On interpolation in theorem proving

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What is interpolation?

- Consider a function f (univariate for simplicity)
- We know the values of f at points $x_1, \ldots, x_n$ on the x-axis (e.g., from sampling or experiments)
- We want to know the values of f at additional intermediate points and build its curve
- This is the problem of interpolation in numerical analysis
- It has many applications in computer graphics (e.g., spline interpolation)

Interpolation in logic

What is interpolation in logic?

Signature

- A finite set of constant symbols: e.g., a, b, c ...
- A finite set of function symbols: e.g., f, g, h ...
- A finite set of predicate symbols: P, Q, R, $\simeq$ ...
- Arities
- Sorts (important but key concepts can be understood without)

An infinite supply of variables: x, y, z, w ...

Logical language

- Terms: a, x, $f(a)$, $f(x)$, $g(a,x)$ ...
- Atoms: R, $P(a)$, $Q(x, g(b))$, ...
- Literals: R, $P(a)$, $Q(x, g(b))$, $\neg R$, $\neg P(a)$, $\neg Q(x, g(b))$, ...
- Formulae: $P(a) \wedge Q(a, g(b))$, $\neg P(a) \vee Q(a, g(b))$, $\neg P(a) \supset Q(g(b), c)$, $\forall x\ P(x)$, $\forall x \exists y\ P(x) \supset Q(y, x)$, ...
- Special formulae: $\perp$, $\top$

Logical language

- Ground term, atom, literal, formula: no occurrences of variables
- Closed formula: all variables are quantified (aka: sentence)

Defined symbols and free symbols

- A symbol is defined if it comes with axioms, e.g., $\simeq$
- Equality ($\simeq$) comes with the congruence axioms
- It is free otherwise, e.g., P
- Aka: interpreted/uninterpreted

Equality and the congruence axioms

- $\forall x.\ x \simeq x$
- $\forall x \forall y.\ x \simeq y \supset y \simeq x$
- $\forall x \forall y \forall z.\ x \simeq y \wedge y \simeq z \supset x \simeq z$
- $\forall x \forall y.\ x \simeq y \supset f(\ldots, x, \ldots) \simeq f(\ldots, y, \ldots)$
- $\forall x \forall y.\ [x \simeq y \wedge P(\ldots, x, \ldots)] \supset P(\ldots, y, \ldots)$

Craig interpolation or interpolation tout court

- Formulæ A and B such that $A \vdash B$
- An interpolant I is a formula that lies between A and B:
  - Derivability: $A \vdash I$ and $I \vdash B$
  - Signature: I made of symbols common to A and B
    where symbol means predicate, function, constant symbol

Trivial cases

- All symbols of A appear in B: then A itself is the interpolant
- All symbols of B appear in A: then B itself is the interpolant

Assume that at least one has at least one symbol that does not appear in the other

Craig's Interpolation Theorem (1957)

- If A and B are closed formulæ with at least one predicate symbol in common
- Then an interpolant I exists and it is also a closed formula
- No predicate symbol in common: either A is unsatisfiable and I is $\perp$ or B is valid and I is $\top$

Theorem proving

- $A \vdash^? B$ is a theorem-proving problem
- Refutational theorem proving
- Equivalently: is $A \wedge \neg B$ inconsistent?
- $A \wedge \neg B \vdash^? \bot$
- $A, \neg B \vdash^? \bot$

Proofs by refutation: reverse interpolant

- A and B inconsistent: $A, B \vdash \perp$
- Then $A \vdash I$ and $B, I \vdash \perp$
- All symbols in I common to A and B

Reverse interpolant of (A, B): interpolant of $(A, \neg B)$
because $A, B \vdash \perp$ means $A \vdash \neg B$ and $B, I \vdash \perp$ means $I \vdash \neg B$

