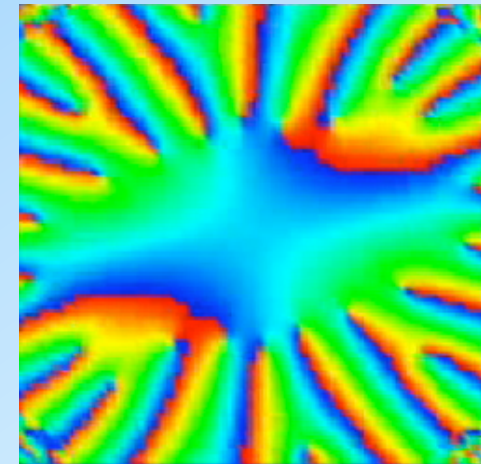
t=0



67ms

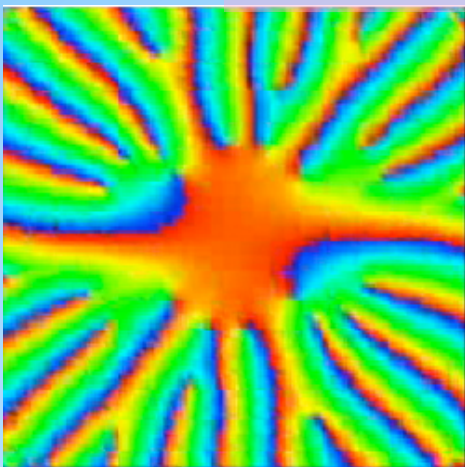


340ms

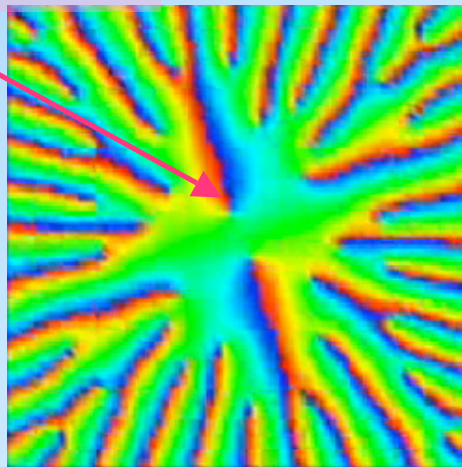


390ms

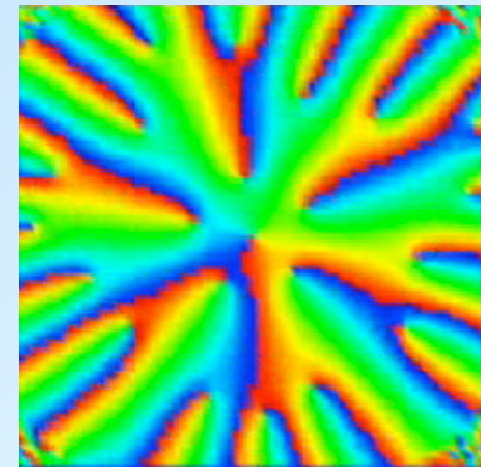
Becoming real vortices



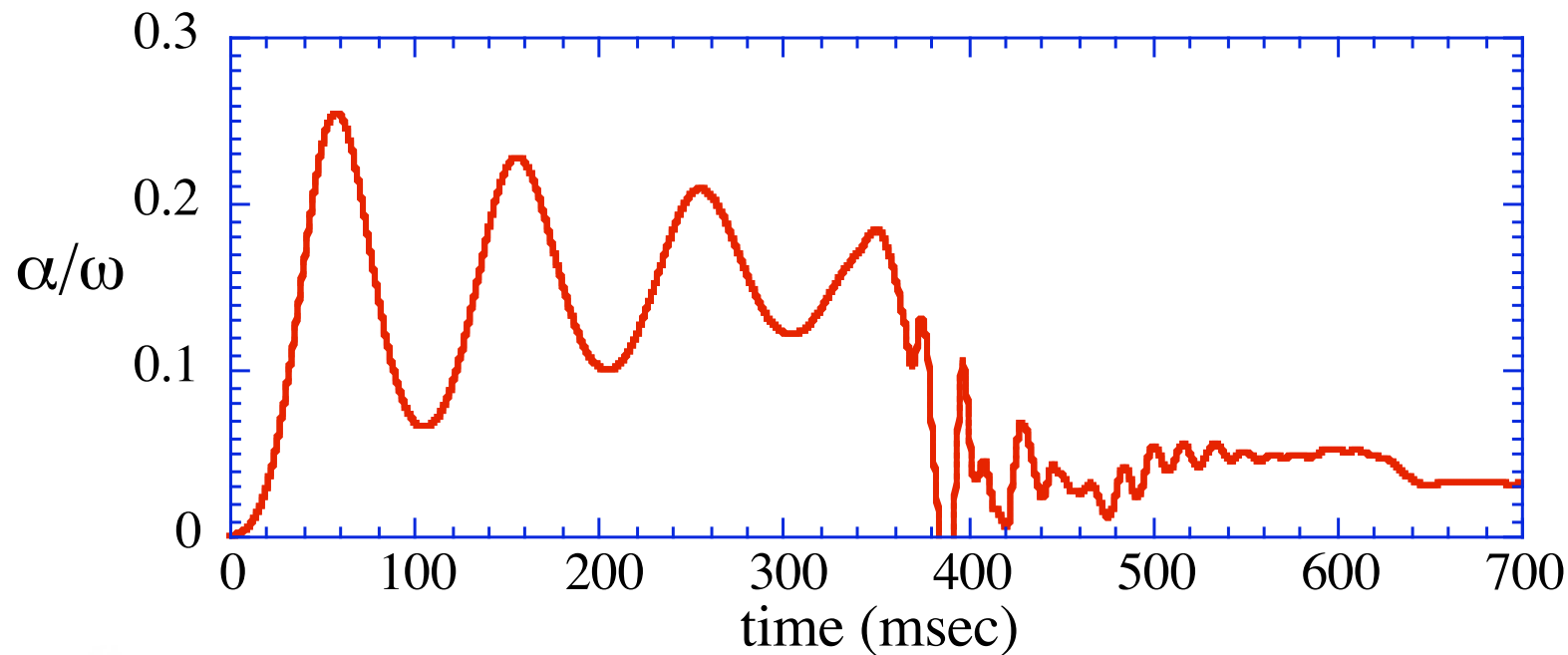
410ms



700ms

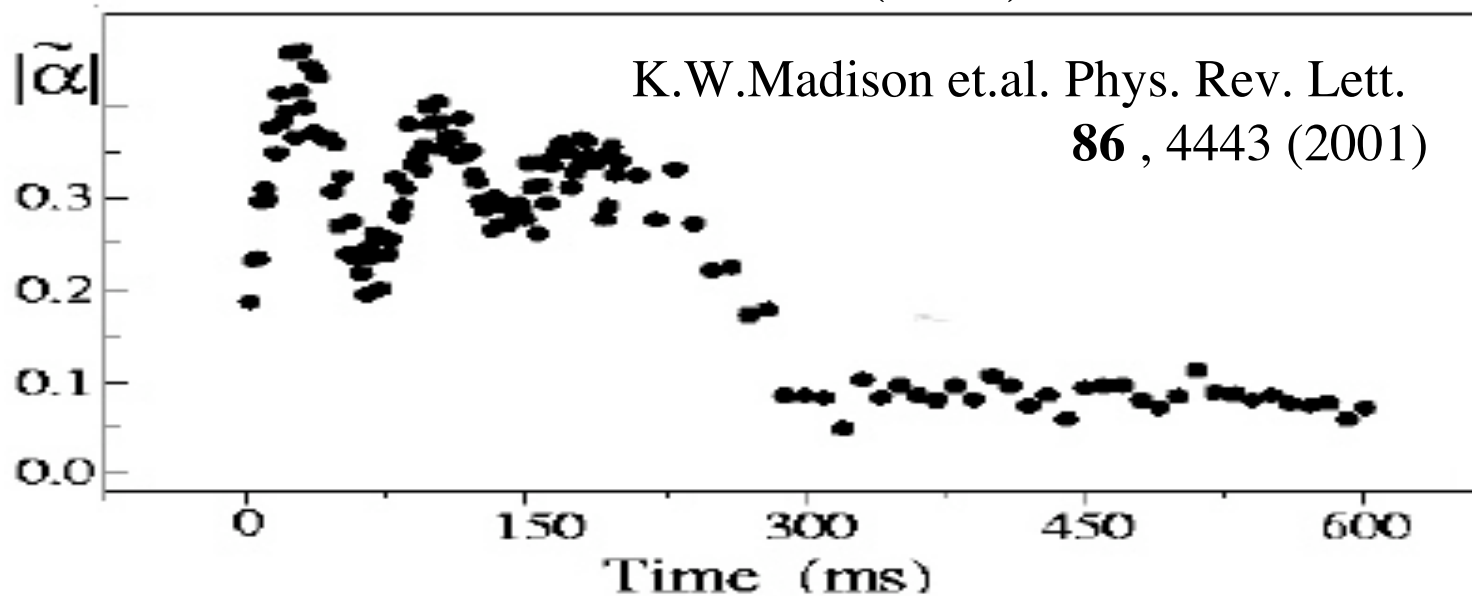


Dynamics of the vortex lattice formation (5)

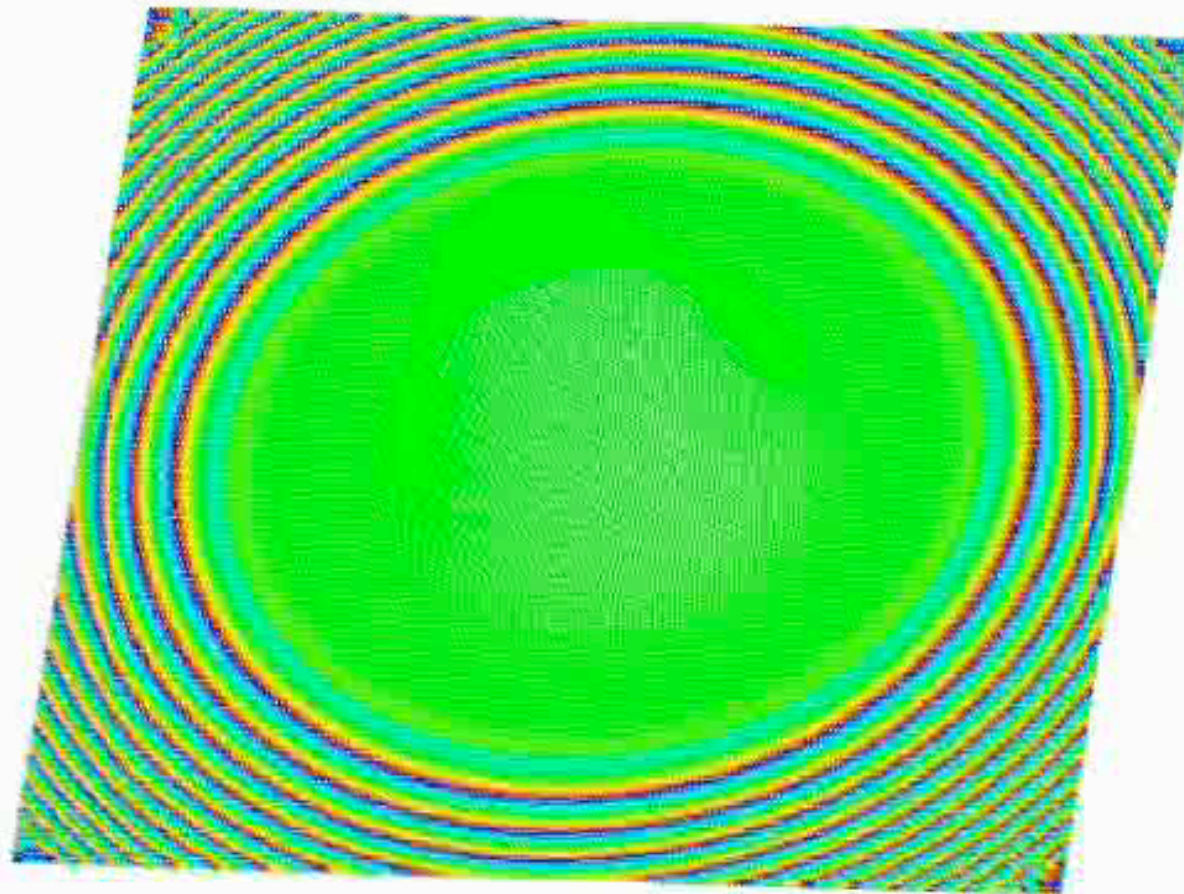


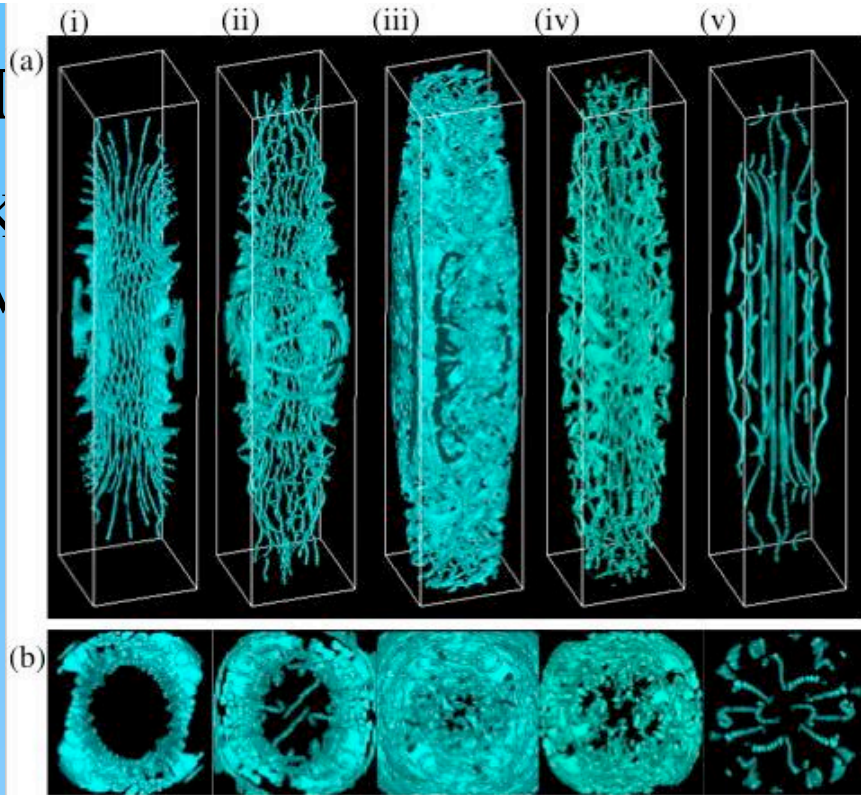
A diagram of an elliptical vortex, shaded in light purple. A vertical red double-headed arrow labeled R_x indicates the vertical semi-axis, and a horizontal red double-headed arrow labeled R_y indicates the horizontal semi-axis.

$$\alpha = \Omega \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}$$



Simultaneous display of the density and the phase



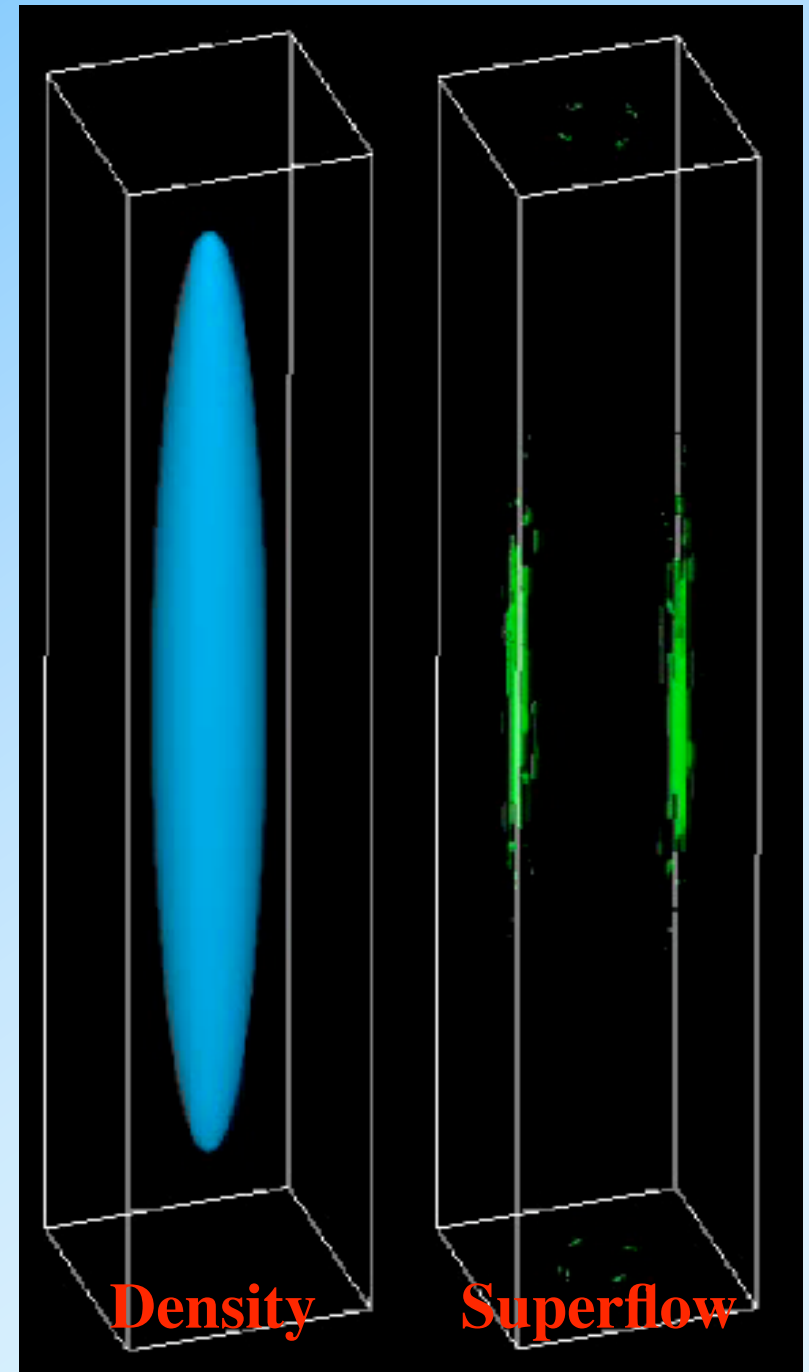


ics

sa,
(5)

The dynamics is qualitatively similar to the two-dimensional case, but we have

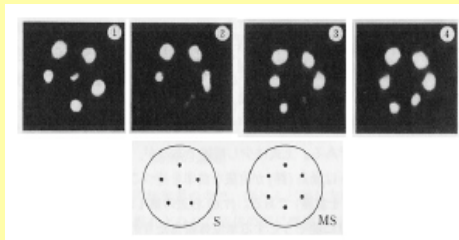
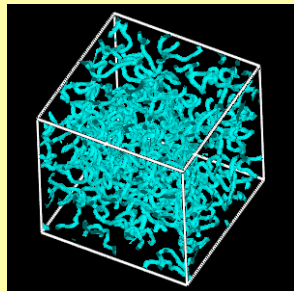
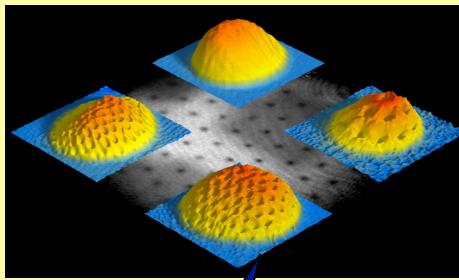
- * Excitation and motion of Kelvin waves
- * Turbulence made of phase defects



Density

Superflow

There are two main cooperative phenomena of quantized vortices; **Vortex array under rotation** and **Vortex tangle (Quantum turbulence)**.

	Vortex array	Vortex tangle
Superfluid He		
Atomic BEC		None

1.

2.

3.

2. Quantum turbulence in superfluid helium

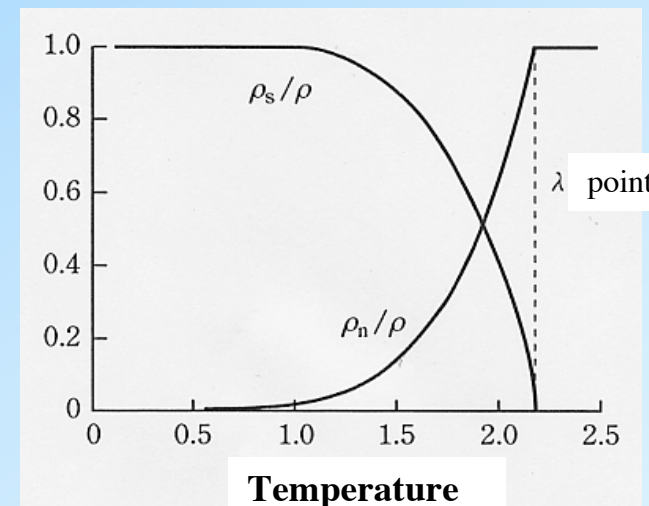
Liquid ^4He enters the superfluid state at 2.17K (λ point) with Bose-Einstein condensation.

Its hydrodynamics is well described by the two-fluid model.

