On the notion of complexity of search in theorem proving

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Abstract

We are concerned with the problem of defining notions of complexity that are suitable for the complexity of search in the infinite search spaces of theorem proving. In this work, our primary motivation is to capture the essence of *contraction* inferences, such as subsumption and simplification, on the complexity of search. Such inference rules contract the search space and, from practical experience, usually produce simpler proofs. However, there has not been any formal analysis which justifies, mathematically, why contraction is effective in controlling the growth of the search space.

In this paper we try to answer this question by providing a general notion of complexity capable of describing and analyzing the behaviour of the search space during a derivation. We begin by presenting a model for representing search. We point out that, unlike the conventional expansion inference rules such as resolution which merely visit the search space, contraction inferences visit and *modify* the search space at the same time. Our model captures this dynamic behaviour by introducing a notion of *marking* into the search graph, which allows us to describe the search plan of a strategy as well as its inference mechanism.

Based on the concept of marked search graph, we introduce a notion of complexity measure for the dynamic search spaces of strategies. Unlike the classical complexity measures which work with finite objects, our method deals with infinite search graphs and derivations which may not halt. Our notion of complexity analysis, based on well-founded orderings instead of natural numbers, captures both the present and the future of a derivation. We define notions of *ancestor-graph* and *dynamic distance* to replace the conventional notions of path and path-lengh. This allows us to "finitize" the future, the portion of the infinite search graph not yet discovered, into a sequence of finite search graphs within bounded distances.

We then apply the above framework to compare contraction-based strategies of different contraction power. We show how they affect the evolution of the respective search spaces during the derivation, and we analyze the variations of the complexity measure.

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