Interpolant of (A, B): reverse interpolant of $(A, \neg B)$

In refutational settings we say interpolant for reverse interpolant

Example

- A is $\forall x.\ P(c, x)$
- B is $\forall x.\ \neg P(x, d)$
- A and B are inconsistent
- Interpolant I is $\exists y \forall x.\ P(y, x)$

Reasoning modulo theory $\mathcal{T}$

- $\vdash_{\mathcal{T}}$ in place of $\vdash$
- All uninterpreted symbols in I common to A and B
- No restrictions on interpreted symbols

Example

- A is $a_1 \not\simeq a_2$
- B is $\forall x \forall y.\ x \simeq y$
- A and B are inconsistent
- Interpolant I is $\exists x \exists y.\ x \not\simeq y$

Clausal theorem proving

- Clause: disjunction of literals where all variables are implicitly universally quantified
- $\neg P(f(z)) \vee \neg Q(g(z)) \vee R(f(z), g(z))$
- No loss of generality: every formula can be transformed into a conjunction, or set, of clauses
- Inconsistency is preserved

Transformation into clausal form

- Eliminate $\equiv$ and $\supset$: ($F \equiv G$ becomes $(F \supset G) \wedge (G \supset F)$ and $F \supset G$ becomes $\neg F \vee G$)
- Reduce the scope of all occurrences of $\neg$ to an atom: ($\neg(F \vee G)$ becomes $\neg F \wedge \neg G$, $\neg(F \wedge G)$ becomes $\neg F \vee \neg G$, $\neg\neg F$ becomes F, $\neg\exists F$ becomes $\forall\neg F$, and $\neg\forall F$ becomes $\exists\neg F$)
- Standardize variables apart
(each quantifier occurrence binds a distinct variable symbol)
- Skolemize $\exists$ and then drop $\forall$
- Distributivity and associativity: $F \vee (G \wedge H)$ becomes $(F \vee G) \wedge (F \vee H)$ and $F \vee (G \vee H)$ becomes $F \vee G \vee H$
- Replace $\wedge$ by comma and get a set of clauses

Skolemization

- Outermost $\exists$:
  - $\exists x\ F[x]$ becomes $F[a]$ (all occurrences of x replaced by a)
    a is a new Skolem constant
  - There exists an element such that F: let this element be named a
- $\exists$ in the scope of $\forall$:
  - $\forall y \exists x\ F[x, y]$ becomes $\forall y\ F[g(y), y]$
    (all occurrences of x replaced by $g(y)$)
    g is a new Skolem function
  - For all y there is an x such that F: x depends on y;
    let g be the map of this dependence

A simple example

- $\neg\{[\forall x\ P(x)] \supset [\exists y\ \forall z\ Q(y,z)]\}$
- $\neg\{\neg[\forall x\ P(x)] \vee [\exists y\ \forall z\ Q(y,z)]\}$
- $[\forall x\ P(x)] \wedge \neg[\exists y\ \forall z\ Q(y,z)]$
- $[\forall x\ P(x)] \wedge [\forall y\ \exists z\ \neg Q(y,z)]$
- $[\forall x\ P(x)] \wedge [\forall y\ \neg Q(y,f(y))]$ where f is a Skolem function
- $\{P(x),\ \neg Q(y,f(y))\}$: a set of two unit clauses

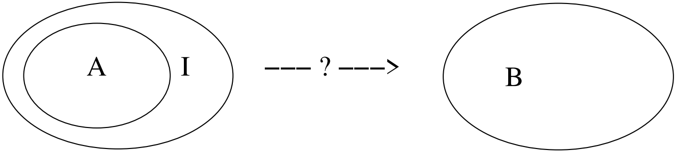
From now on we work with clauses

Why interpolation?