The two-fluid model

The system is a mixture of inviscid superfluid and viscous normal fluid.

$$\rho = \rho_s + \rho_n \quad \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$



	density	velocity	viscosity	entropy
Superfluid	$\rho_s(T)$	$\mathbf{v}_s(\mathbf{r})$	0	0
Normal fluid	$\rho_n(T)$	$\mathbf{v}_n(\mathbf{r})$	$\eta_n(T)$	$s_n(T)$

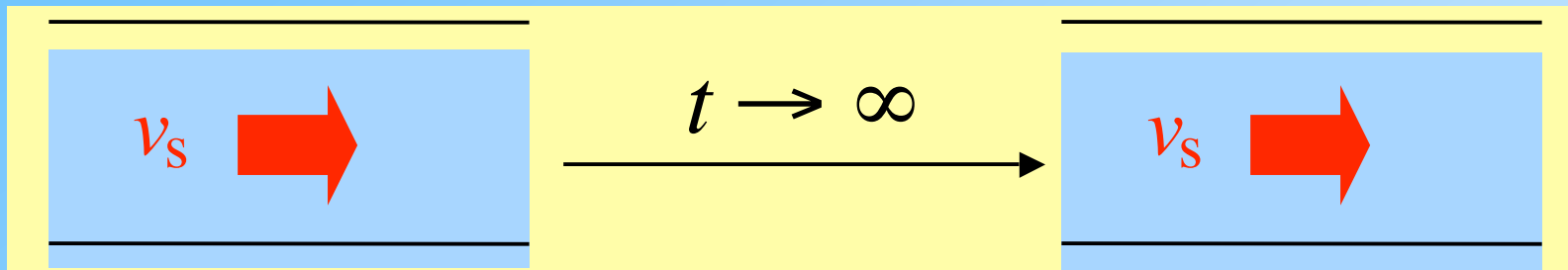
The two-fluid model could explain various phenomena of superfluidity which were observed experimentally.

Thermo-mechanical effect, film flow, *etc.*

However,

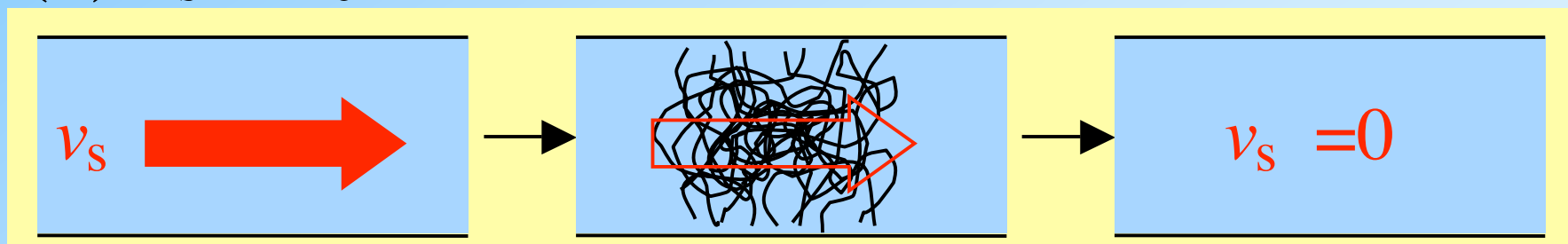
The superfluidity breaks down when it flows fast.

(i) $v_s < v_c$ (some critical velocity)



There is no interaction between two fluids, and the superfluid can flow forever without decaying.

(ii) $v_s > v_c$



A tangle of quantized vortices grows. The two fluids interact through the mutual friction due to the tangle, and the superflow decays.

1955 **Feynman** proposed that “superfluid turbulence” consists of a tangle of quantized vortices.

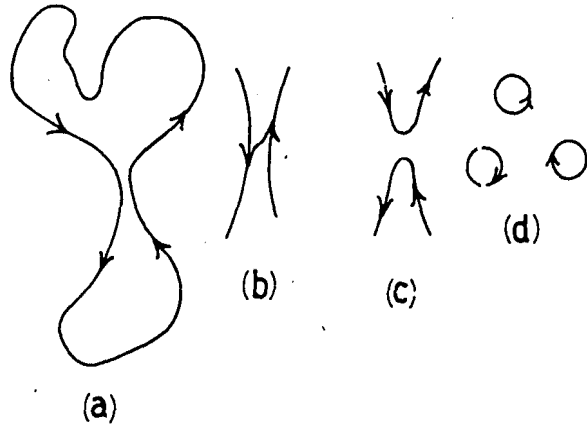
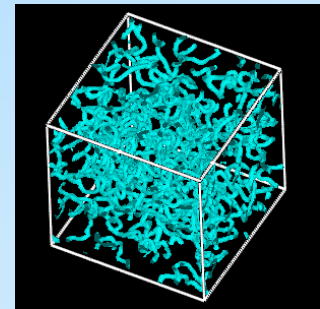


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

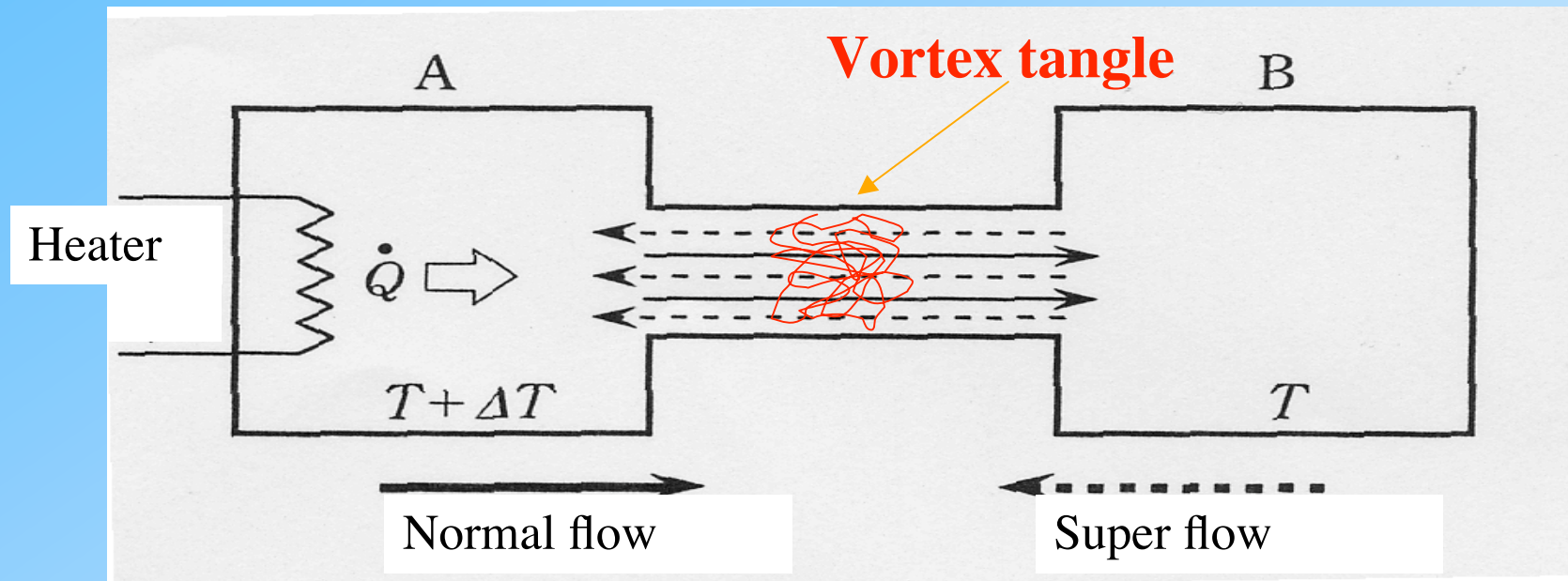
Progress in Low Temperature Physics
Vol.I (1955), p.17



1955 - 1957 **Hall and Vinen** observed “superfluid turbulence”.

The mutual friction between the vortex tangle and the normal fluid causes the dissipation of the flow.

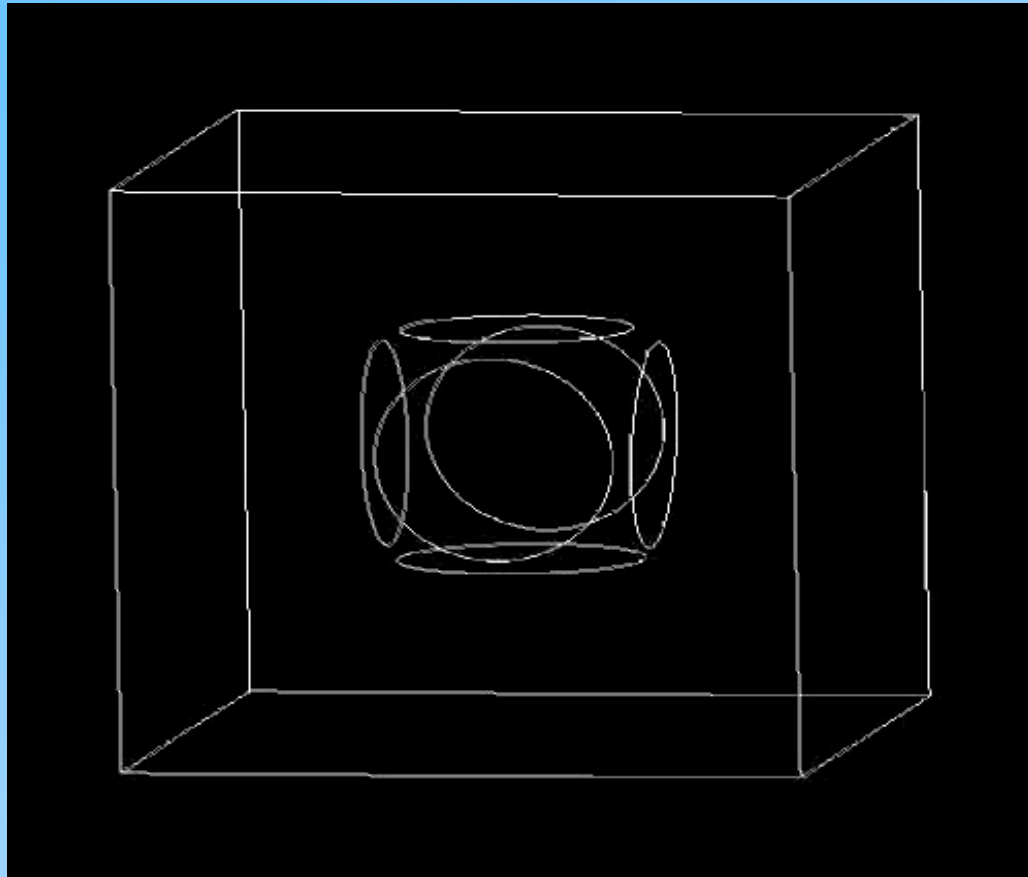
Lots of experimental studies were done chiefly for thermal counterflow of superfluid ^4He .



1980's **K. W. Schwarz** Phys.Rev.B38, 2398(1988)

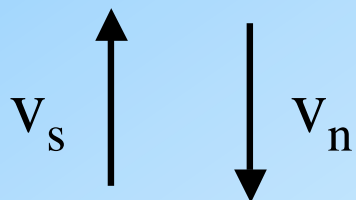
Made the direct numerical simulation of the three-dimensional dynamics of quantized vortices and succeeded in explaining quantitatively the observed temperature difference ΔT .

Development of a vortex tangle in a thermal counterflow



Schwarz, Phys.Rev.B38, 2398(1988).

Schwarz obtained numerically the statistically steady state of a vortex tangle which is sustained by the competition between the applied flow and the mutual friction. The obtained vortex density $L(v_{ns}, T)$ agreed quantitatively with experimental data.



The research of counterflow turbulence has been successful.

Most studies of superfluid turbulence are based on thermal counterflow.

⇒ No analogy

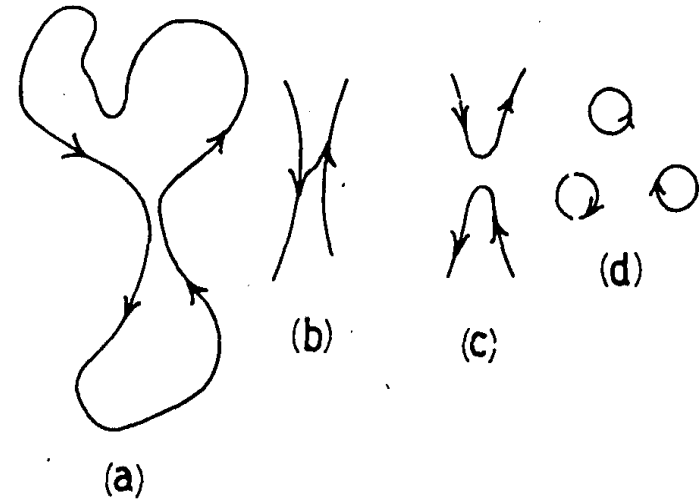


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

When Feynman showed the above figure, he thought of a cascade process in classical turbulence.

What is the relation between superfluid turbulence and classical turbulence ?

How can we study the similarity and the difference between QT and CT?

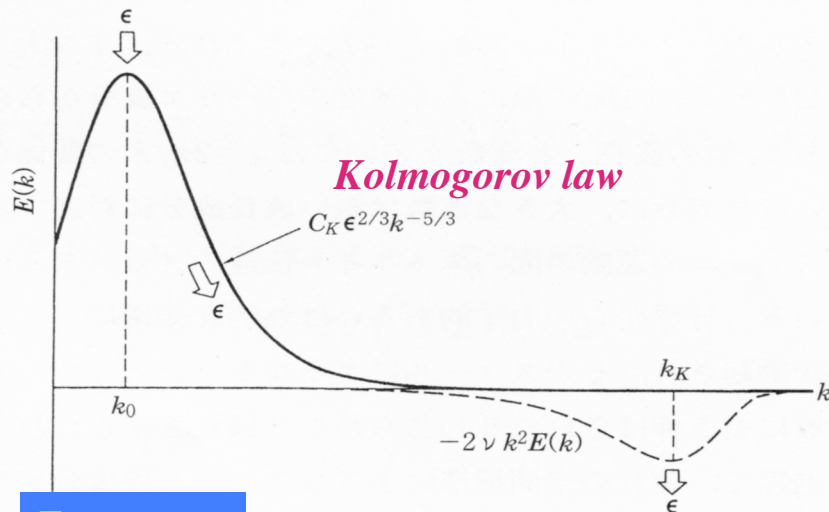
Let's focus on the most important statistical law in CT, namely, the Kolmogorov law.

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

Energy spectra of fully developed turbulence

Energy spectrum of the velocity field

$$E = \frac{1}{2} \int \mathbf{v}^2 d\mathbf{r} = \int E(k) dk$$



Energy-containing range

Inertial range

Energy-dissipative range

Energy spectrum of turbulence

The

Richardson cascade process

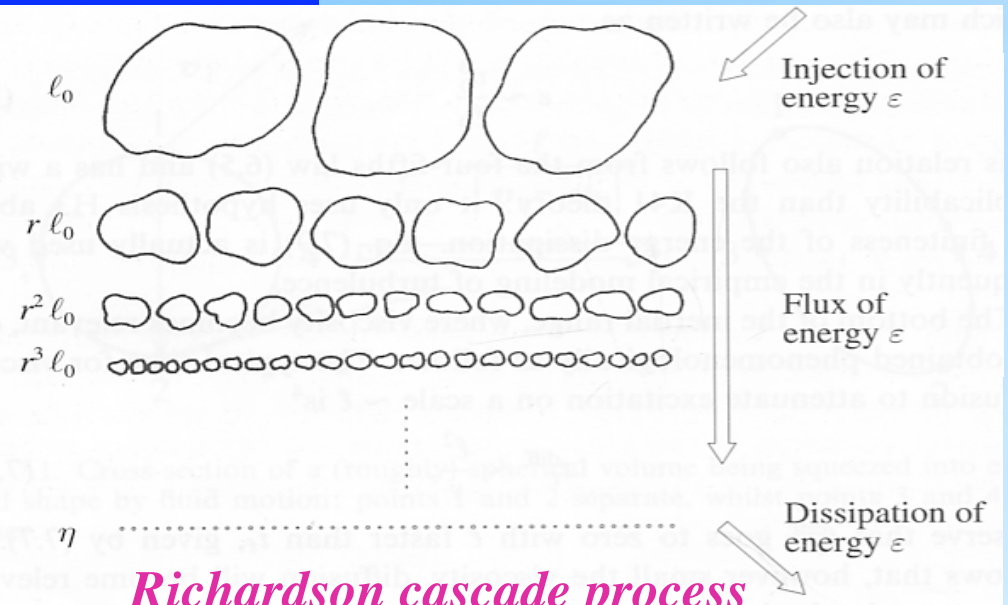
Inertial range

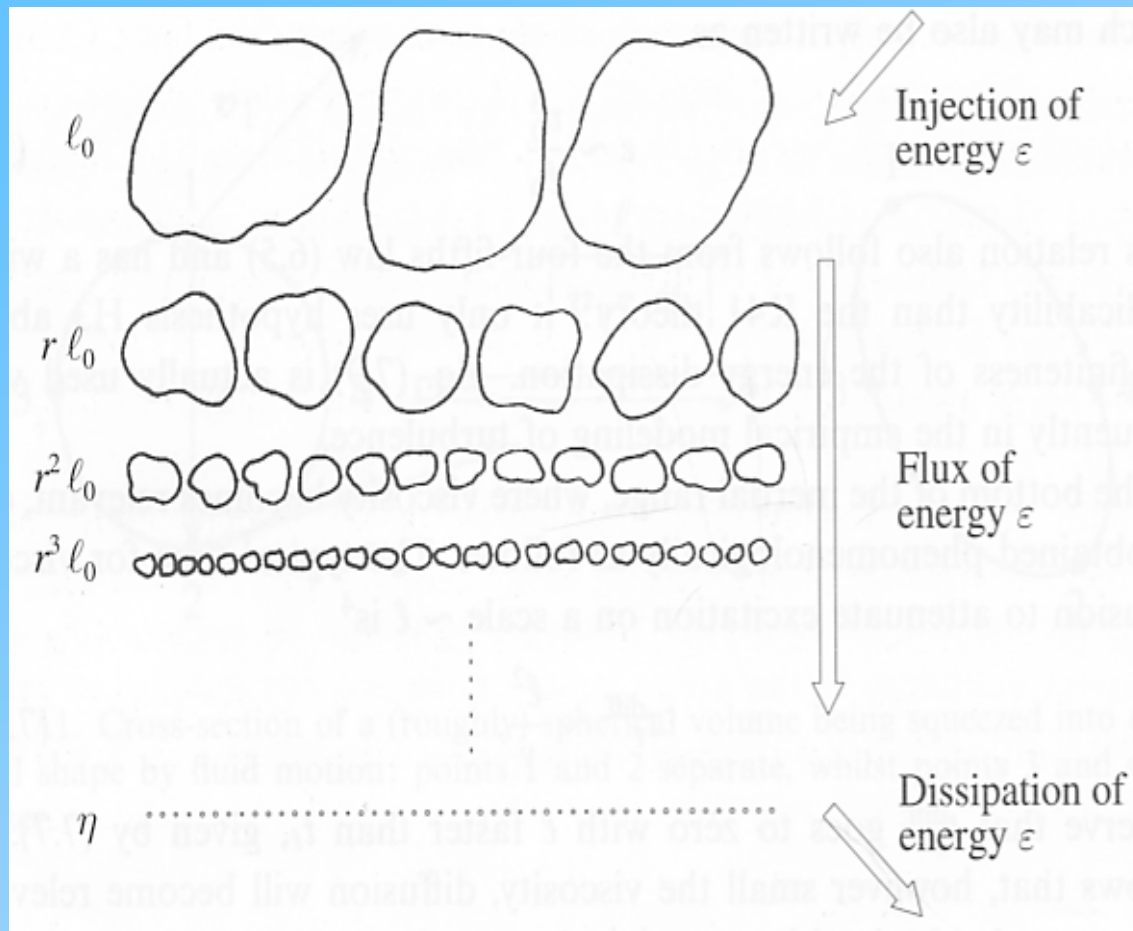
Dissipation does not work. The nonlinear interaction transfers the energy from low k region to high k region.

Kolmogorov law : $E(k) = C \epsilon^{2/3} k^{-5/3}$

Energy-dissipative range

The energy is dissipated with the rate ϵ at the Kolmogorov wave number $k_c = (\epsilon/\nu^3)^{1/4}$.





Everybody believes that turbulence is sustained by this Richardson cascade. However, this is only a cartoon; nobody has ever confirmed it clearly.

One of the reasons is that it is too difficult to identify each eddy in a fluid.

Quantum turbulence may give a prototype of turbulence, much simpler than conventional turbulence.

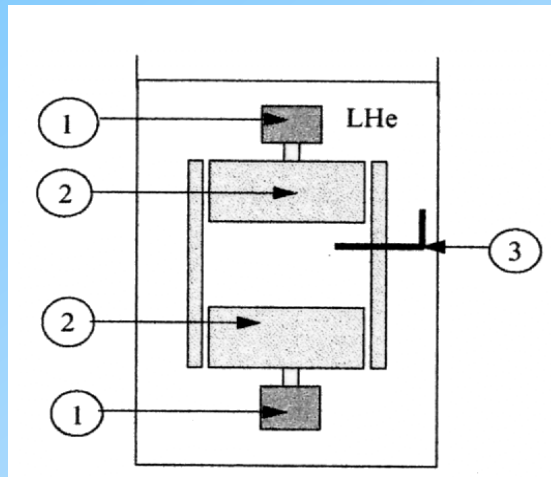
Can such quantized vortices still produce the essence of turbulence, for example, the Kolmogorov law?

