Two-stage Interpolation Systems*

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The application of interpolation to *invariant generation* has led to the study of interpolation in first-order logic with equality, for proofs produced by superposition-based theorem provers, or SMT-solvers with instantiation procedures. In this context, the proofs to be interpolated in general are *not ground*, interpolants naturally contain *quantifiers*, and the capability of generating interpolants with quantifiers is an advantage, because invariants often need them.

Given an interpolation problem (A, B), where $A, B \vdash \perp$, the formulæ A and B are reduced to clausal form and the resulting set of clauses is given to a theorem prover that generates a refutation. An *interpolation system* takes the refutation and extracts a (reverse) interpolant: it associates a *partial interpolant* to every clause, in such a way that the partial interpolant of the empty clause is an interpolant of (A, B). The interpolation system is defined by defining how it builds the partial interpolant of the conclusion from those of the premises, for each inference rule. An interpolation system is *complete* for an inference system, if for all its refutations it extracts an interpolant.

The state of the art for interpolation systems is represented by the *color-based approach*: the interpolation system tracks non-shared symbols, called *colored* (e.g., A-colored and B-colored), to exclude them from the interpolant, and determine which literals descend from A or B, in order to ensure that the interpolant is entailed by A and inconsistent with B. We show that the color-based style cannot handle non-ground proofs with substitutions. Also it cannot handle *model-based theory combination* of non-convex theories.

We present a *two-stage approach*, which separates the issues of ensuring that the reverse interpolant is entailed by A and inconsistent with B, and that it contains only shared symbols. The first stage addresses the first issue: the interpolation system works by tracking literals to compute a *provisional interpolant*, which is entailed by A and inconsistent with B, but may contain colored symbols. The second stage addresses the second issue: a mechanism called *lifting* replaces terms with colored symbols in the provisional interpolant by quantified variables. We prove that the lifting of a provisional interpolant is an interpolant, so that this two-stage mechanism forms a *complete interpolation system*.

We obtain complete interpolation systems for non-ground proofs by superposition, and by DPLL($\Gamma + \mathcal{T}$), which integrates superposition (Γ) into DPLL($\mathcal{T}$) with model-based theory combination, regardless of whether theories are convex.

* Abstract of: Maria Paola Bonacina and Moa Johansson, *On interpolation in automated theorem proving*, where references can be found, available at `http://profs.sci.univr.it/~bonacina/interpolation.html`, and submitted for publication.