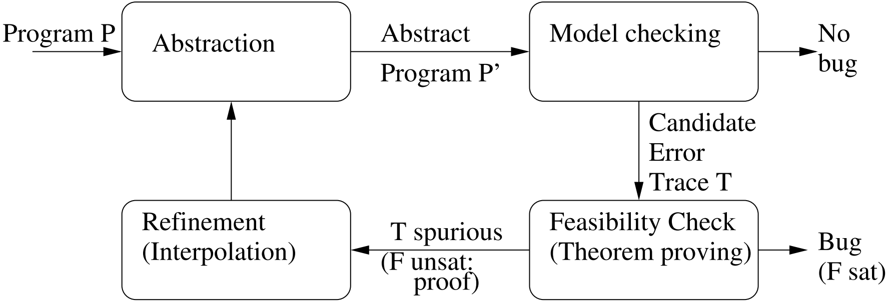
- Interpolant is a formula in between formulæ
- Formulæ represent states that satisfy them
- States of an automaton, of a transition system, of a program
- Interpolant may give information on intermediate states

Image computation in model checking

- Transition system with transition relation
- Forward reachability: computing images
- Backward reachability: computing pre-images
- Interpolant: over-approximation of an image/pre-image
- Interpolation to accelerate convergence towards fixed point



Abstraction refinement in software model checking



$F = A \cup B$; add predicates from interpolant I of (A, B): exclude T

Automated invariant generation

- Loop: *pre* `while C do T` *post*
  - $\forall s.\ pre[s] \supset I(s)$
  - $\forall s, s'.\ I(s) \wedge C[s] \wedge T[s, s'] \supset I(s')$
  - $\forall s.\ I(s) \wedge \neg C[s] \supset post(s)$
- Invariant I made of symbols common to *pre* and *post*; no symbols local to the loop body `T`
- A: k-unfolding of loop; B: post-condition violated
- $A, B \vdash \bot$
- Interpolant of (A, B): candidate invariant

Why interpolation?

- Interpolant is an explanation of $A, B \vdash \bot$
- Conflict-driven reasoning: explaining conflicts,
  where a conflict is an inconsistency between a formula to be satisfied and a candidate model

Example of explanation by interpolation I

$F = \{x \geq 2,\ \neg(x \geq 1) \vee y \geq 1,\ x^2 + y^2 \leq 1 \vee xy > 1\}$

- Caveat: x and y here are constant symbols logically
- $M = \emptyset$
- $M \; = \; x \geq 2$
- $M \; = \; x \geq 2, \;\; x \geq 1$
- $M \; = \; x \geq 2, \;\; x \geq 1, \;\; y \geq 1$
- $M \; = \; x \geq 2, \;\; x \geq 1, \;\; y \geq 1, \;\; x^2 + y^2 \leq 1$
- $M \; = \; x \geq 2, \;\; x \geq 1, \;\; y \geq 1, \;\; x^2 + y^2 \leq 1, \;\; x \leftarrow 2$
- Conflict: no value for y such that $4 + y^2 \leq 1$

Example of explanation by interpolation II

$F = \{x \geq 2,\ \neg(x \geq 1) \vee y \geq 1,\ x^2 + y^2 \leq 1 \vee xy > 1\}$

- $x^2 + y^2 \leq 1$ implies $-1 \leq x \wedge x \leq 1$ which is inconsistent with $x = 2$
- $-1 \leq x \wedge x \leq 1$ is an interpolant because x is shared
- Learn $\neg(x^2 + y^2 \leq 1) \vee x \leq 1$
- Undo $x \leftarrow 2$ and add $x \leq 1$
- $M \ = \ x \geq 2, \quad x \geq 1, \quad y \geq 1, \quad x^2 + y^2 \leq 1, \quad x \leq 1$

Interpolation in propositional logic

Interpolation in propositional logic

Terminology for interpolation: Colors

Uninterpreted symbol:

- A-colored: occurs in A and not in B
- B-colored: occurs in B and not in A
- Transparent: occurs in both

Alternative terminology: A-local, B-local, global

Terminology for interpolation: Colors

Ground term/literal/clause:

- All transparent symbols: transparent
- A-colored (at least one) and transparent symbols: A-colored
- B-colored (at least one) and transparent symbols: B-colored
- Otherwise: AB-mixed

Interpolation system

- A and B sets of clauses
- Given: a refutation of $A \cup B$
- Interpolation system: extracts interpolant of (A, B)
- How? Computing a partial interpolant $PI(C)$ for each clause C in refutation
- Defined in such a way that $PI(\Box)$ is interpolant of (A, B)

Partial interpolant

- Clause C in refutation of $A \cup B$
- $A \wedge B \vdash C$
- $A \wedge B \vdash C \vee C$
- $A \wedge \neg C \vdash \neg B \vee C$
- Interpolant of $A \wedge \neg C$ and $\neg B \vee C$
- Reverse interpolant of $A \wedge \neg C$ and $B \wedge \neg C$
- The signatures of $A \wedge \neg C$ and $B \wedge \neg C$ are not necessarily those of A and B unless C is transparent
- Use projections

Symmetric projections

C: disjunction (conjunction) of literals

- $C|_A$: A-colored and transparent literals
- $C|_B$: B-colored and transparent literals
- $C|_{A,B}$: transparent literals
- $\bot$ ($\top$) if empty

If C has no AB-mixed literals: $C = C|_A \vee C|_B$

Asymmetric projections

C: disjunction (conjunction) of literals

- $C\backslash_B = C|_A \setminus C|_{A,B}$ (A-colored only)
- $C\downarrow_B = C|_B$ (transparent go with B-colored)

If C has no AB-mixed literals: $C = C\backslash_B \vee C\downarrow_B$

Partial interpolant

- Clause C in refutation of $A \cup B$
- Partial interpolant $PI(C)$: interpolant of
  $A \wedge \neg(C|_A)$ and
  $B \wedge \neg(C|_B)$
- If C is $\Box$: $PI(C)$ interpolant of (A, B)
- Requirements:
  - $A \wedge \neg(C|_A) \vdash PI(C)$
  - $B \wedge \neg(C|_B) \wedge PI(C) \vdash \bot$
  - $PI(C)$ transparent
- Or as above with asymmetric projections

Complete interpolation system

An interpolation system is complete for an inference system if

- For all sets of clauses A and B such that $A \cup B$ is unsatisfiable
- For all refutations of $A \cup B$ by the inference system

It generates an interpolant of (A, B)

There may be more than one

Inductive approach to interpolation

- The interpolation system is defined inductively
- By defining the partial interpolant of the consequence given the partial interpolants of the premises for each inference rule
- Prove complete:
  show that its partial interpolants are indeed such

Propositional resolution: example

$$\frac{P \vee \neg Q \vee \neg R, \;\; \neg P \vee O}{O \vee \neg Q \vee \neg R}$$

where O, P, Q, and R are propositional atoms
(aka propositional variables, aka 0-ary predicates)

Propositional resolution

$$\frac{S \cup \{L \vee C,\ \neg L \vee D\}}{S \cup \{L \vee C,\ \neg L \vee D,\ C \vee D\}}$$

- L is an atom
- C and D are disjunctions of literals
- L and $\neg L$ are the literals resolved upon
- $C \vee D$ is called resolvent

First-order ground resolution

$$\frac{P(c, g(a)) \vee \neg R(c, b),\ \ \neg P(c, g(a)) \vee Q(a, g(a))}{\neg R(c, b) \vee Q(a, g(a))}$$

Same as propositional resolution: map ground atoms into propositional atoms

Example in propositional logic

$A = \{a \vee e,\ \neg a \vee b,\ \neg a \vee c\} \quad B = \{\neg b \vee \neg c \vee d,\ \neg d,\ \neg e\}$

1. $a \vee e$ resolves with $\neg e$ to yield a
2. a resolves with $\neg a \vee c$ to yield c
3. a resolves with $\neg a \vee b$ to yield b
4. b resolves with $\neg b \vee \neg c \vee d$ to yield $\neg c \vee d$
5. c resolves with $\neg c \vee d$ to yield d
6. d resolves with $\neg d$ to yield $\Box$