Having this sort of motivation, the studies of QT have entered a new stage since the middle of 90's !

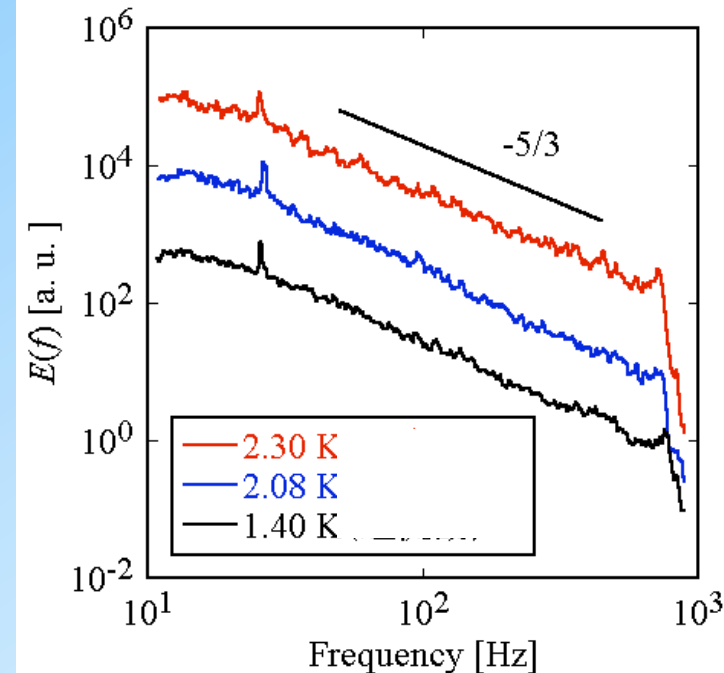
New study of quantum turbulence (1)

- Maurer and Tabeling, Europhysics. Letters. 43, 29(1998) $1.4\text{K} < T < T_\lambda$

Measurements of local pressure in flows driven by two counterrotating disks finds the **Kolmogorov spectrum**.



Energy spectra



- Stalp, Skrbek and Donnely, Phys.Rev.Lett. 82, 4831(1999) $1.4 < T < 2.15\text{K}$

Decay of grid turbulence. The data of the second sound attenuation was consistent with a classical model with the **Kolmogorov spectrum**.

New study of quantum turbulence (2)

- Vinen, Phys.Rev.B61, 1410(2000)

Considering the relation between ST and CT

The Oregon's result is understood by the coupled dynamics of the superfluid and the normal fluid due to the mutual friction. Length scales are important, compared with the characteristic vortex spacing in a tangle.

- Kivotides, Vassilicos, Samuels and Barenghi, Europhysics Lett. 57, 845(2002)

When superfluid is coupled with the normal-fluid turbulence that obeys the Kolmogorov law, its spectrum follows the Kolmogorov law too.

**What happens at very low temperatures?
Is there still the similarity or not?**

Energy spectra of quantum turbulence (QT)

There are three works which directly study the energy spectrum of QT at zero temperature.

Decaying Kolmogorov turbulence in a model of superflow

C. Nore, M. Abid and M.E.Brachet, Phys.Fluids 9, 2644 (1997)

The Gross-Pitaevskii (GP) model

Energy Spectrum of Superfluid Turbulence with No Normal-Fluid Component

T. Araki, M.Tsubota and S.K.Nemirovskii, Phys.Rev.Lett.89, 145301(2002)

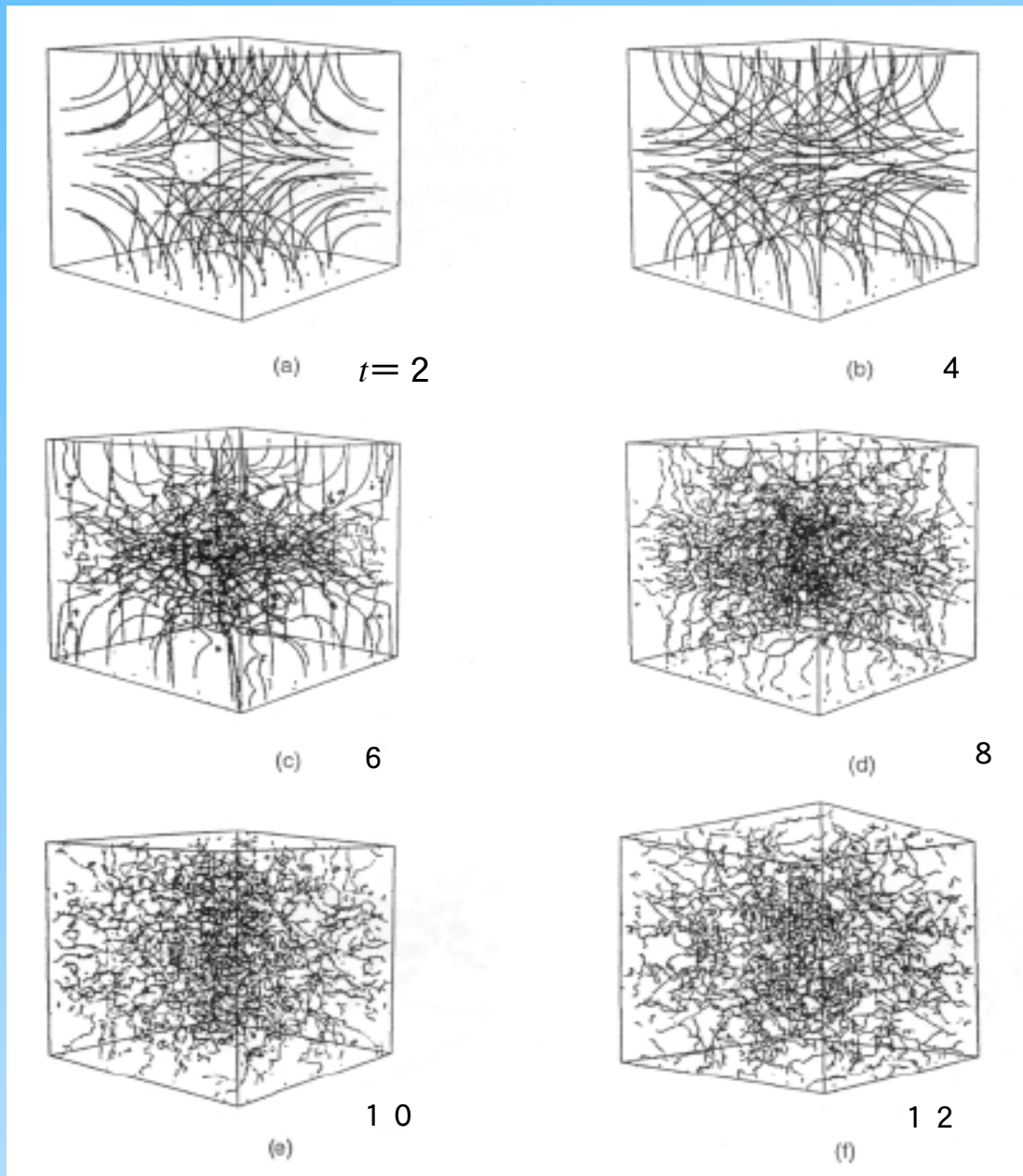
The vortex-filament model

Kolmogorov Spectrum of Superfluid Turbulence: Numerical Analysis of the Gross-Pitaevskii Equation with a Small-Scale Dissipation

M. Kobayashi and M. Tsubota,

Phys. Rev. Lett. 94, 065302 (2005), J. Phys. Soc. Jpn.74, 3248 (2005).

C. Nore, M. Abid and M.E.Brachet, Phys.Fluids 9, 2644(1997)



By using the GP model, they obtained a vortex tangle with starting from the Taylor-Green vortices.

We should note that the GP model describes a compressible fluid.

In order to study the Kolmogorov spectrum, it is necessary to decompose the total energy into some components. (Nore *et al.*, 1997)

Total energy

$$E = \frac{1}{\int d\mathbf{x} \rho} \int d\mathbf{x} \Phi^* \left[-\nabla^2 + \frac{g}{2} |\Phi|^2 \right] \Phi \quad \Phi = \sqrt{\rho} \exp(i\theta)$$

$$E = E_{\text{int}} + E_q + E_{\text{kin}}$$

The kinetic energy

$$E_{\text{kin}} = \frac{1}{\int d\mathbf{x} \rho} \int d\mathbf{x} (|\Phi| \nabla \theta)^2 \text{ is divided into}$$

the compressible part

$$E_{\text{kin}}^c = \frac{1}{\int d\mathbf{x} \rho} \int d\mathbf{x} [(|\Phi| \nabla \theta)^c]^2 \text{ with } \text{rot} (|\Phi| \nabla \theta)^c = 0$$

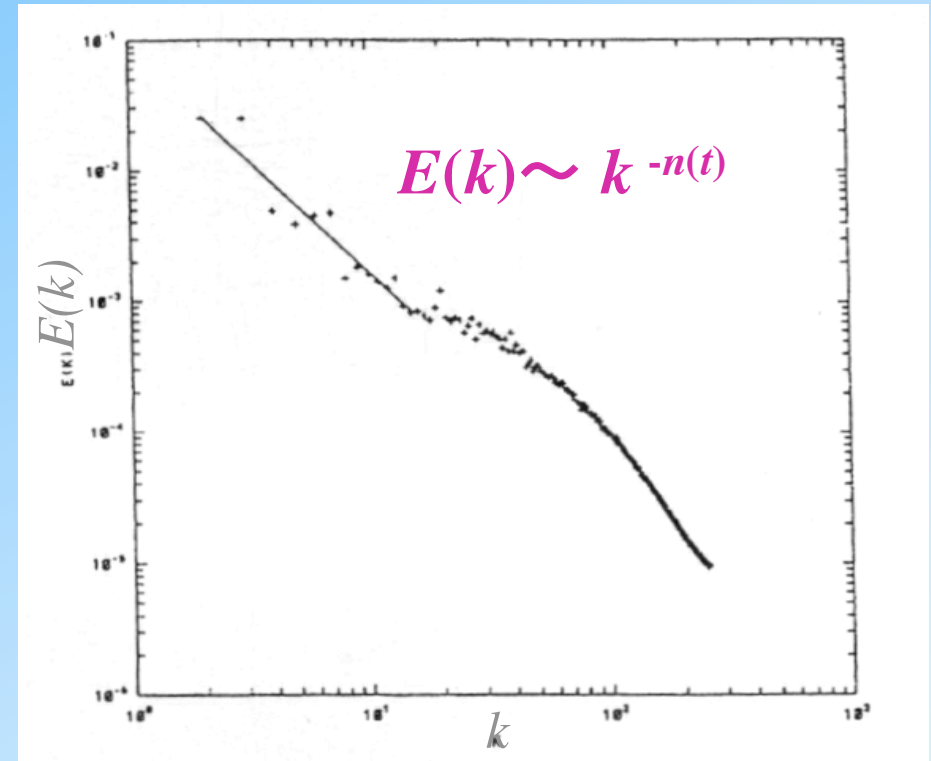
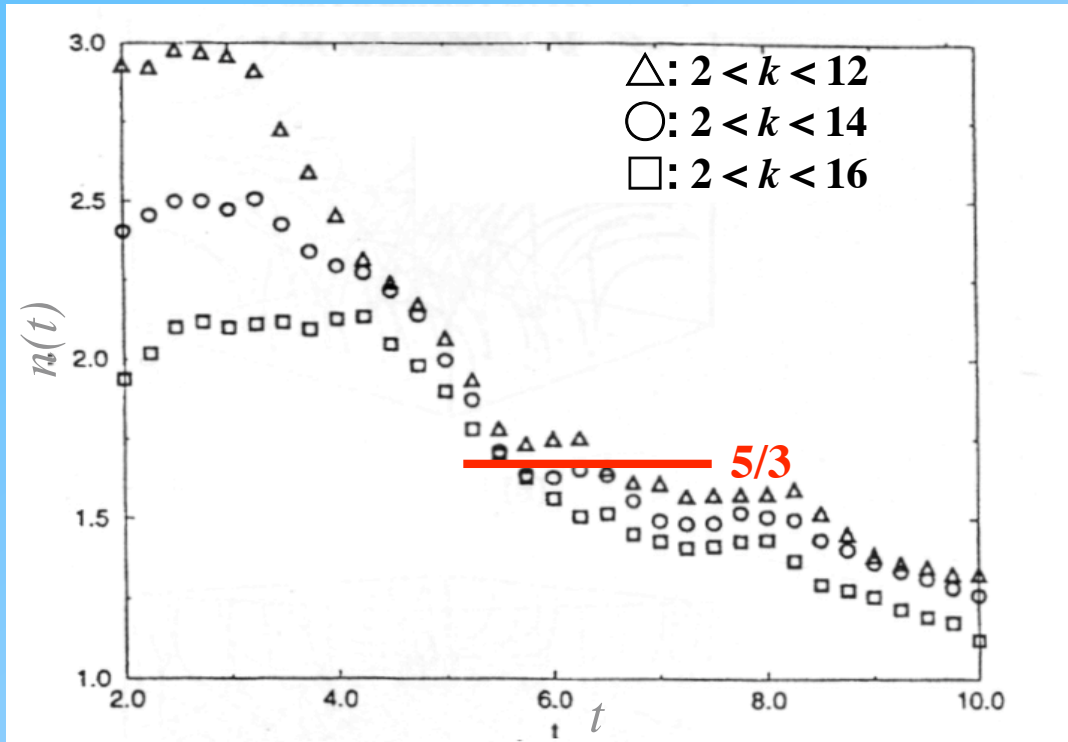
and

the incompressible part

$$E_{\text{kin}}^i = \frac{1}{\int d\mathbf{x} \rho} \int d\mathbf{x} [(|\Phi| \nabla \theta)^i]^2 \text{ with } \text{div} (|\Phi| \nabla \theta)^i = 0.$$

This incompressible kinetic energy E_{kin}^i should obey the Kolmogorov spectrum.

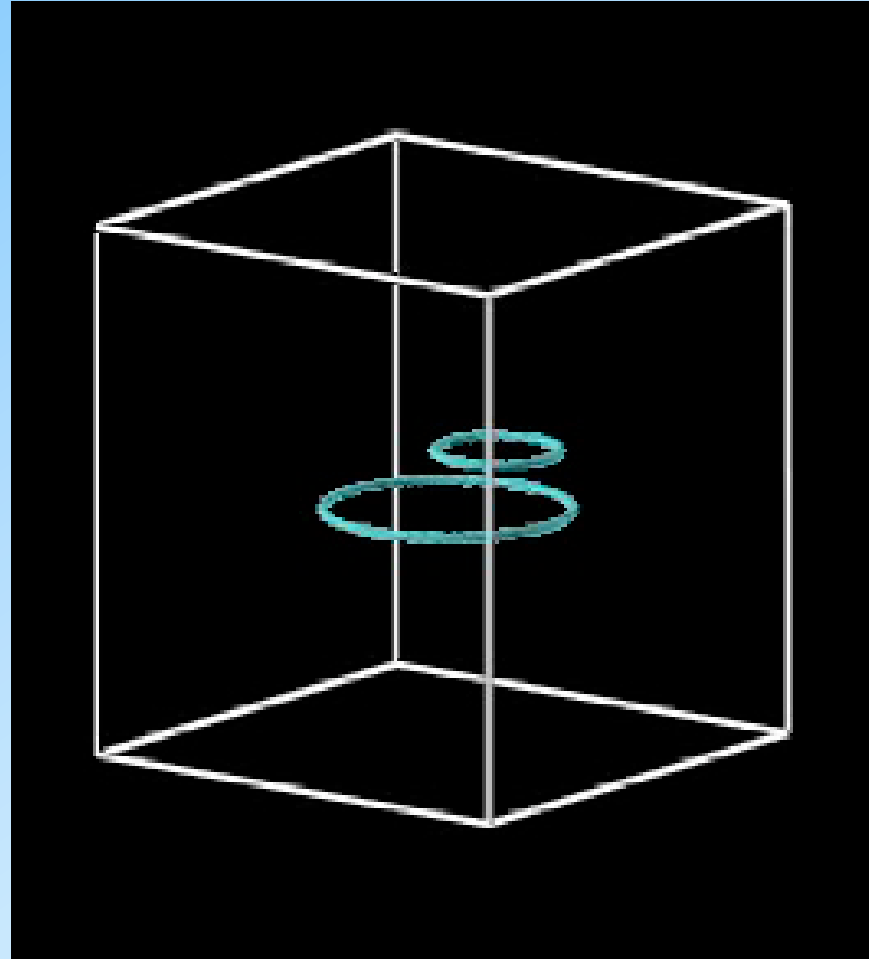
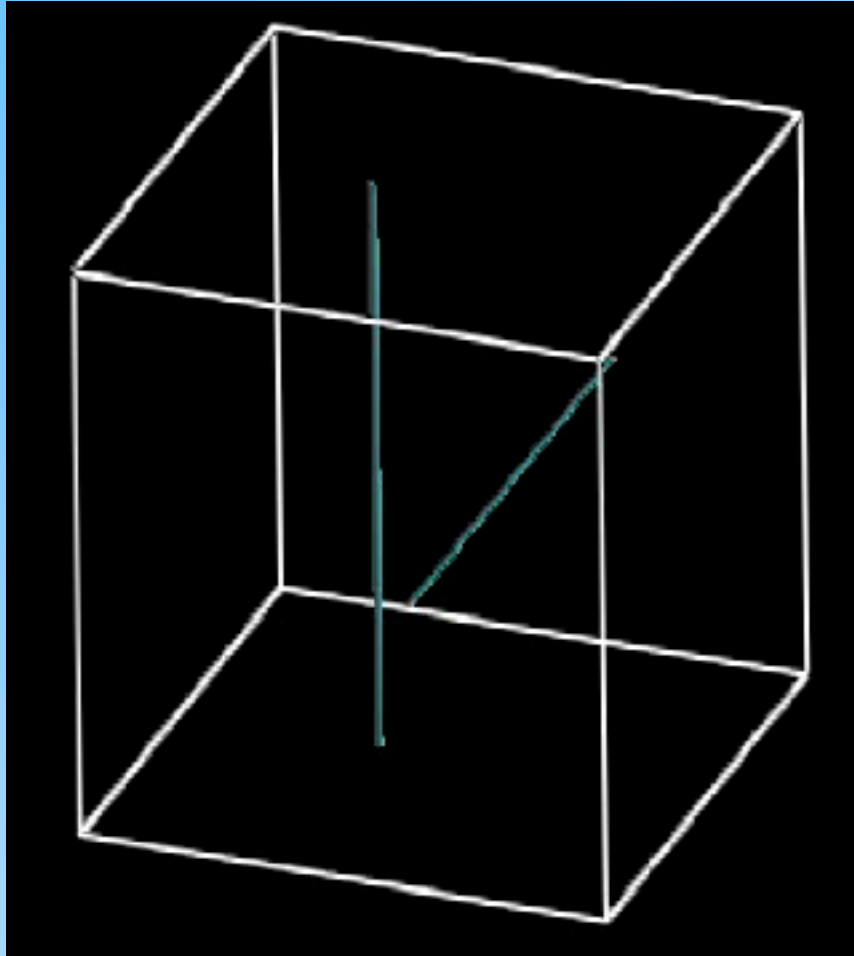
C. Nore, M. Abid and M.E.Brachet, Phys.Fluids 9, 2644(1997)