Goal: interpolate this refutation to get an interpolant of (A, B)

Propositional interpolation systems

- Literals in proof are input literals
- Input literals are either A-colored or B-colored or transparent
- No AB-mixed literals

The HKPYM interpolation system

C clause in refutation of $A \cup B$ by propositional resolution:

- $C \in A$: $PI(C) = \bot$
- $C \in B$: $PI(C) = \top$
- $C \vee D$ propositional resolvent of $p_1 : C \vee L$ and $p_2 : D \vee \neg L$:
  - L A-colored: $PI(C \vee D) = PI(p_1) \vee PI(p_2)$
  - L B-colored: $PI(C \vee D) = PI(p_1) \wedge PI(p_2)$
  - L transparent: $PI(C \vee D) = (L \vee PI(p_1)) \wedge (\neg L \vee PI(p_2))$

Symmetric projections

[Huang 1995] [Krajíček 1997] [Pudlàk 1997] [Yorsh, Musuvathi 2005]

Example with HKPYM

$A = \{a \vee e,\ \neg a \vee b,\ \neg a \vee c\} \quad B = \{\neg b \vee \neg c \vee d,\ \neg d,\ \neg e\}$

1. $a \vee e\ [\bot]$ resolves with $\neg e\ [\top]$ to yield $a\ [e]$:
   $PI(a) = (e \vee \bot) \wedge (\neg e \vee \top) = e$
2. $a\ [e]$ resolves with $\neg a \vee c\ [\bot]$ to yield $c\ [e]$: $PI(c) = e \vee \bot = e$
3. $a\ [e]$ resolves with $\neg a \vee b\ [\bot]$ to yield $b\ [e]$: $PI(b) = e \vee \bot = e$
4. $b\ [e]$ resolves with $\neg b \vee \neg c \vee d\ [\top]$ to yield $\neg c \vee d\ [b \vee e]$:
   $PI(\neg c \vee d) = (b \vee e) \wedge (\neg b \vee \top) = b \vee e$
5. $c\ [e]$ resolves with $\neg c \vee d\ [b \vee e]$ to yield $d\ [e \vee (c \wedge b)]$:
   $PI(d) = (c \vee e) \wedge (\neg c \vee b \vee e) = e \vee (c \wedge b)$
6. $d\ [e \vee (c \wedge b)]$ resolves with $\neg d\ [\top]$ to yield $\Box\ [e \vee (c \wedge b)]$:
   $PI(\Box) = (e \vee (c \wedge b)) \wedge \top = e \vee (c \wedge b)$

The MM interpolation system

C clause in refutation of $A \cup B$ by propositional resolution:

- $C \in A$: $PI(C) = C|_{A,B}$
- $C \in B$: $PI(C) = \top$
- $C \vee D$ propositional resolvent of $p_1: C \vee L$ and $p_2: D \vee \neg L$:
  - L A-colored: $PI(C \vee D) = PI(p_1) \vee PI(p_2)$
  - L B-colored or transparent: $PI(C \vee D) = PI(p_1) \wedge PI(p_2)$

Asymmetric projections

[McMillan 2003]

Example with MM

$A = \{a \vee e,\ \neg a \vee b,\ \neg a \vee c\}$ $\quad B = \{\neg b \vee \neg c \vee d,\ \neg d,\ \neg e\}$

1. $a \vee e\ [e]$ resolves with $\neg e\ [\top]$ to yield $a\ [e]$: $PI(a) = e \wedge \top = e$
2. $a\ [e]$ resolves with $\neg a \vee c\ [c]$ to yield $c\ [e \vee c]$: $PI(c) = e \vee c$
3. $a\ [e]$ resolves with $\neg a \vee b\ [b]$ to yield $b\ [e \vee b]$: $PI(b) = e \vee b$
4. $b\ [e \vee b]$ resolves with $\neg b \vee \neg c \vee d\ [\top]$ to yield $\neg c \vee d\ [e \vee b]$: $PI(\neg c \vee d) = (e \vee b) \wedge \top = e \vee b$
5. $c\ [e \vee c]$ resolves with $\neg c \vee d\ [e \vee b]$ to yield $d\ [e \vee (c \wedge b)]$: $PI(d) = (e \vee c) \wedge (e \vee b) = e \vee (c \wedge b)$
6. $d\ [e \vee (c \wedge b)]$ resolves with $\neg d\ [\top]$ to yield $\Box\ [e \vee (c \wedge b)]$: $PI(\Box) = (e \vee (c \wedge b)) \wedge \top = e \vee (c \wedge b)$

Comparison of HKPYM and MM

- In this example the final interpolant is the same, although at each step the HKPYM partial interpolant implies the MM partial interpolant
- In general: MM interpolants imply HKPYM interpolants [D'Silva, Kroening, Purandare, Weissenbacher 2010]
- But there is no general result as to whether weaker or stronger is preferable

Interpolation and equality

Interpolation and equality

Equational reasoning

Replacing equals by equals as in ground rewriting:

$$\frac{S \cup \{f(a,a) \simeq a,\ P(f(a,a)) \vee Q(a)\}}{S \cup \{f(a,a) \simeq a,\ P(a) \vee Q(a)\}}$$

It can be done as $f(a,a) \succ a$: replacing equals by equals needs an ordering in order to know in which direction apply the equality

Monotonicity

- $\succ$ ordering
- $s \succ t$
- Example: $f(a, i(a)) \succ e$
- Monotonicity: $r[s] \succ r[t]$ for all contexts r
  (A context is an expression, here a term or atom, with a hole)
- $f(f(a, i(a)), b) \succ f(e, b)$

Subterm property

- $\succ$ ordering
- $s[t] \succ t$
- Example: $f(a, i(a)) \succ i(a)$

Well-foundedness

- No infinite descending chain $s_0 \succ s_1 \succ \ldots s_i \succ s_{i+1} \succ \ldots$
- Monotonicity and the subterm property suffice to ensure well-foundedness on ground terms

Equality changes the picture for interpolation

- Propositional logic: no AB-mixed literals and colors are stable
- Equality: what if AB-mixed equality $t_a \simeq t_b$ is derived?
  t_a: A-colored ground term; t_b: B-colored ground term
- Rewriting: t_a and t_b in normal form, $t_a \succ t_b$:
  rewrite t_a as t_b; t_b should become transparent
- A-colored/B-colored/transparent cannot change dynamically!

Equality-interpolating theory

- (A, B): there exist transparent ground terms
- If $A \wedge B \models_{\mathcal{T}} t_a \simeq t_b$
  t_a: A-colored ground term and t_b: B-colored ground term
- Then $A \wedge B \models_{\mathcal{T}} t_a \simeq t \wedge t_b \simeq t$ for some transparent ground term t called equality-interpolating term

[Yorsh, Musuvathi 2005]

Separating ordering

Ordering $\succ$ on terms and literals:
separating if $s \succ r$ whenever r is transparent and s is not
([McMillan 2008], [Kovàcs, Voronkov 2009])

Rewriting: t_a and t_b rewritten to t

Separating implies no AB-mixed literals

- Γ: inference system with resolution, superposition, simplification, subsumption ...
- Lemma: If the ordering $\succ$ is separating, ground Γ-refutations contain no AB-mixed literals
  - $s \simeq r$ and $l[s]$ not AB-mixed, and $s \succ r$
  - either s and r same color or r transparent
  - $l[r]$ not AB-mixed