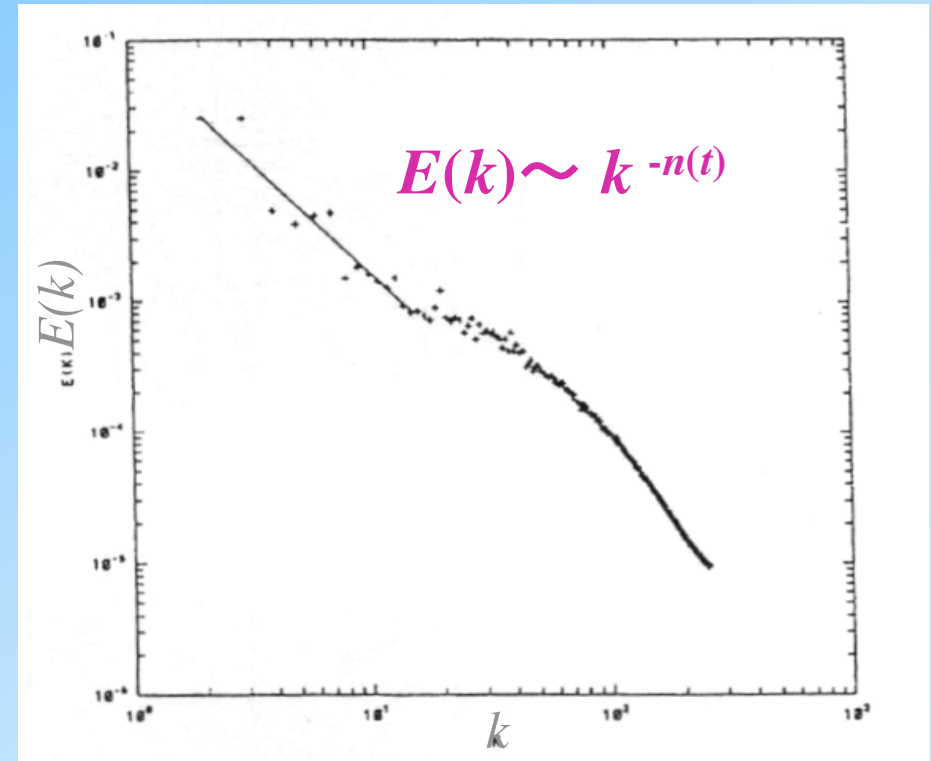
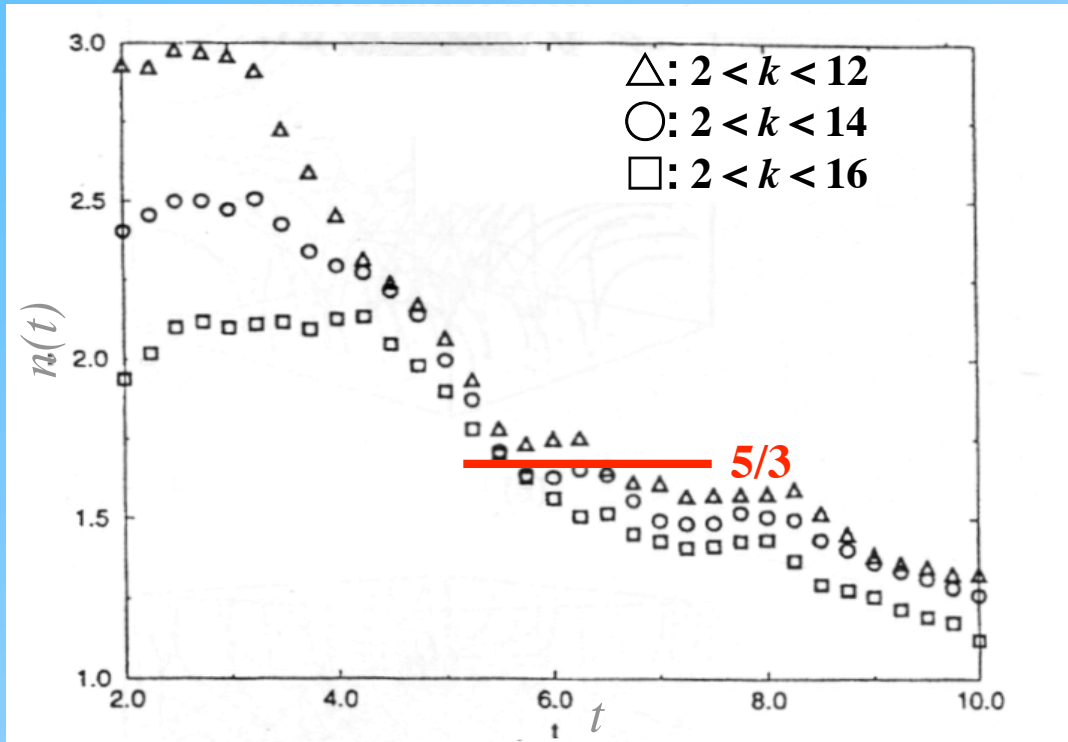
The right figure shows the energy spectrum at a moment. The left figure shows the development of the exponent $n(t)$. The exponent $n(t)$ goes through $5/3$ on the way of the dynamics.

In the late stage, however, the exponent deviates from $5/3$, because the sound waves resulting from vortex reconnections disturb the cascade process of the inertial range.

Reconnection of quantized vortices (Analysis of the Gross-Pitaevskii equation)



C. Nore, M. Abid and M.E.Brachet, Phys.Fluids 9, 2644(1997)



The right figure shows the energy spectrum at a moment. The left figure shows the development of the exponent $n(t)$. The exponent $n(t)$ goes through $5/3$ on the way of the dynamics.

In the late stage, however, the exponent deviates from $5/3$, because the sound waves resulting from vortex reconnections disturb the cascade process of the inertial range.

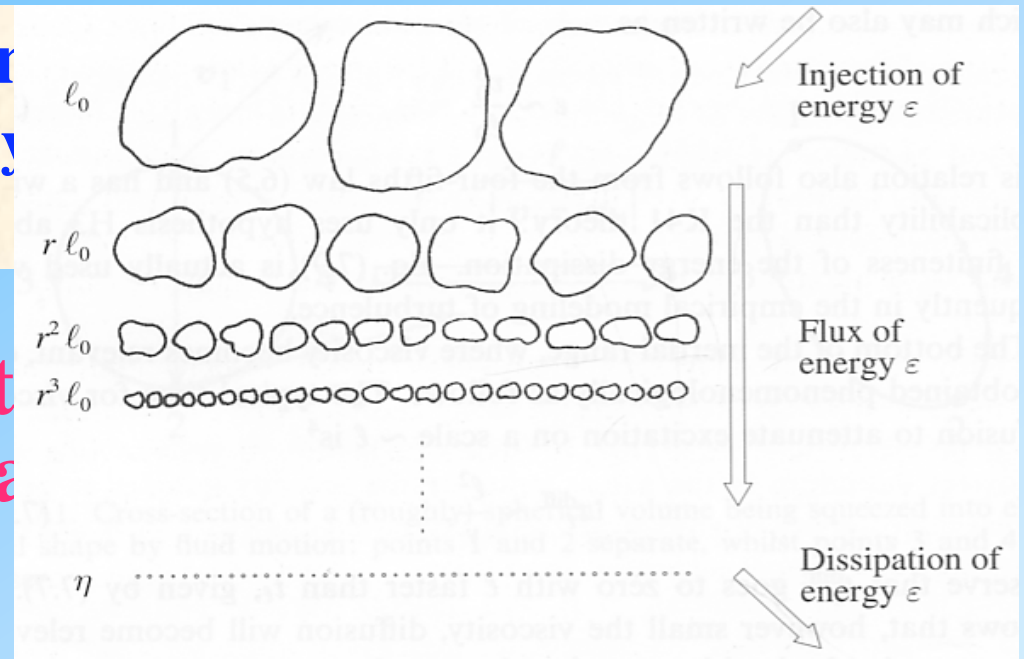
Kolmogorov spectrum of quantum turbulence
M. Kobayashi and M. Tsubota, Phys. Rev. Lett. 94, 055302 (2005)
J. Phys. Soc. Jpn. 74, 3248 (2005)

1. We solved the GP equation in Fourier space in order to use the fast Fourier transformation.

2. We made a steady state of turbulence. In order to do that,

2-1 We introduced a dissipative term which dissipates the Fourier component of the high wave number, namely, phonons of short wave length.

2-2 We excited the system at a large scale by moving a random potential.



The GP equation in the real space

$$i \frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = \left[-\nabla^2 - \mu + g |\Phi(\mathbf{r}, t)|^2 \right] \Phi(\mathbf{r}, t)$$

The GP equation in the Fourier space

$$i \frac{\partial}{\partial t} \Phi(\mathbf{k}, t) = (k^2 - \mu) \Phi(\mathbf{k}, t) + \frac{g}{V^2} \sum_{\mathbf{k}_1, \mathbf{k}_2} \Phi(\mathbf{k}_1, t) \Phi^*(\mathbf{k}_2, t) \Phi(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2, t)$$

$$\xi^2 = 1/g|\Phi|^2 \quad \text{healing length giving the vortex core size}$$

To solve the GP equation numerically with high accuracy, we use the Fourier spectral method in space with the periodic boundary condition in a cube.

How to dissipate the energy at small scales?

The GP equation with the small scale dissipation

$$\{i - \gamma(k)\} \frac{\partial}{\partial t} \Phi(\mathbf{k}, t) = (k^2 - \mu) \Phi(\mathbf{k}, t) + \frac{g}{V^2} \sum_{\mathbf{k}_1, \mathbf{k}_2} \Phi(\mathbf{k}_1, t) \Phi^*(\mathbf{k}_2, t) \Phi(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2, t)$$

$\xi^2 = 1/g|\Phi|^2$:healing length giving the vortex core size

$$\gamma(k) = \gamma_0 \theta(k - 2\pi / \xi)$$

We introduce the dissipation that works only in the scale smaller than ξ .

Since there is no vortex motion at the scales smaller than ξ , this dissipation must work for only short-wavelength sound waves.

cf. M. Kobayashi and M. Tsubota, PRL **97**, 145301 (2006)

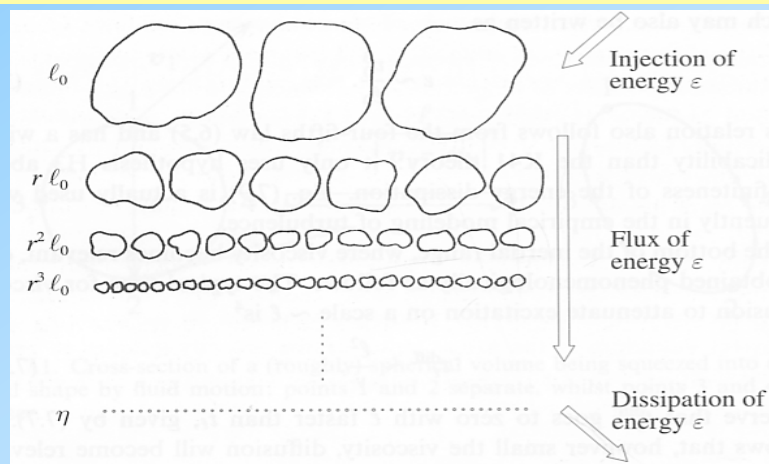
How to inject the energy at large scales?

This is done by moving the random potential satisfying the space-time correlation:

$$\langle V(\mathbf{x}, t) V(\mathbf{x}', t') \rangle = V_0^2 \exp \left[-\frac{(\mathbf{x} - \mathbf{x}')^2}{2X_0^2} - \frac{(t - t')^2}{2T_0^2} \right]$$

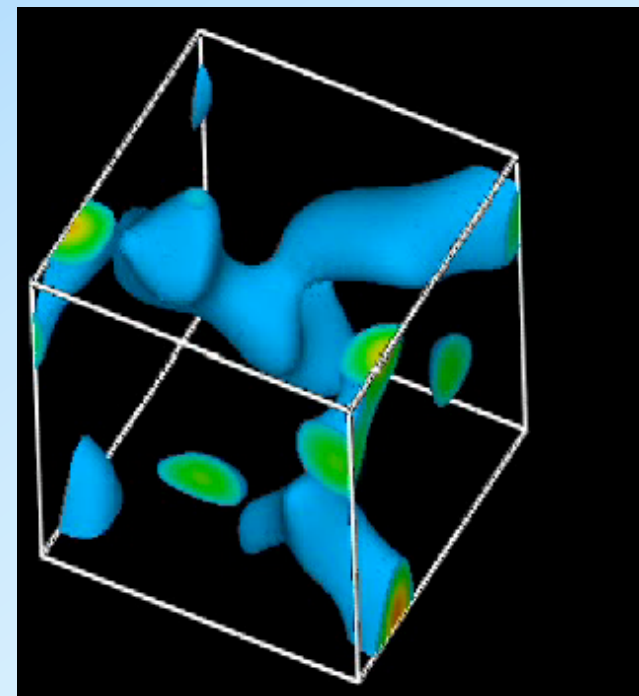
The variable X_0 determines the scale of the energy-containing range.

$V_0=50$, $X_0=4$ and $T_0=6.4 \times 10^{-2}$



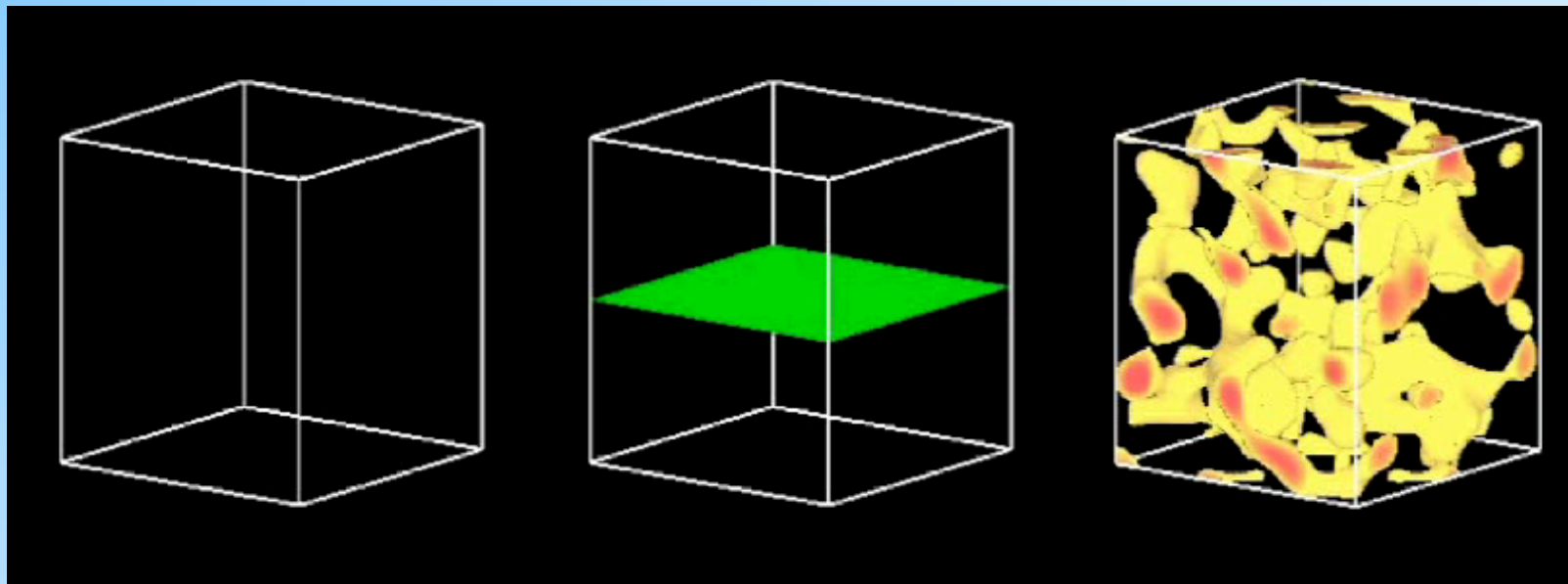
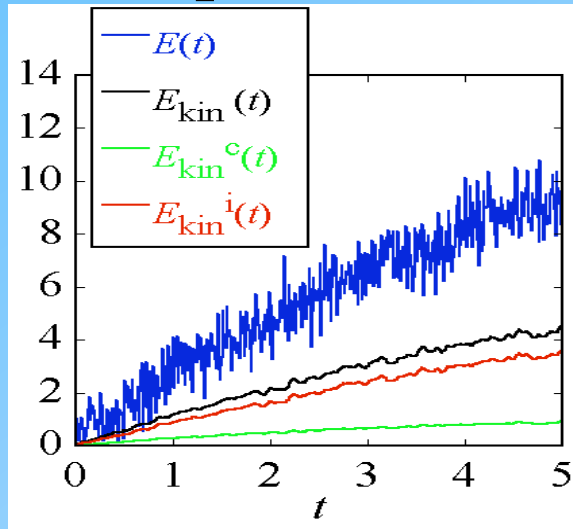
X_0

η



Thus steady turbulence is obtained.(1)

Time development of each energy component



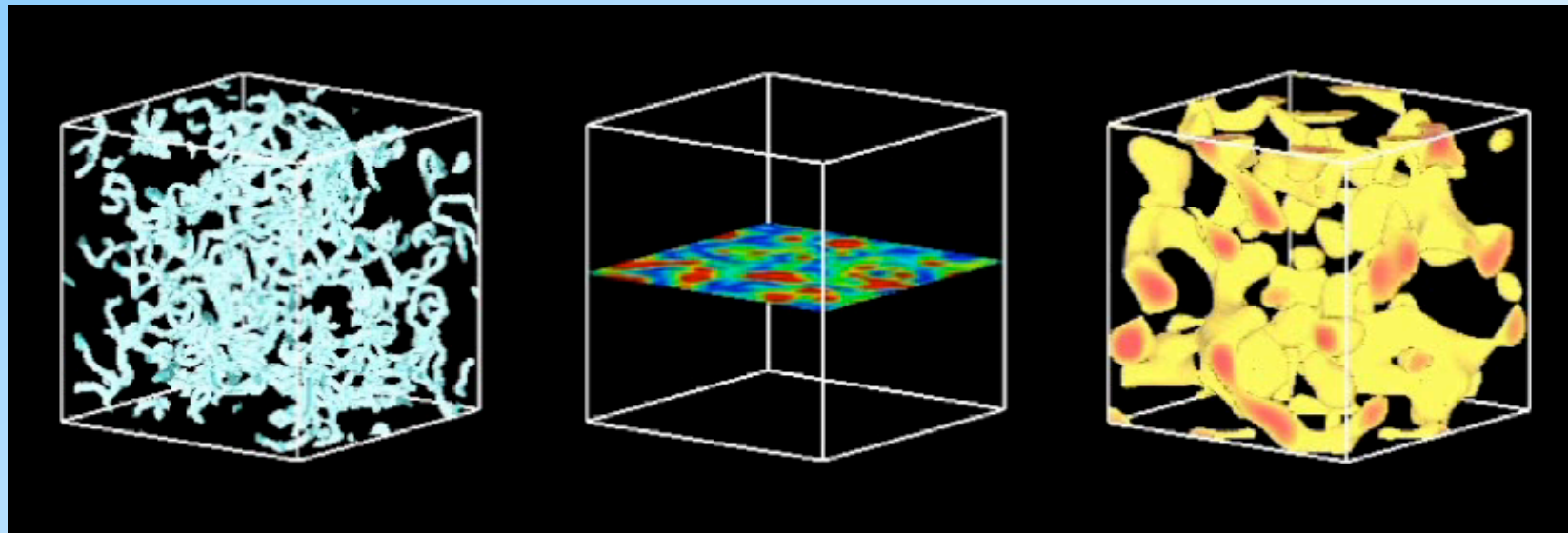
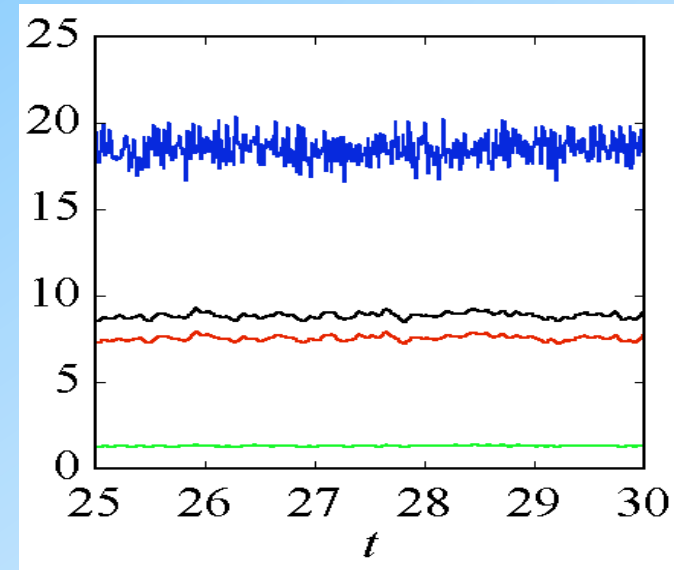
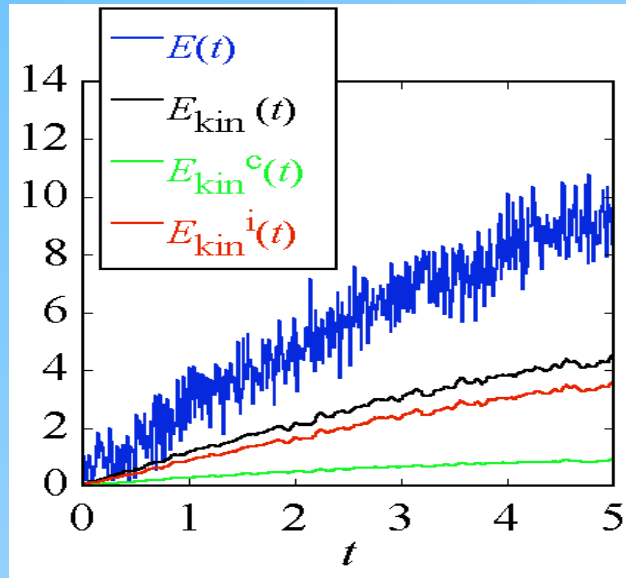
Vortices

Phase in a central plane

Moving random potential

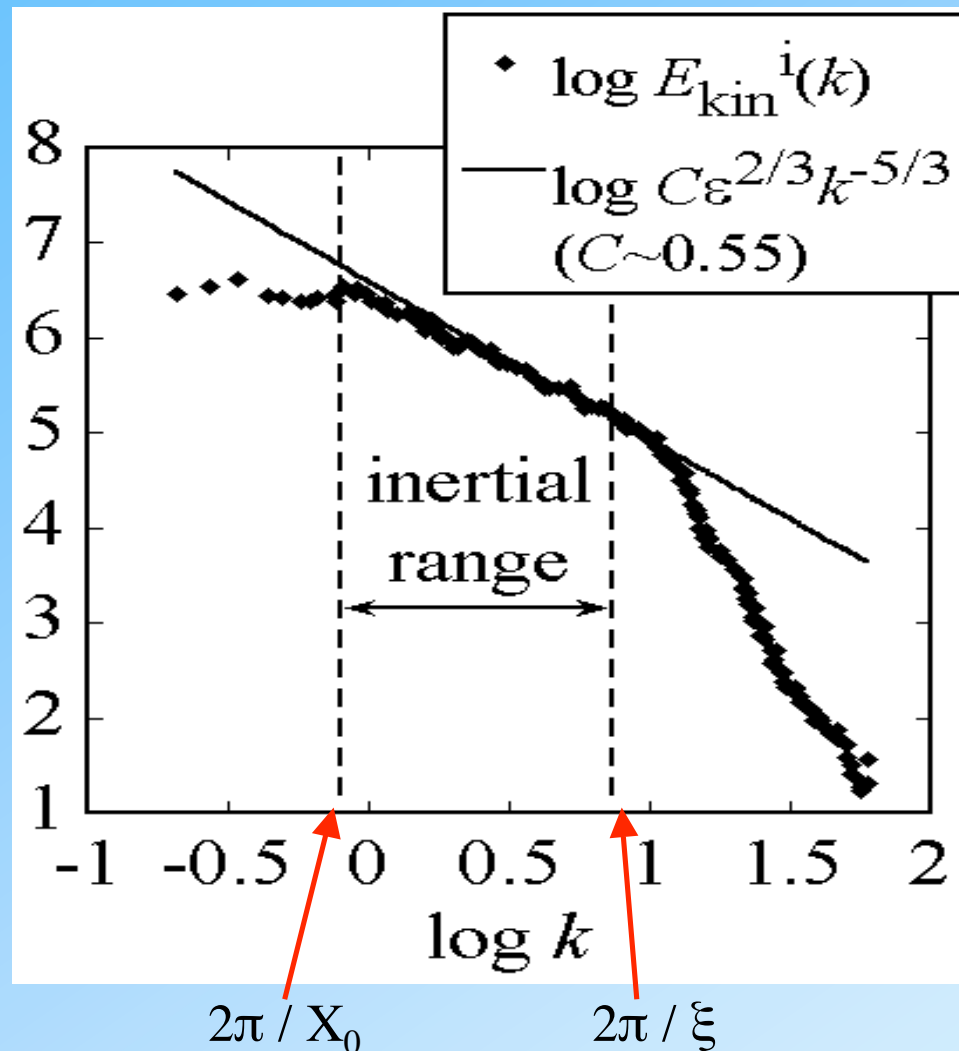
Thus steady turbulence is obtained.(2)

Time development of each energy component



Energy spectrum of the steady turbulence

The energy spectrum obeys the Kolmogorov form.