EUF is equality-interpolating

- Theorem: The quantifier-free fragment of the theory of equality is equality-interpolating
  - Γ with $\succ$ separating ordering
  - (A, B): there exist transparent ground terms
  - If $A \wedge B \models t_a \simeq t_b$
  - $A \cup B \cup \{t_a \not\simeq t_b\} \vdash_\Gamma \bot$ by refutational completeness of Γ
  - No AB-mixed equalities as $\succ$ is separating
  - Valley proof $t_a \xrightarrow{*} t \xleftarrow{*} t_b$ contains at least a transparent term
  - t must be transparent

Interpolation system GΓI

C clause in ground Γ-refutation of $A \cup B$:

- Base cases and resolution: same as in HKPYM
- $c\colon C \vee l[r] \vee D$ generated from $p_1\colon C \vee s \simeq r$ and $p_2\colon l[s] \vee D$
  - $s \simeq r$ A-colored: $PI(c) = PI(p_1) \vee PI(p_2)$
  - $s \simeq r$ B-colored: $PI(c) = PI(p_1) \wedge PI(p_2)$
  - $s \simeq r$ transparent: $PI(c) = (s \simeq r \vee PI(p_1)) \wedge (s \not\simeq r \vee PI(p_2))$

Example

$A = \{P(c),\ \neg P(e)\} \qquad B = \{c \simeq e\} \qquad c \succ e$

P is A-colored, c and e are transparent

1. $c \simeq e$ $[\top]$ simplifies $P(c)$ $[\bot]$ into $P(e)$ $[c \not\simeq e]$
   $PI(P(e)) = (c \simeq e \vee \top) \wedge (c \not\simeq e \vee \bot) = c \not\simeq e$
2. $\neg P(e)$ $[\bot]$ resolves with $P(e)$ $[c \not\simeq e]$ to yield $\Box$ $[c \not\simeq e]$
   $PI(\Box) = \bot \vee c \not\simeq e = c \not\simeq e$

Example

$$A = \{Q(f(a)),\ f(a) \simeq c\} \qquad B = \{\neg Q(f(b)),\ f(b) \simeq c\}$$

a is A-colored, b is B-colored, all other symbols are transparent

1. $f(a) \simeq c$ $[\bot]$ simplifies $Q(f(a))$ $[\bot]$ into $Q(c)$ $[\bot]$
   where $f(a) \succ c$ in any separating ordering
   $PI(Q(c)) = \bot \vee \bot = \bot$
2. $f(b) \simeq c$ $[\top]$ simplifies $\neg Q(f(b))$ $[\top]$ into $\neg Q(c)$ $[\top]$
   where $f(b) \succ c$ in any separating ordering
   $PI(\neg Q(c)) = \top \wedge \top = \top$
3. $Q(c)$ $[\bot]$ resolves with $\neg Q(c)$ $[\top]$ to yield $\Box$ $[Q(c)]$
   $PI(\Box) = (Q(c) \vee \bot) \wedge (\neg Q(c) \vee \top) = Q(c)$

Completeness

- Theorem: If the ordering is separating, $G\Gamma I$ is a complete interpolation system for ground Γ-refutations
- The proof shows that the partial interpolants built by $G\Gamma I$ satisfy the requirements for partial interpolants.

References

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- Maria Paola Bonacina and Moa Johansson. Towards interpolation in an SMT solver with integrated superposition. 9th SMT Workshop, Snowbird, Utah, USA, July 2011; TR UCB/EECS-2011-80, 9-18, 2011
- Maria Paola Bonacina and Moa Johansson. On interpolation in decision procedures. In Proc. of the 20th TABLEAUX Conference, Bern, Switzerland, July 2011; Springer, LNAI 6793, 1–16, 2011

Discussion

- Generality: interpolants for more logics, theories, inference systems
- Quality: better interpolants; stronger? weaker? shorter?
- Non-ground proofs theories?
  Two-stage approach:
  Maria Paola Bonacina and Moa Johansson. On interpolation in automated theorem proving. Journal of Automated Reasoning, 54(1):69-97, 2015