Quantum turbulence is found to show the essence of classical turbulence!

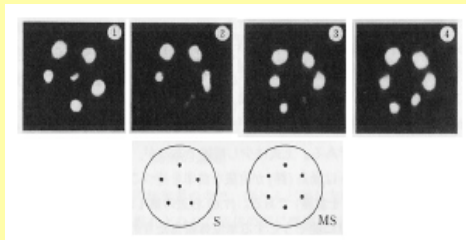
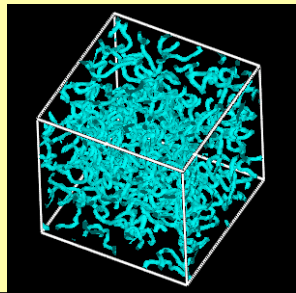
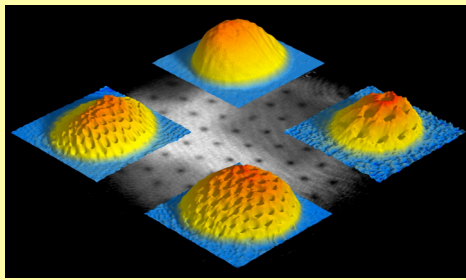
The inertial range is sustained by the genuine Richardson cascade of quantized vortices.

M. Kobayashi and M. Tsubota,
J. Phys. Soc. Jpn. 74, 3248 (2005)

3. Quantum turbulence in a trapped Bose-Einstein condensate

M. Kobayashi and M. Tsubota, Phys. Rev. A76, 045603 (2007)

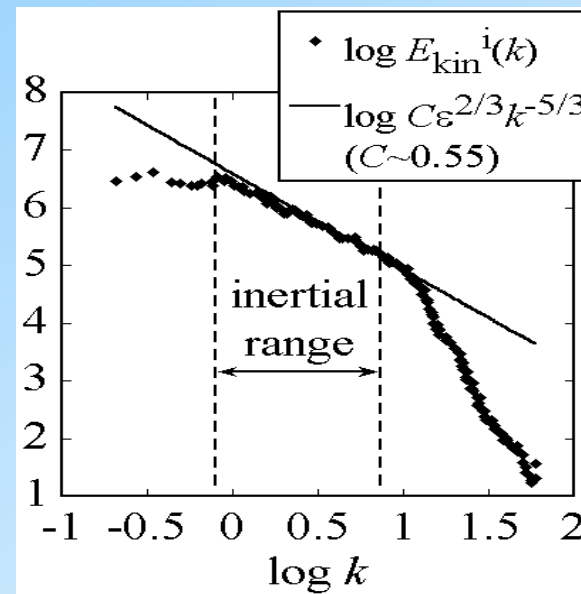
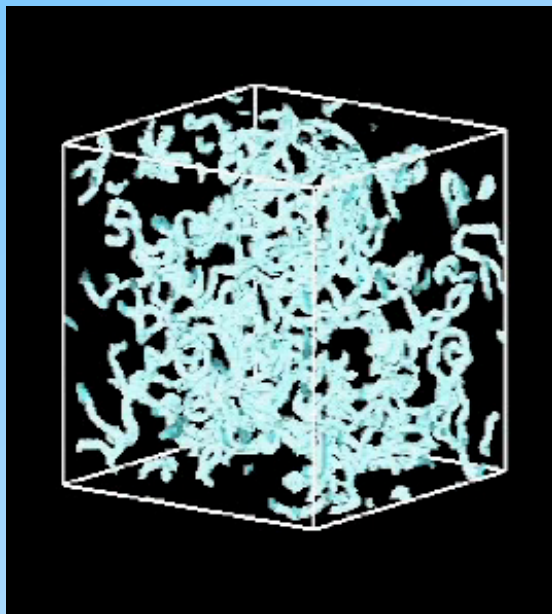
Two main cooperative phenomena of quantized vortices are **Vortex array** and **Vortex tangle**.

	Vortex array	Vortex tangle
Superfluid He		
Atomic BEC		None

Is it possible to make turbulence in a trapped BEC?

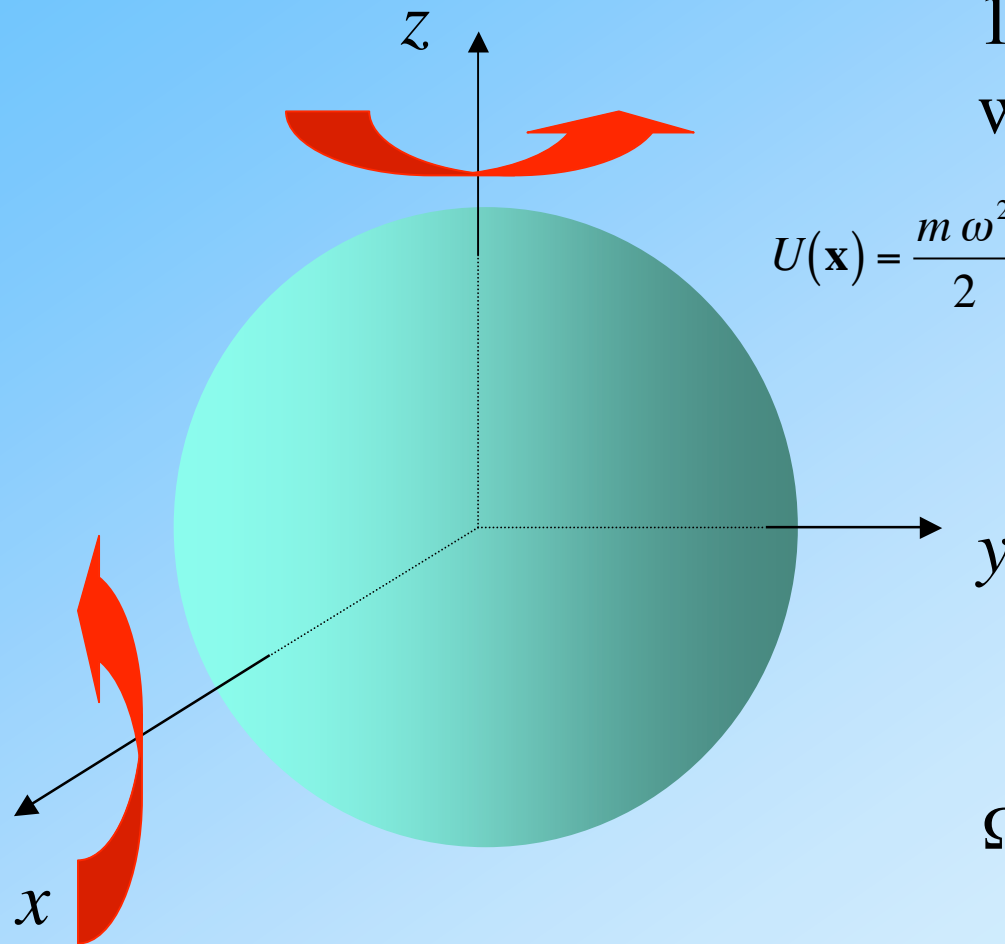
(1) We cannot apply some flow to the system.

(2) This is a finite-size system. Can we make turbulence with enough inertial range?



Then the coherence length is not much smaller than the system size. However, we could confirm the Kolmogorov law.

How to make turbulence in a trapped BEC



1. Trap the BEC in a weakly elliptic potential.

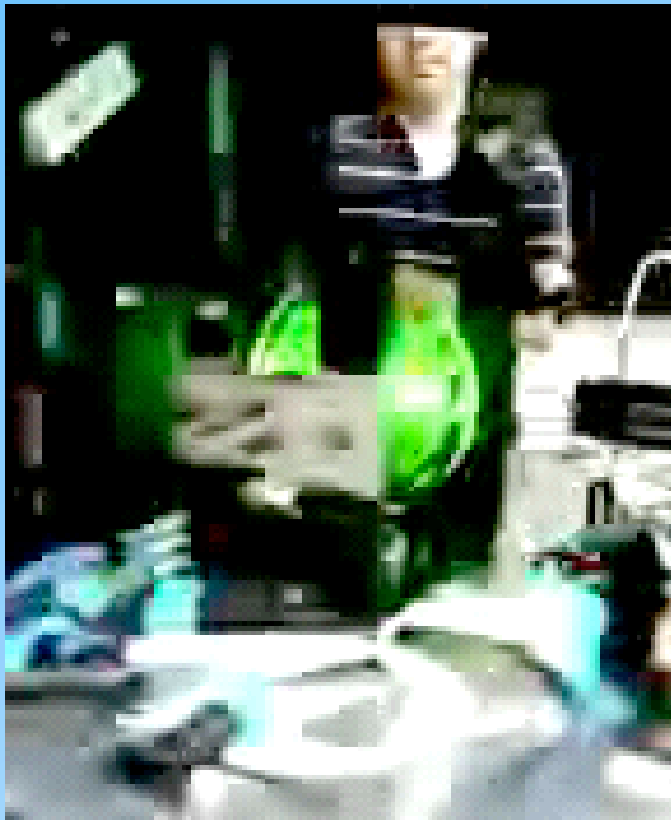
$$U(\mathbf{x}) = \frac{m \omega^2}{2} \left[(1 - \varepsilon_1)(1 - \varepsilon_2)x^2 + (1 + \varepsilon_1)(1 - \varepsilon_2)y^2 + (1 + \varepsilon_2)z^2 \right]$$

2. Rotate the system first around the x -axis, next around the z -axis.

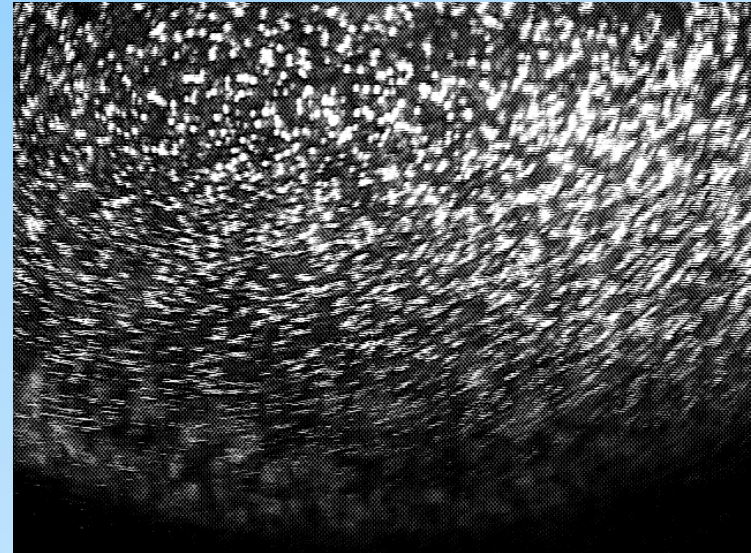
$$\Omega(t) = (\Omega_x, \Omega_z \sin \Omega_x t, \Omega_z \cos \Omega_x t)$$

Actually this idea has been already used in CT.

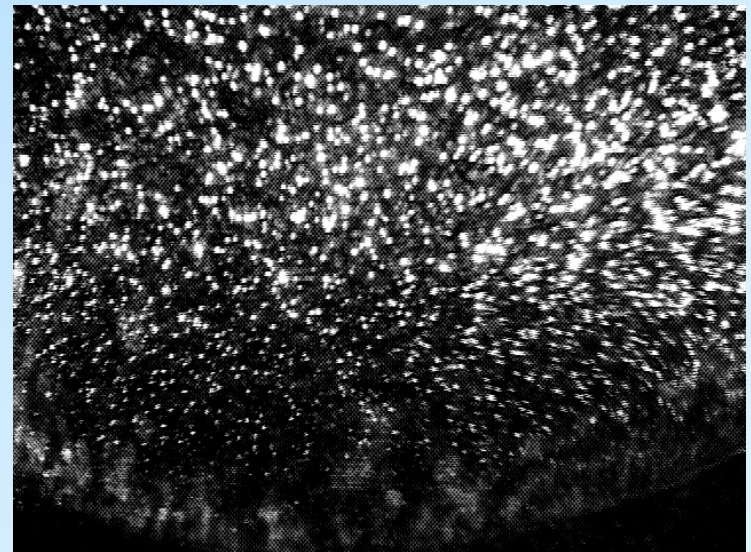
S. Goto, N. Ishii, S. Kida, and M. Nishioka, Phys. Fluids 19, 061705 (2007)



Rotation
around
one axis



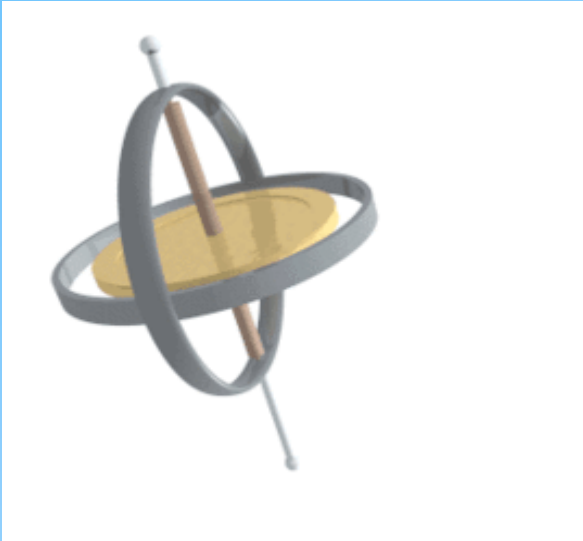
Rotation
around
two axes



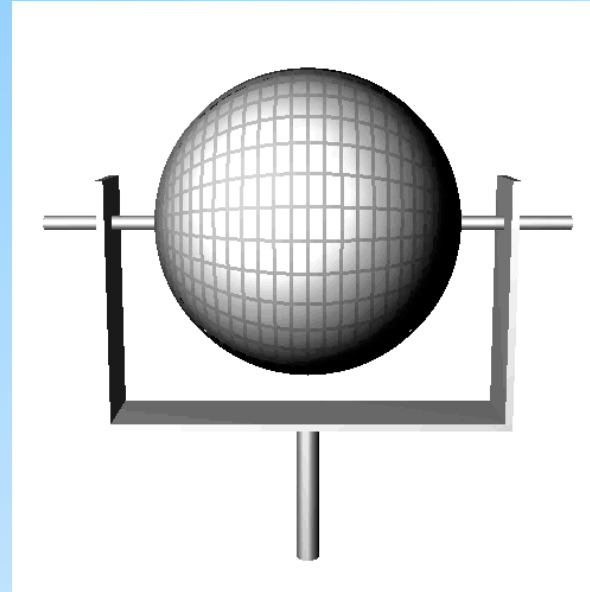
Are these two rotations represented by their sum? No!

Precession

Spin axis itself rotates around another axis.



Precessing motion of a gyroscope

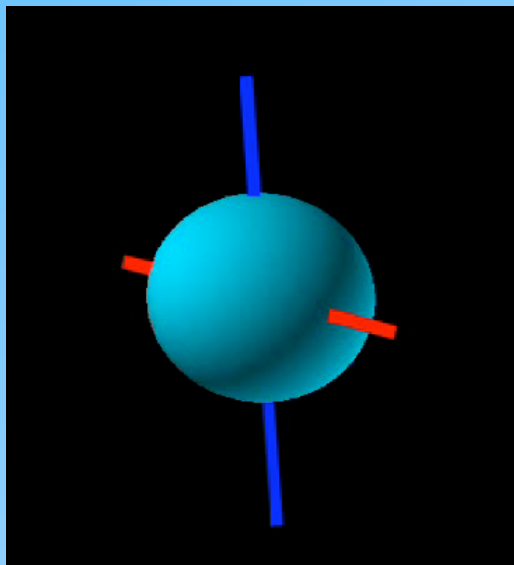


Ω_x

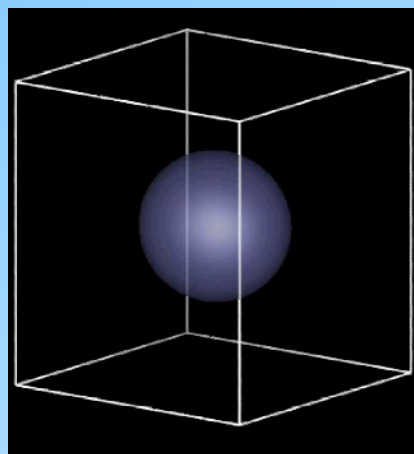
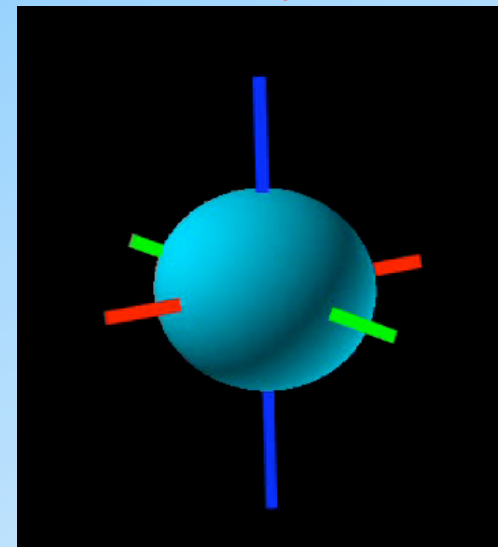
Ω_z

We consider the case where the spinning and precessing rotational axes are perpendicular to each other. Hence, the two rotations do not commute, and thus cannot be represented by their sum.

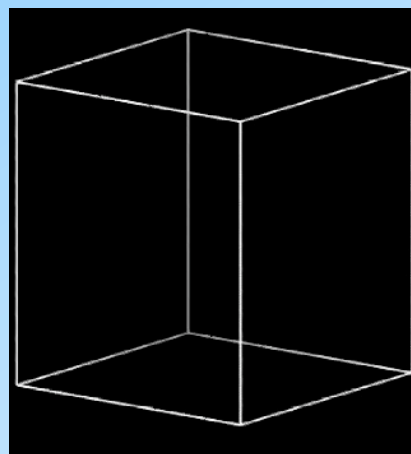
Two precessions ($\omega_x \times \omega_z$)



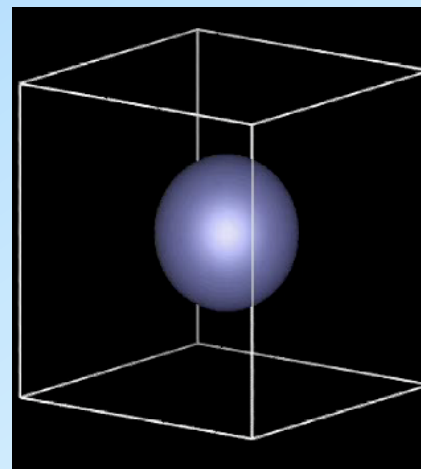
Three precessions ($\omega_y \times \omega_x \times \omega_z$)



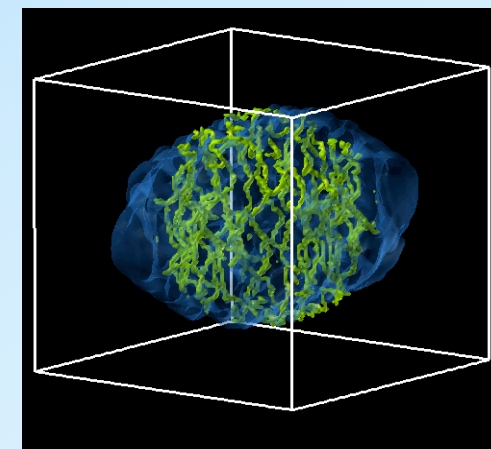
Condensate density



Quantized vortices



Condensate density

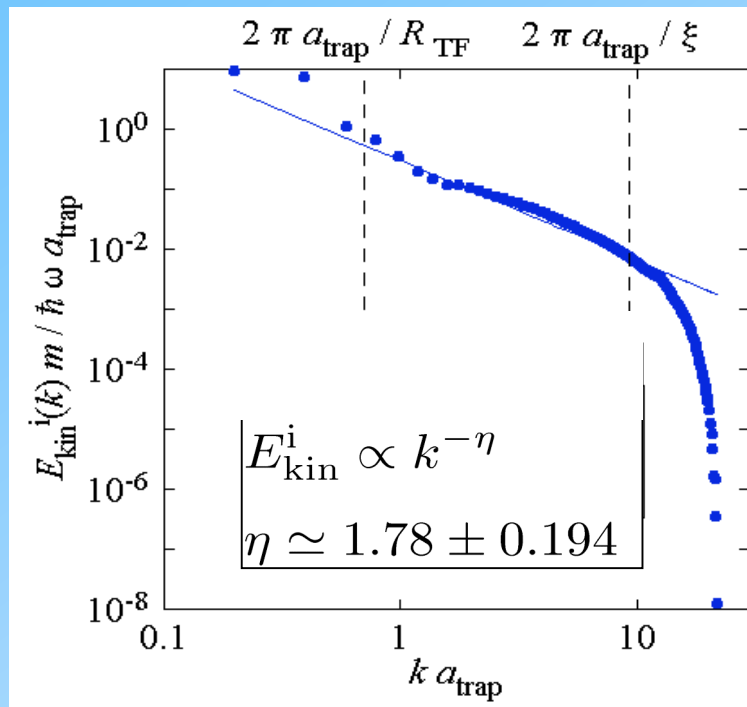


Quantized vortices

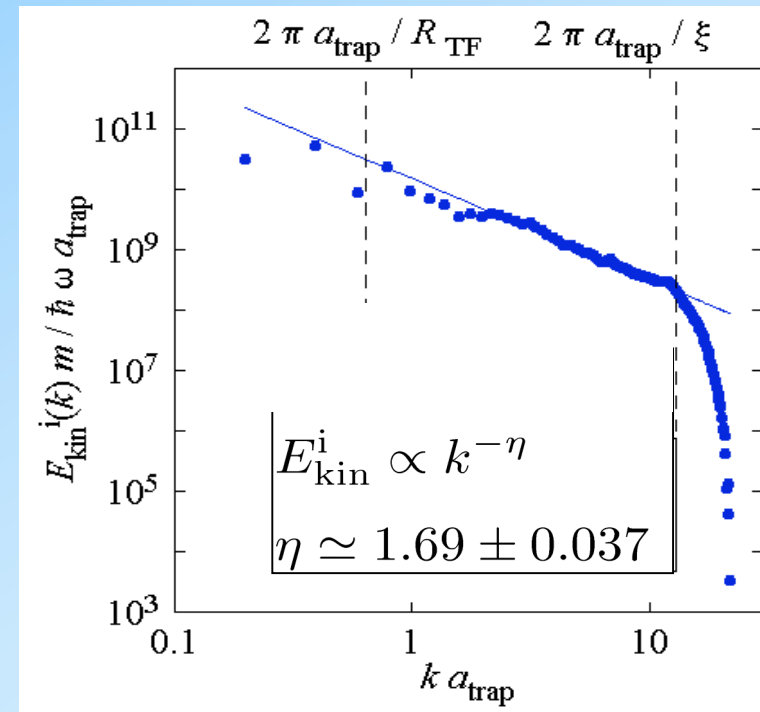
Rotation around two axes and three axes

-comparison of the energy spectra-

Rotation around two axes



Rotation around three axes

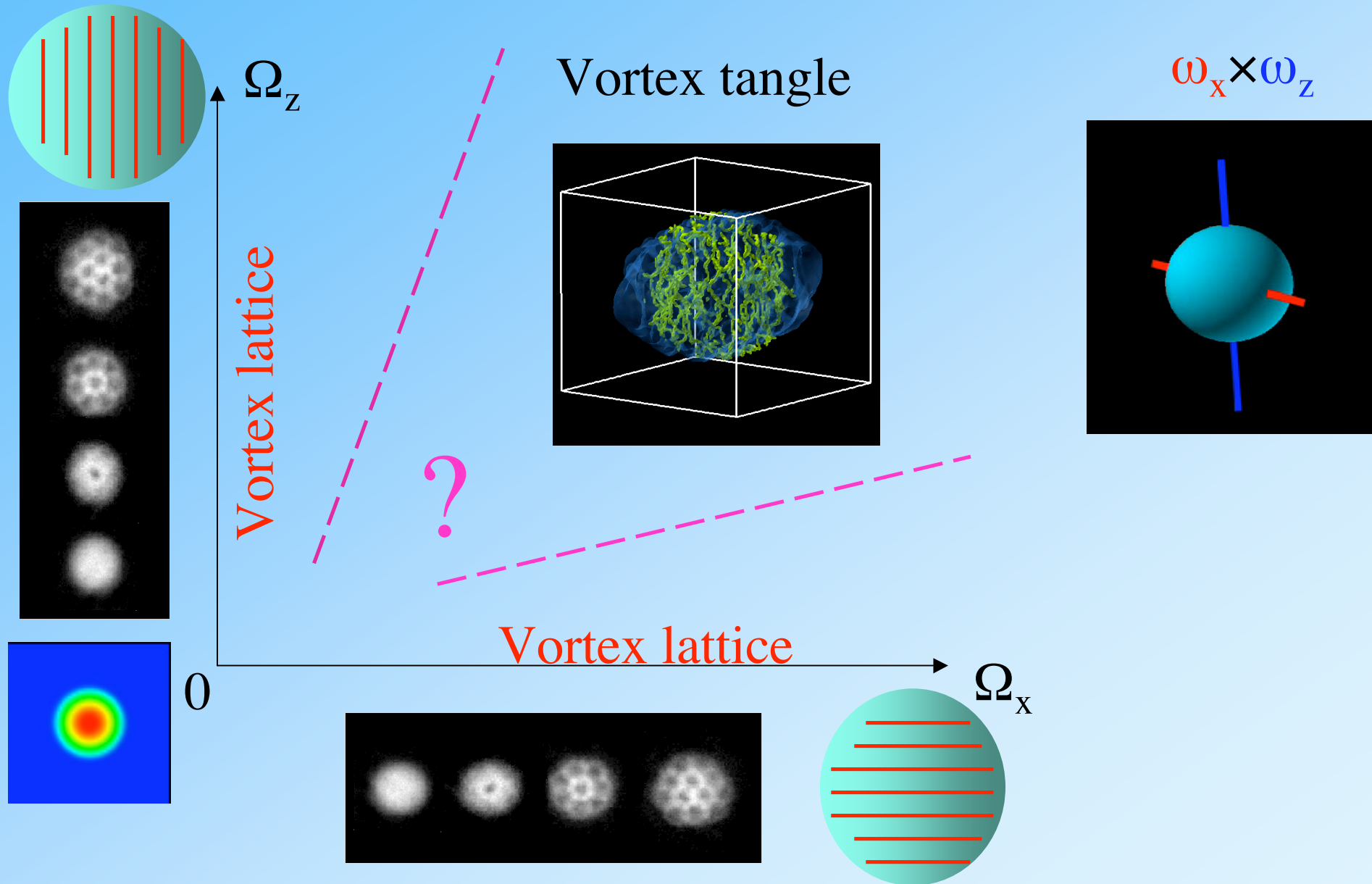


Rotation around three axes makes more isotropic QT whose η is closer to $5/3$.

What can we learn from QT of atomic BEC?

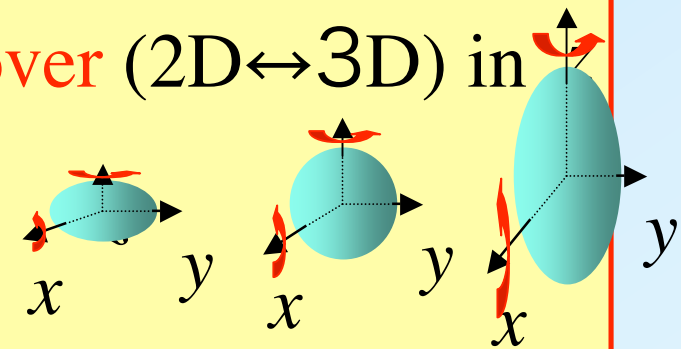
- Controlling the transition to turbulence by changing the rotational frequency or interaction parameters, etc.
- We can visualize quantized vortices. We can consider the relation of real space cascade of vortices and the wavenumber space cascade (Kolmogorov's law).
- Changing the trapping potential or the rotational frequency leads to dimensional crossover ($2D \leftrightarrow 3D$) in turbulence.

Controlling the transition to turbulence



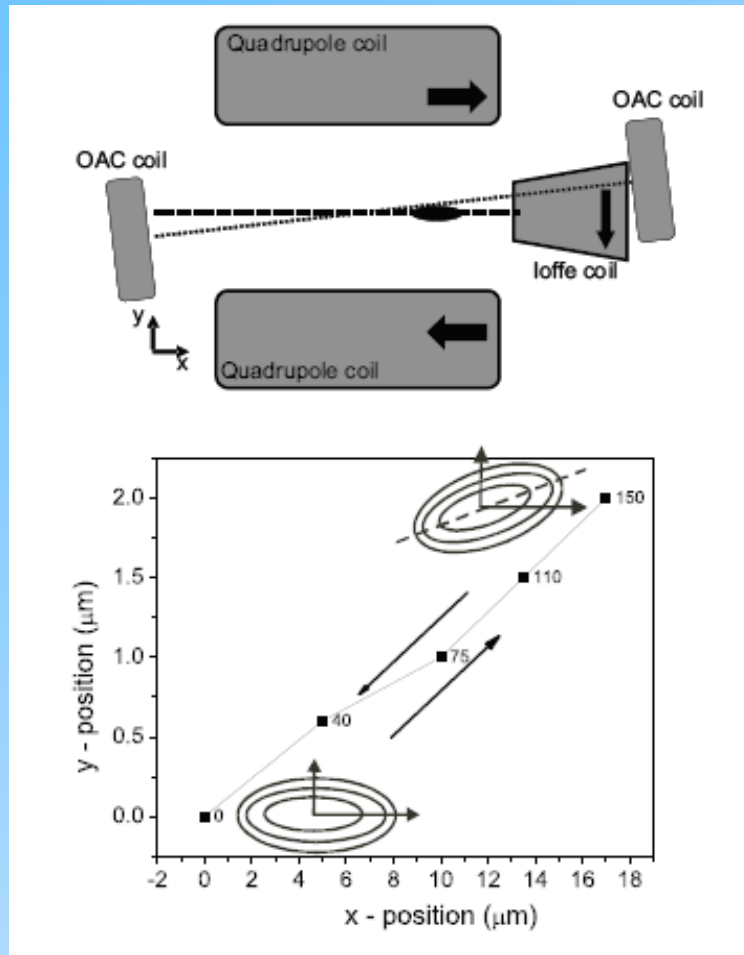
What can we learn from quantum turbulence of atomic BEC?

- **Controlling the transition to turbulence** by changing the rotational frequency or interaction parameters, etc.
- We can **visualize quantized vortices**. We can consider the relation of real space cascade of vortices and the wavenumber space cascade (Kolmogorov's law).
- Changing the trapping potential or the rotational frequency leads to **dimensional crossover** ($2D \leftrightarrow 3D$) in turbulence.

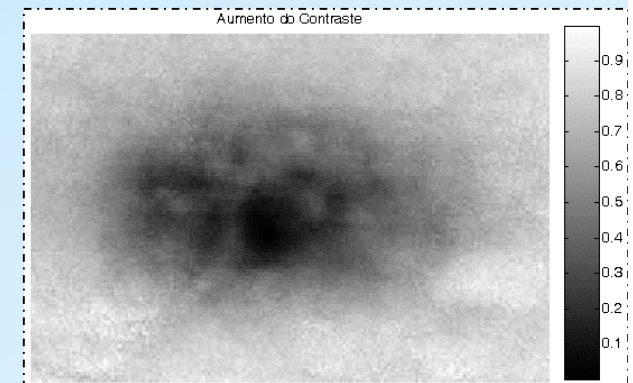
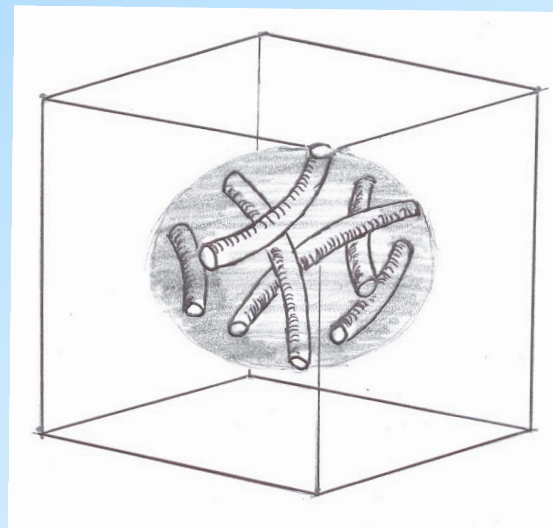
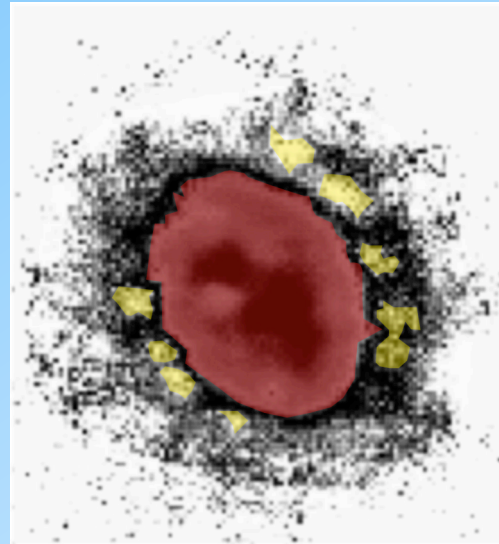


Very recently quantum turbulence was realized also in atomic BECs!

Henn *et al.*, PRL103, 045301(2009)



Coupled large amplitude oscillation



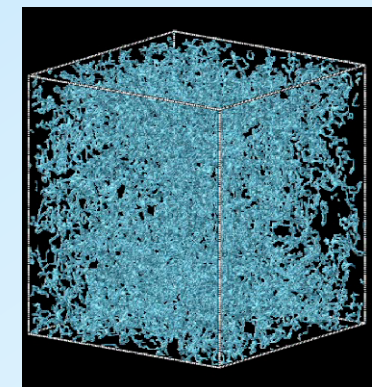
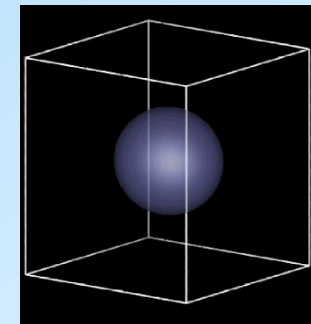
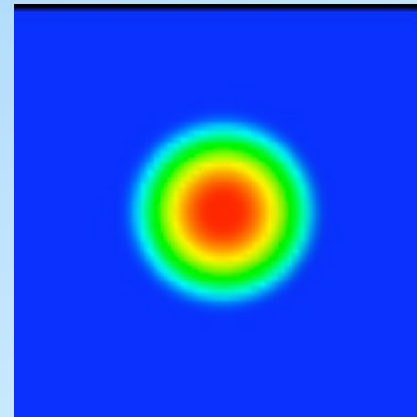
Summary

16th century

We discussed the recent interests in quantum turbulence.

Quantum turbulence consists of quantized vortices which are stable topological defects.

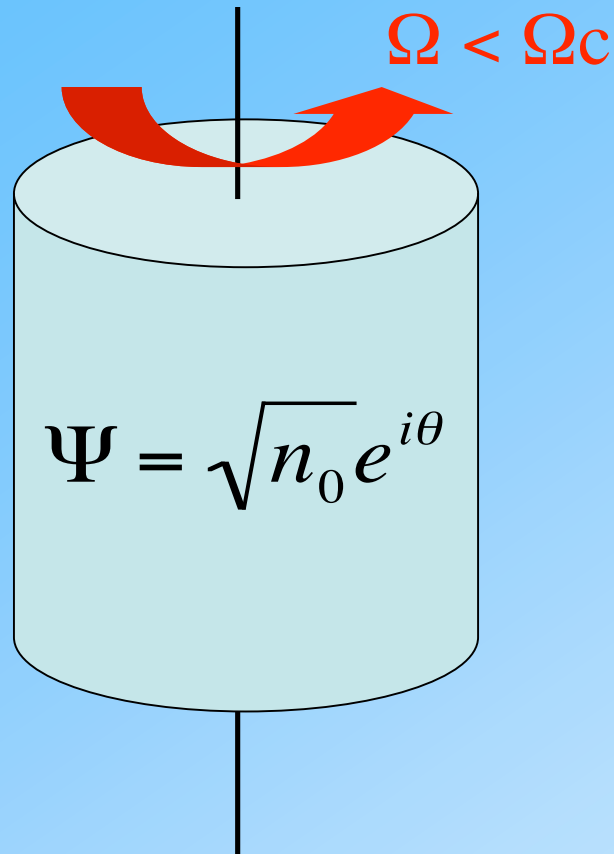
Quantum turbulence may give a prototype of turbulence much simpler than conventional turbulence.



21st century

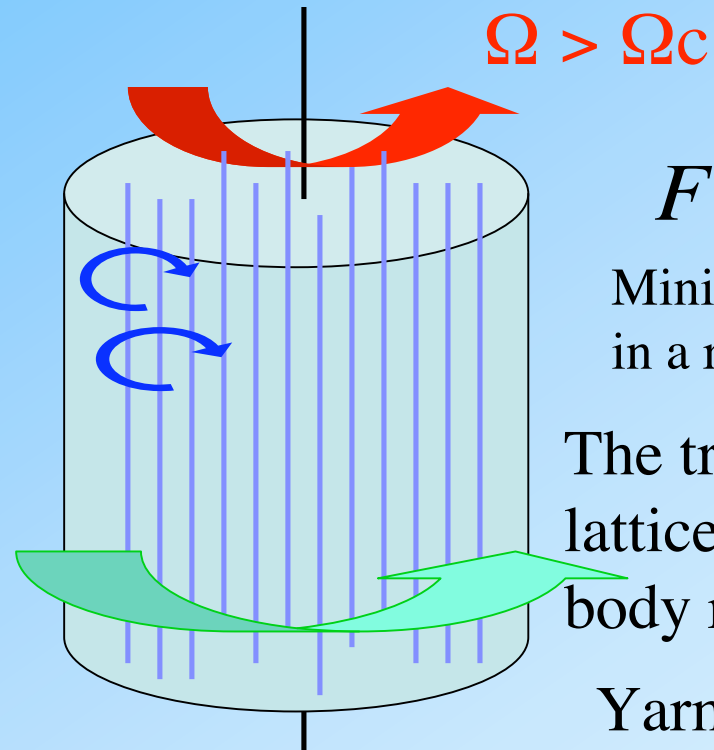


Rotating superfluid and the vortex lattice



$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \theta$$

$$\nabla \times \mathbf{v}_s = 0$$

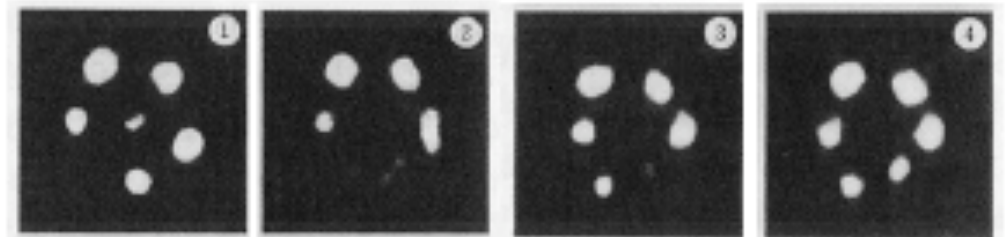


$$F = E - \Omega L_z$$

Minimizing the free energy
in a rotating frame.

The triangular vortex
lattice sustains the solid
body rotation.

Yarmchuck and Packard



Vortex lattice
observed in rotating
superfluid helium



Comments

4-1. Do we need dissipation for the vortex lattice formation in a rotating BEC?

4-2. Full Biot-Savart calculation for thermal counterflow

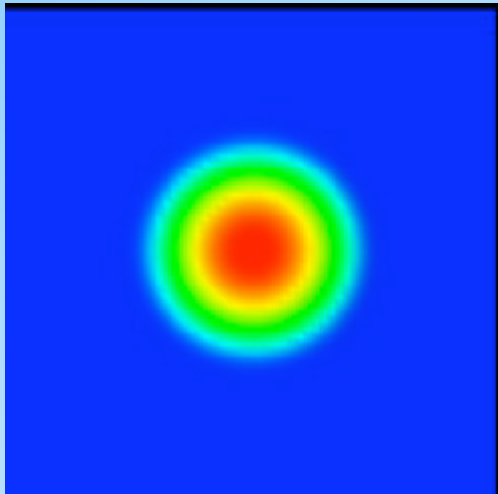
4-1. Do we need dissipation for the vortex lattice formation in a rotating BEC?

M. Kobayashi, K. Kasamatsu, M. Ueda, M. Tsubota, in preparation.

There is some controversy about this problem.

We can conclude that we do need dissipation.

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + (V_{\text{trap}} + U_{\text{stir}}) + g|\Psi|^2 - \mu - \Omega L_z \right] \Psi$$



C. Lobo, A. Sinatra, Y. Castin, Phys. Rev. Lett. 92, 020403(2004)

The authors say that a vortex lattice is formed even without dissipation.

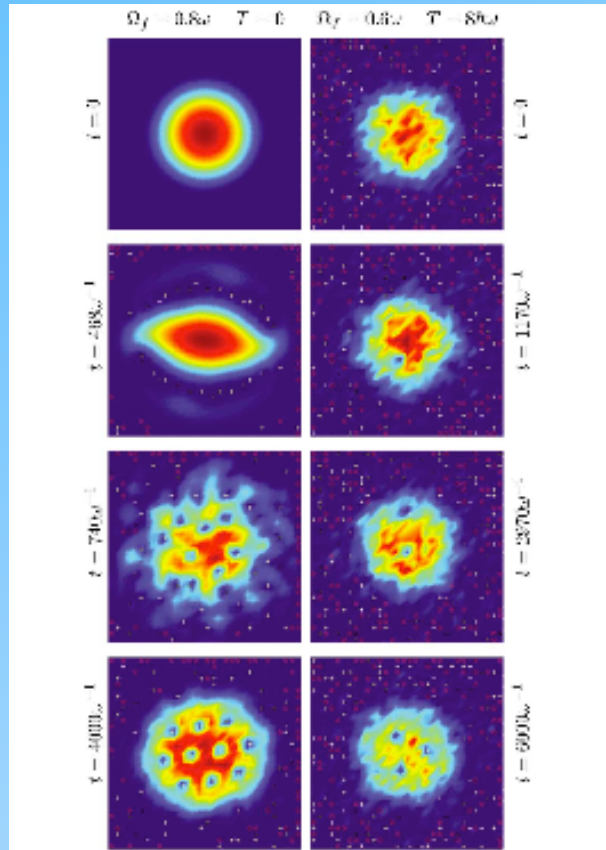


FIG. 1 (color online). Cut along the radial plane ($z = 0$) of the system spatial density at different times. Crosses (circles) indicate the position of vortices of positive (negative) charge [14]. Left column: $T = 0$, $\Omega_f = 0.8\omega$. Top to bottom: initial state; near instability; turbulent behavior; end of simulation. Right column: $k_B T = 8\hbar\omega$, $\Omega_f = 0.6\omega$. Top to bottom: initial state; entry of first vortex; entry of second vortex; end of simulation with a three-vortex lattice.

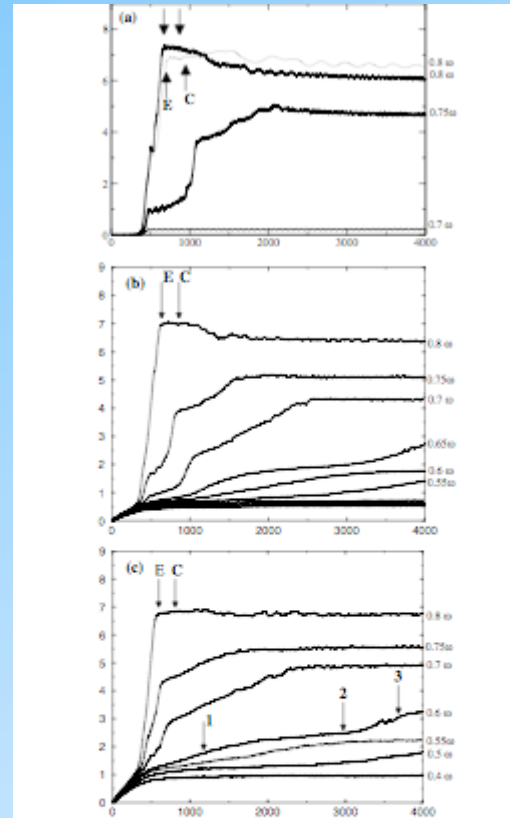


FIG. 2. Total angular momentum of the system in units of $\hbar$ per atom as a function of ωt . The arrows marked E and C indicate the entry of the vortices into the condensate and the crystallization of the lattice for $\Omega_f = 0.8\omega$. (a) $T = 0$, solid lines from bottom to top: $\Omega_f/\omega = 0.7(0)$, $0.75(7)$, $0.8(10)$; dashed line: $\Omega_f/\omega = 0.8(10)$ with a grid size of $64 \times 64 \times 256$. All other curves were done on a $32 \times 32 \times 128$ grid. In parenthesis is the number of vortices in the lattice at the end of the simulation. (b) $k_B T = 4\hbar\omega$, $\Omega_f/\omega = 0.4(0)$, $0.45(0)$, $0.5(0)$, $0.55(1)$, $0.6(1)$, $0.65(2)$, $0.7(6)$, $0.75(7)$, $0.8(10)$. (c) $k_B T = 8\hbar\omega$, $\Omega_f/\omega = 0.4(0)$, $0.5(1)$, $0.55(1)$, $0.6(3)$, $0.7(7)$, $0.75(7)$, $0.8(10)$. The arrows correspond to the approximate entry time of the vortices for $\Omega_f = 0.6\omega$ as shown in Fig. 1. Note that the total angular momentum shows no signature of the entries.

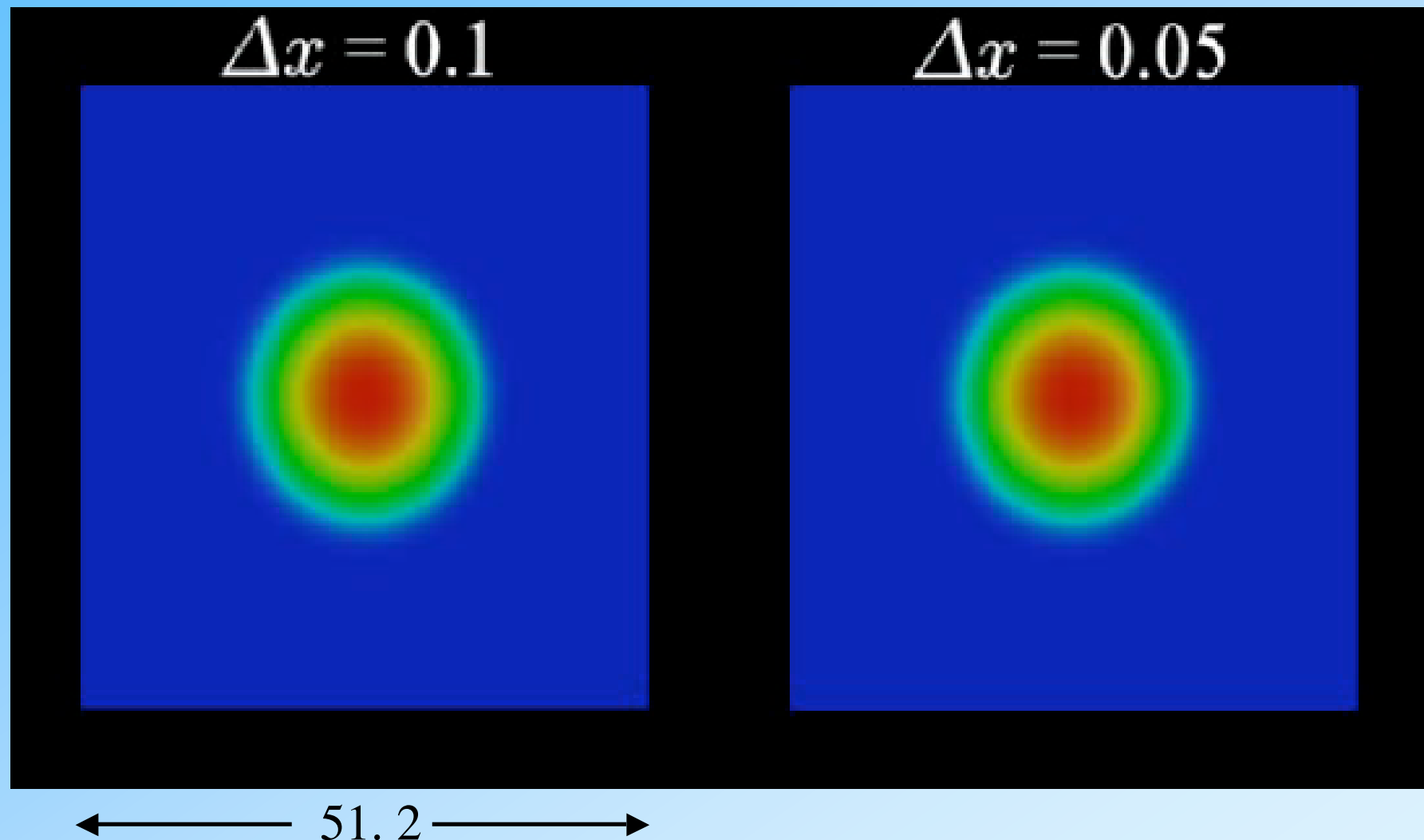
3D simulation

Grid size

$32 \times 32 \times 128$

$64 \times 64 \times 256$

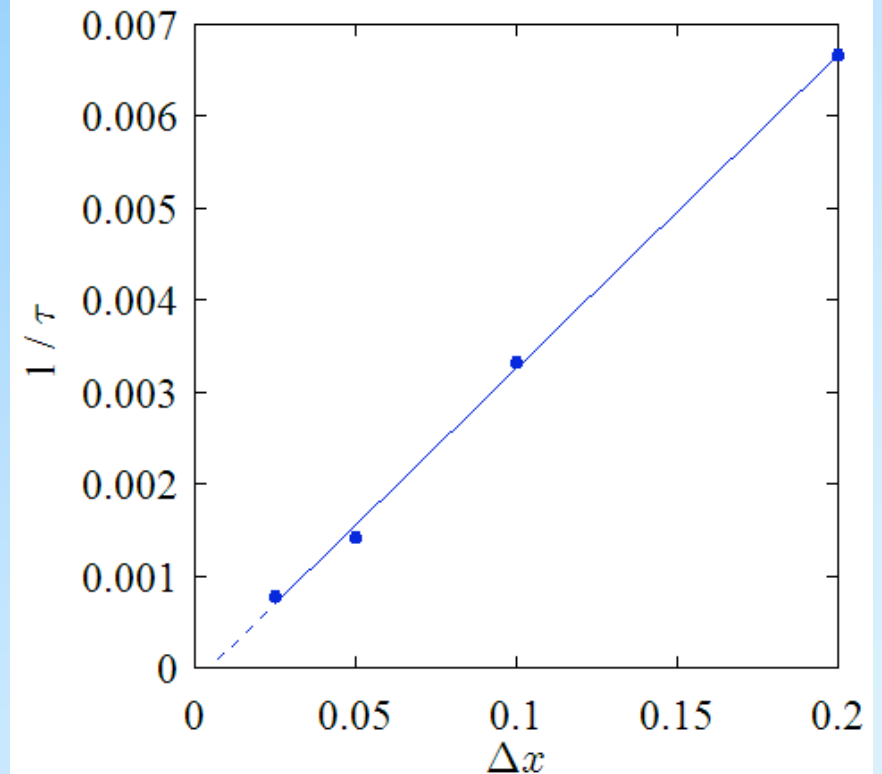
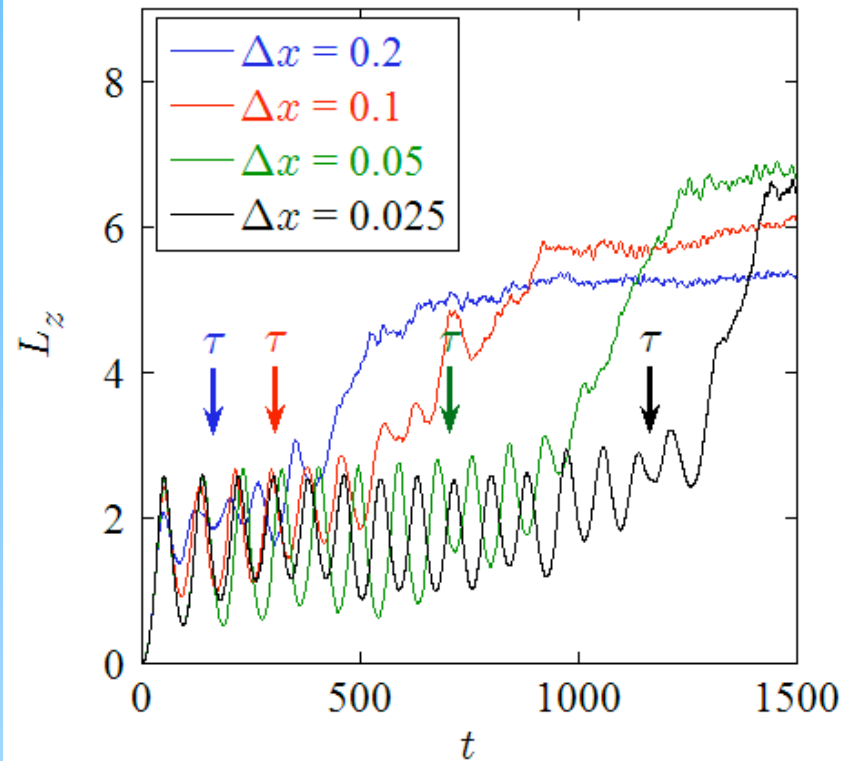
2D Simulation without dissipation for different mesh sizes



The dynamics depends strongly on the mesh size!

The finer the mesh becomes, the longer it takes for the vortex lattice to form.

Angular momentum



At τ , the vortices enter the condensate.

τ becomes infinite for $\Delta x \rightarrow 0$.

Vortices never enter the condensate for infinitely small mesh!

Therefore, the statement “a vortex lattice is formed even without dissipation” is doubtful.

Conclusion

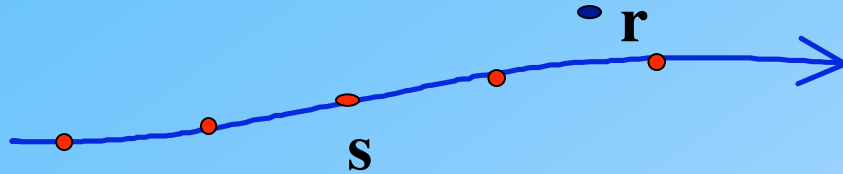
The system needs some dissipation in order to form a vortex lattice.

4-2. Full Biot-Savart calculation for thermal counterflow

Makoto Tsubota, Hiroyuki Adachi

Osaka City university, Japan

Vortex filament model (Schwarz)



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered.

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi} \mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi} \int_L \frac{(\mathbf{s}_1 - \mathbf{r}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{r}|^3} + \mathbf{v}_{s,a}(\mathbf{s})$$

$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_0 + \alpha \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0) - \alpha' \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0)]$$

The approximation neglecting the nonlocal term is called the LIA(Localized Induction Approximation).

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi} \mathbf{s}' \times \mathbf{s}'' + \mathbf{v}_{s,a}(\mathbf{s})$$

Schwarz's simulation(1) PRB38, 2398(1988)

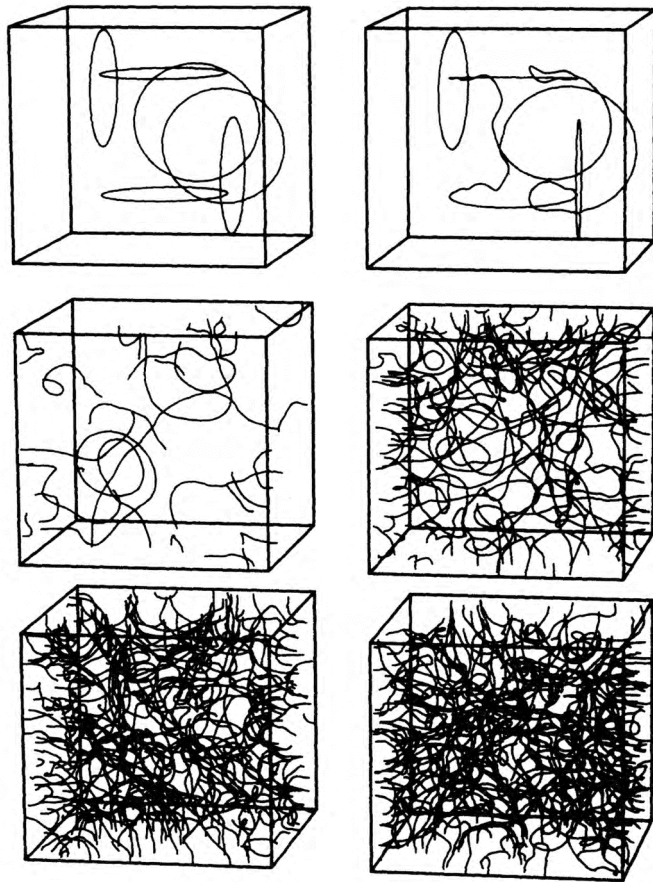


FIG. 4. Case study of the development of a vortex tangle in a real channel. Here, $\alpha=0.10$, corresponding to a temperature of about 1.6 K, and $v_{s,0}=75$ into the front face of the channel section shown. Upper left: $t_0=0$, no reconnections; upper right: $t_0=0.0028$, three reconnections; middle left: $t_0=0.05$, 18 reconnections; middle right: $t_0=0.20$, 844 reconnections; lower left: $t_0=0.55$, 12 128 reconnections; lower right: $t_0=2.75$, 124 781 reconnections.

Schwarz simulated the counterflow turbulence by the vortex filament model and obtained the steady state.

However, this simulation was unsatisfactory.

1. All calculations were performed by the LIA.

Schwarz's simulation(2) PRB38, 2398(1988)

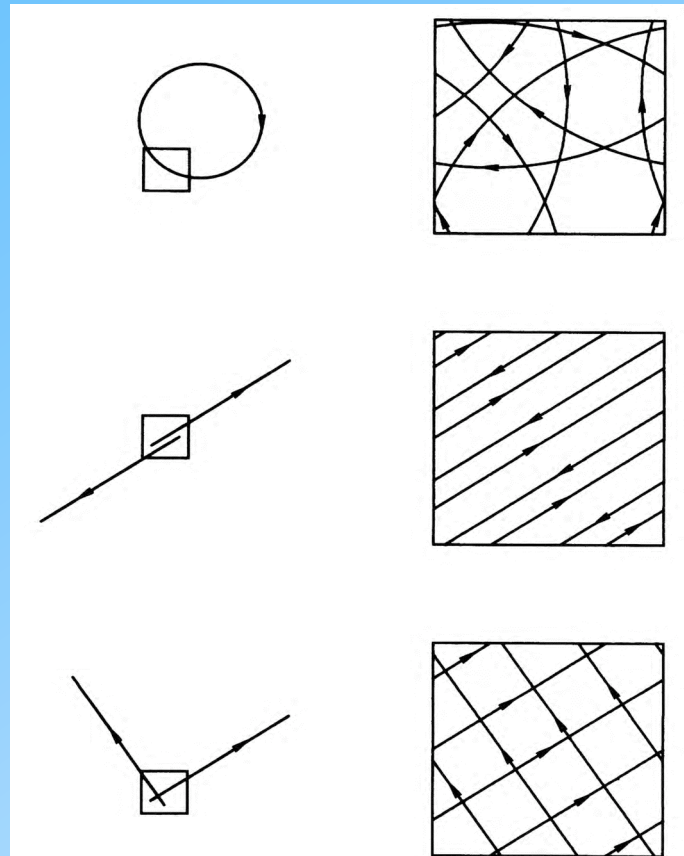


FIG. 8. Mapping of various vortex configurations into the computational volume, showing the appearance of the unit cell when all space is filled by the repetition of these objects. The end points of the lines represent equivalent points in the unit cell. Top row: closed loops; middle row: parallel infinite lines characteristic of a dead-end fluctuation; bottom row: infinite lines after randomizing procedure designed to reestablish three-dimensional behavior. The illustrations are intended to be purely schematic.

However, this simulation was unsatisfactory.

1. All calculation was performed by the LIA.
2. He used an artificial mixing procedure in order to obtain the steady state.

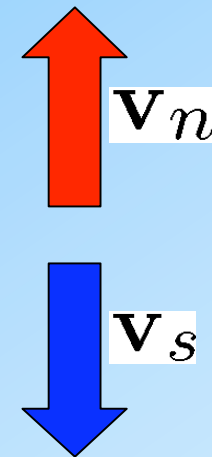
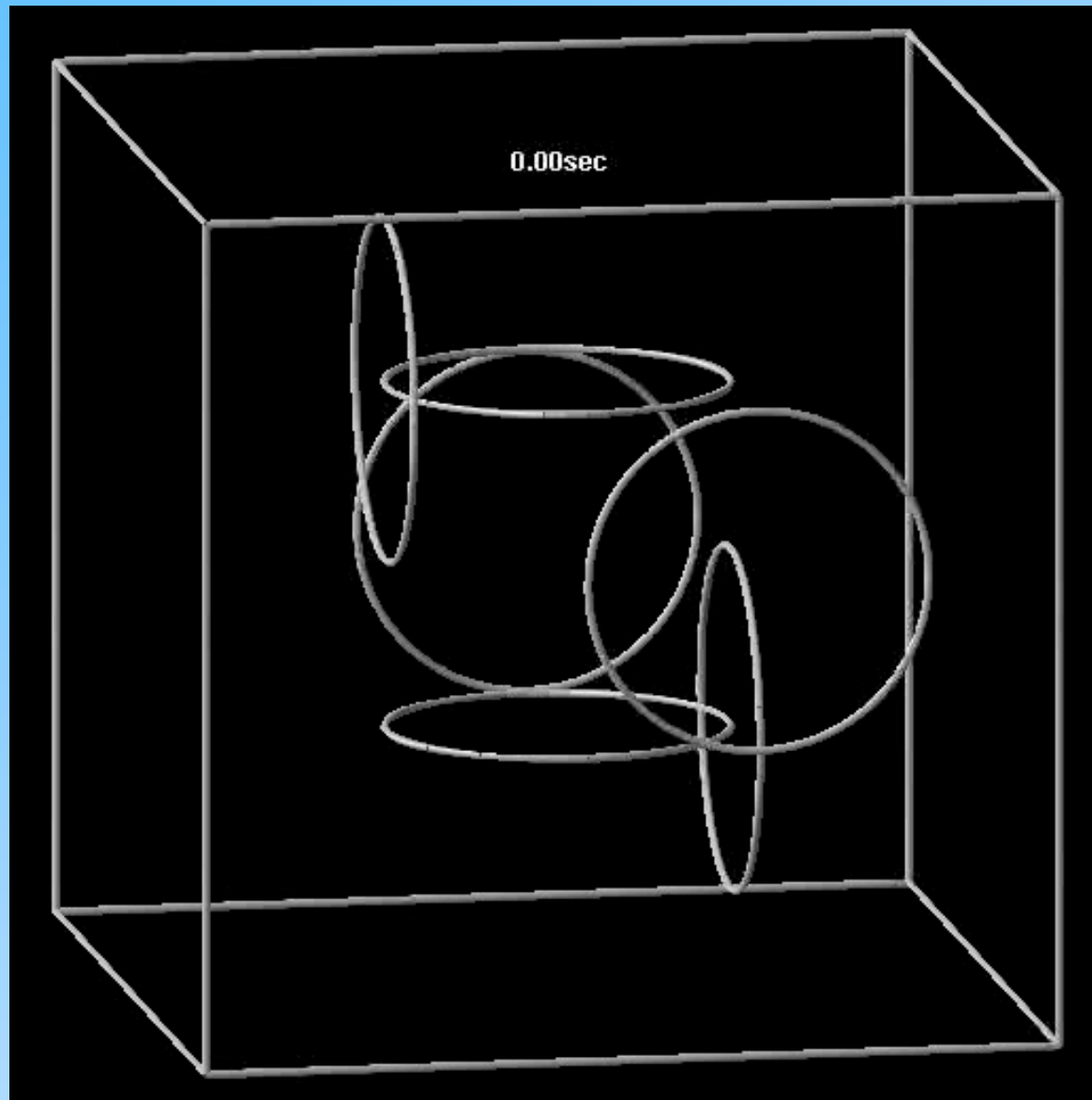
After Schwarz, there has been no progress on the counterflow simulation.

In this work we made the steady state of counterflow turbulence by fully nonlocal simulation.

An important criterion of the steady state is to obtain

$$L = \gamma^2 |v_n - v_s|^2 \quad \gamma = \frac{\pi B \rho_n \chi_1}{\kappa \rho \chi_2}$$

Simulation by the full Biot-Savart law



BOX $(0.1\text{cm})^3$

$T=1.6(\text{K})$

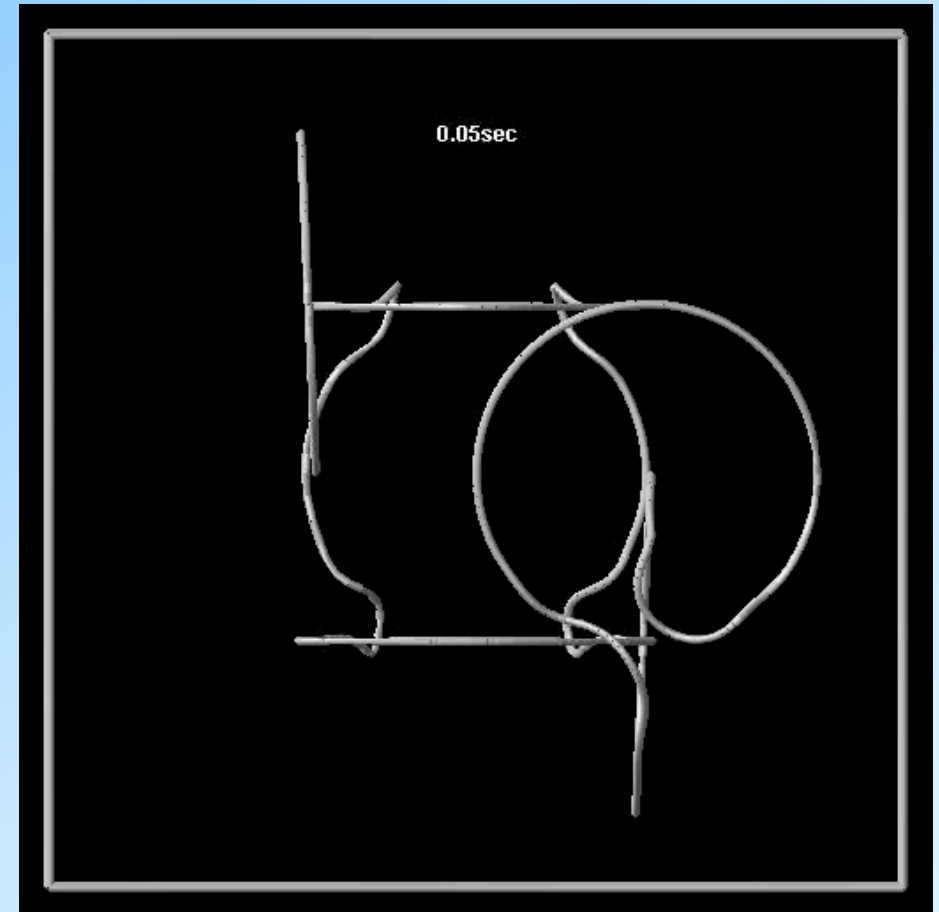
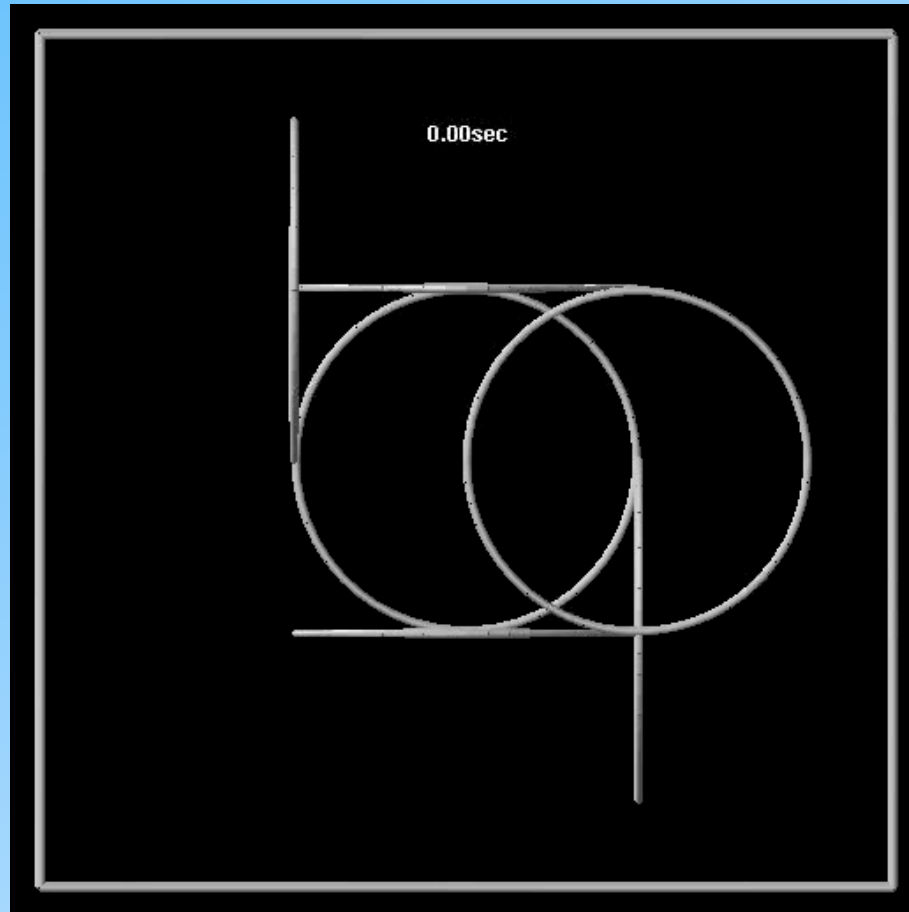
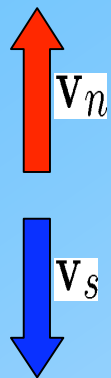
$V_{ns}=0.367\text{cm/s}$

Periodic boundary conditions for
all three directions

Comparison between LIA and full Biot-Savart

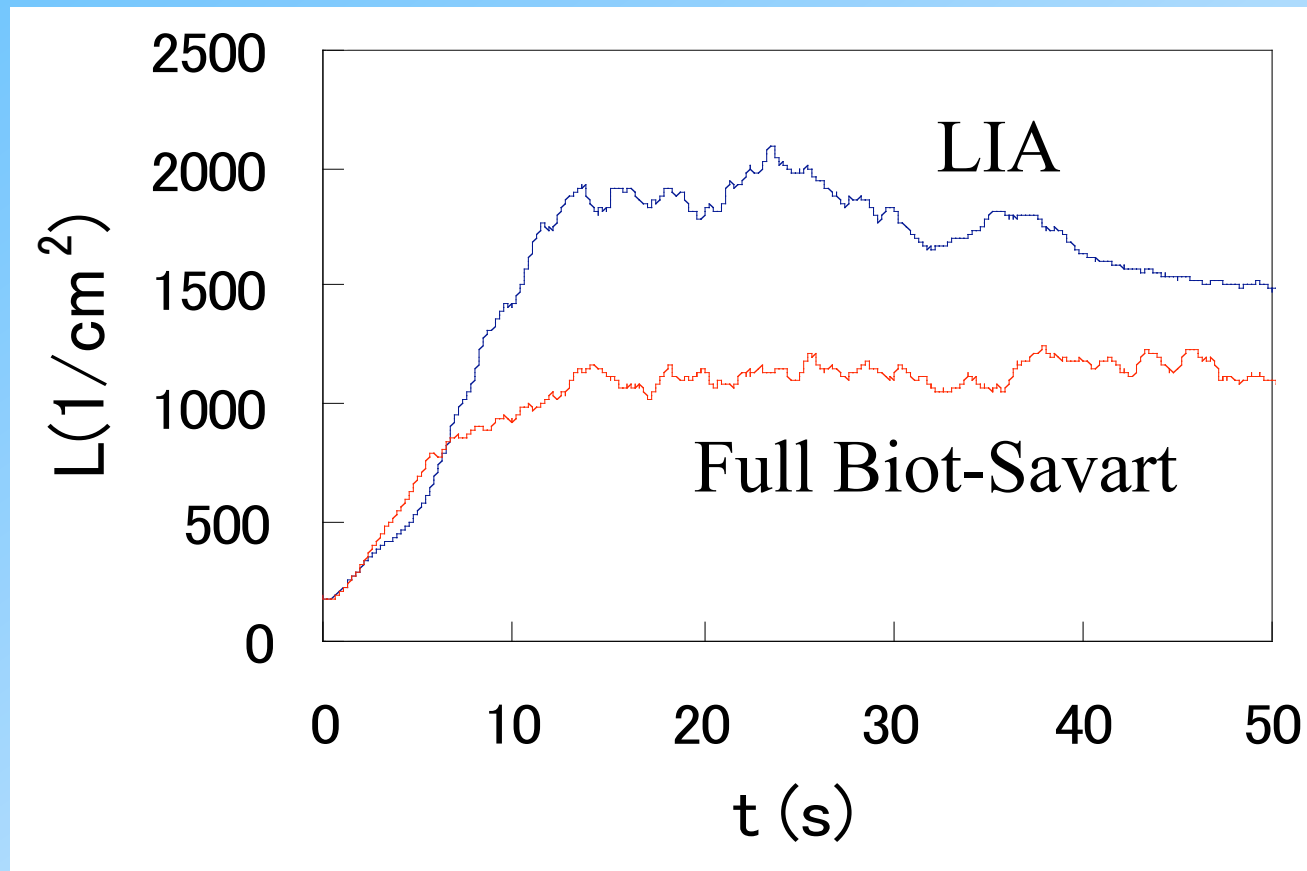
Full Biot-Savart

LIA



Vortices become anisotropic, forming layer structures.

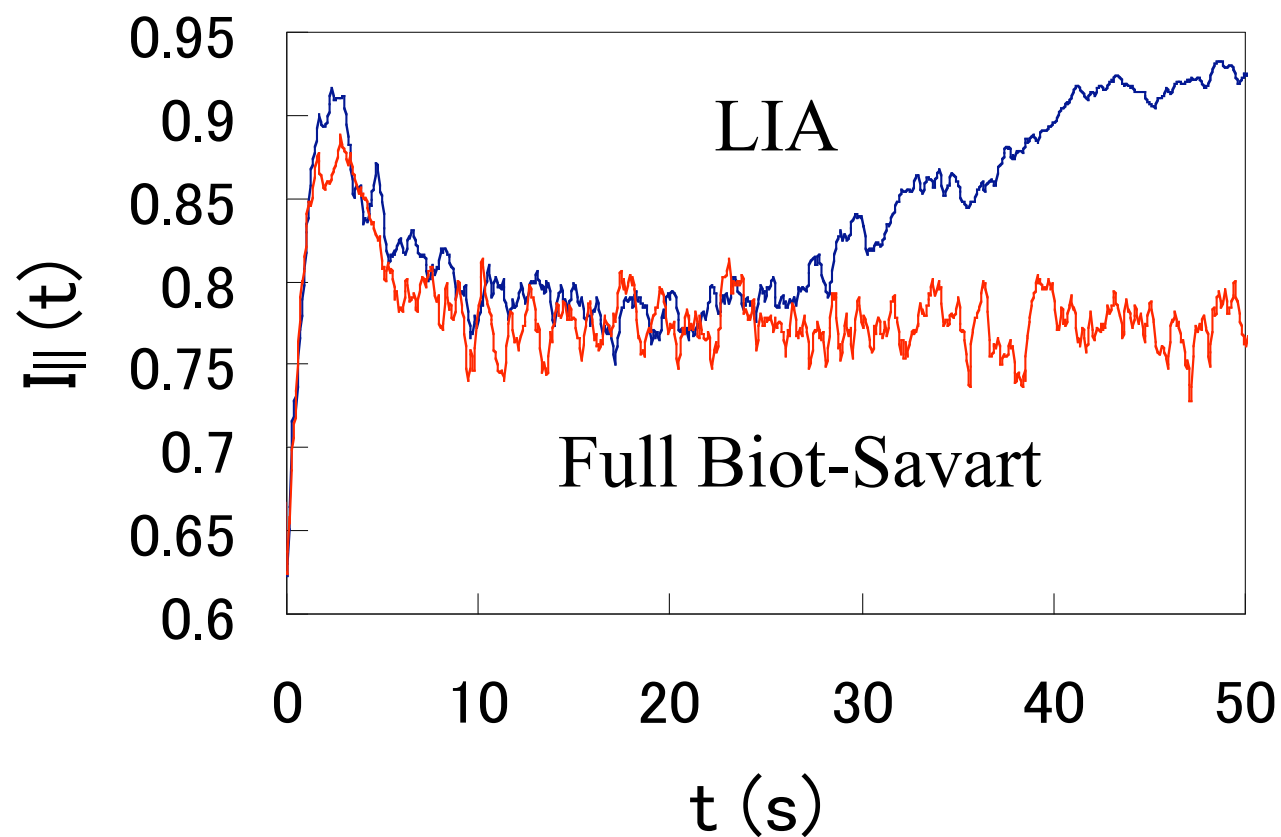
Developments of the line-length density between LIA and Full Biot-Savart



$T=1.6$ K、 $V_{ns}=0.367$ cm/s、 $\text{box}=(0.2 \text{ cm})^3$

Anisotropic parameter

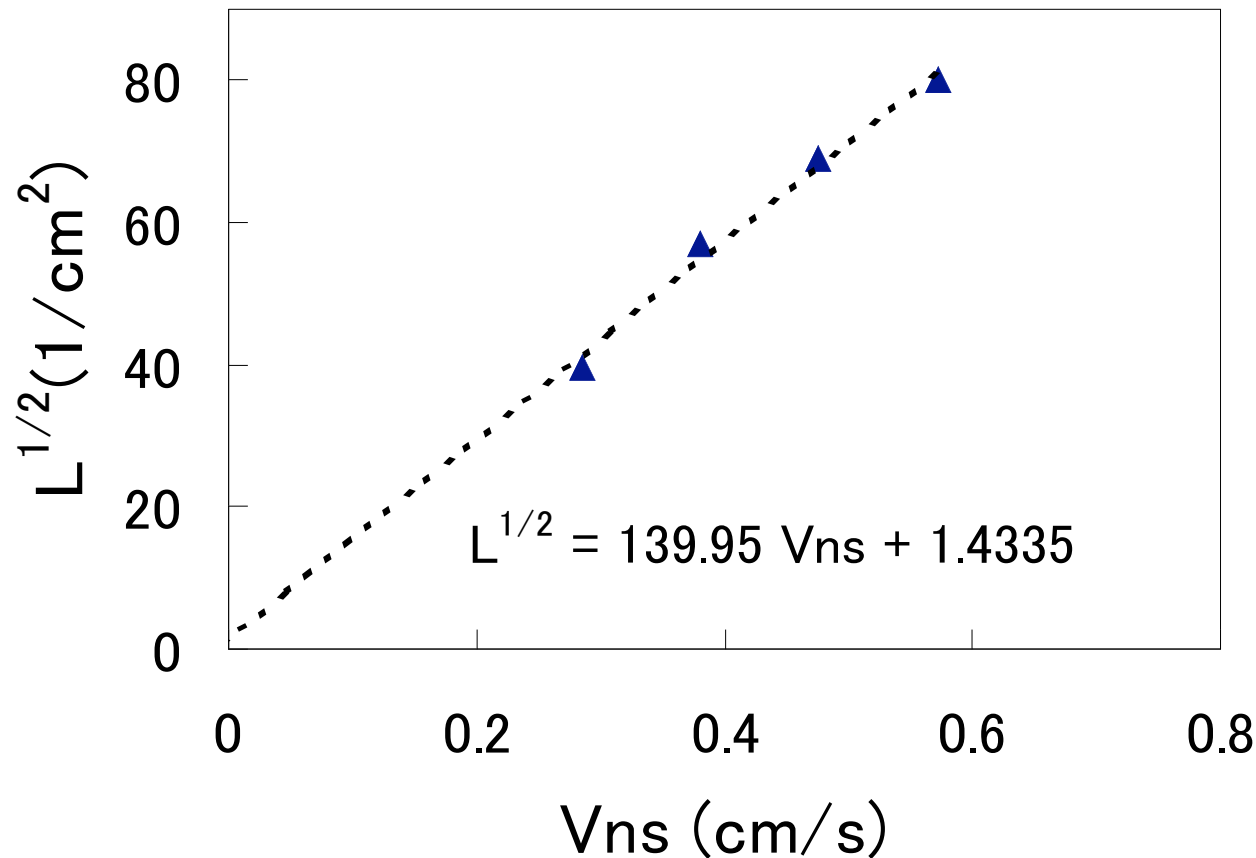
$$I_{||} = \frac{1}{\Omega L} \int_{\mathcal{L}} [1 - (\mathbf{s}' \cdot \hat{\mathbf{r}}_{||})^2] d\xi$$



$T=1.6$ K、 $V_{ns}=0.367$ cm/s、 $\text{box}=(0.2 \text{ cm})^3$

L vs. Vns by full Biot-Savart simulation

T=1.9K、box (0.1cm)³



We first obtained a statistically steady state by full Biot-Savart simulation.

$$L = \gamma^2 v_{ns}^2$$

Numerical results $\gamma \simeq 140$

Experiment $\gamma \simeq 130$