EXERCISE 1

EXERCISE 1.

CONSIDERING THE FOLLOWING SENTENCES :

1. *JULIET LOVE ROMEO, HER FATHER AND HER MOTHER.*
2. *ROMEO LOVES JULIET AND ALSO EVERYBODY LOVED BY JULIET*.
3. *THE FATHER OF ROMEO LOVES ROMEO, BUT DOES NOT LOVE THE FATHER OF JULIET.*
4. *THE FATHER OF ROMEO LOVES EVERYBODY LOVED BY ROMEO*.

(i) INDICATE THE LEXICON OF SENTENCES 1-4, NAMELY, THE NAMES, THE FUNCTION AND THE PREDICATE OCCURRING IN THEM. DEFINE A FORMAL LANGUAGE "L" WITH COSTANTS, FUNCTION SYMBOLS CORRESPONDING TO THIS LEXICON AND FORMALIZE 1-4.

(ii) INDICATE THE CLOSED ATOMIC SENTENCES CONTAINED IN 1-4, i.e SENTENCES WHICH DO NOT CONTAIN QUANTIFIERS OR CONNECTIVES.

(iii) FORMALIZE 1-4.

(iv) DO YOU THINK THAT 1-4 ENTAIL A CONTRADDICTION? IF SO, CAN YOU INDICATE A CLOSED ATOMIC SENTENCE SENTENCE SUCH THAT BOTH THIS SENTENCE AND ITS NEGATION FOLLOW FROM 1-4?

(i) **LEXICON**

Names { Romeo, Juliet }
Predicate { _Love_ }
Functions { father of _, mother of_ }

FORMAL LANGUAGE

L = **(** *{ R, J } ; { L (_ , _)}; { f(_), m(_) }*)
where *Romeo = R, Juliet = J; Love = L ; father of = f, mother of = m*

(ii) – (iii) **CLOSED ATOMIC FORMULAS and FORMALIZATION**

1.*JULIET LOVES ROMEO, HER FATHER AND HER MOTHER.*

Closed atomic formulas:

Juliet loves Romeo = L(J,R)
Juliet loves her father = L(J, f(J))
Juliet loves her mother = L(J, m(J))

Formalization:

$$L(\mathrm{J}, \mathrm{R}) \wedge L(\mathrm{J}, f(\mathrm{J})) \wedge L(\mathrm{J}, m(\mathrm{J}))$$

2. *ROMEO LOVES JULIET AND ALSO EVERYBODY LOVED BY JULIET.*

Closed atomic formulas:

Romeo loves Juliet = $L(\mathrm{R},\mathrm{J})$

Formalization:

$$L(\mathrm{R}, \mathrm{J}) \wedge (\forall x.\ L(\mathrm{J}, x) \rightarrow L(\mathrm{R}, x))$$

3. *THE FATHER OF ROMEO LOVES ROMEO BUT DOES NOT LOVE THE FATHER OF JULIET.*

Closed atomic formulas:

The father of Romeo loves Romeo = $L(\, f(\mathrm{R}), \mathrm{R})$
The father of Romeo loves the father of Juliet = $L(\, f(\mathrm{R}), f(\mathrm{J}))$

Formalization:

$$L(\, f(\mathrm{R}), \mathrm{R}) \wedge \neg L(\, f(\mathrm{R}), f(\mathrm{J}))$$

4. *THE FATHER OF ROMEO LOVES EVERYBODY LOVED BY ROMEO.*

Closed atomic formulas: none.

Formalization:

$$(\forall x.\ L(\mathrm{R}, x) \rightarrow L(\, f(\mathrm{R}), x))$$

(iii) **CONTRADICTION**

The contradiction is in the fourth sentence; the right sequence is:

1. *Juliet loves Romeo, her father and her mother.*
2. *Romeo loves Juliet and also everybody loved by Romeo.*
3. *The father of Romeo loves Romeo.*
4. *The father of Romeo loves everybody loved by Romeo, but does not love the father of Juliet.*

The fourth sentence expresses a universal quantification with an **exception**; it can be formalized as follows:

$$\forall x.\ (L(\mathrm{R}, x) \wedge \neg\ x = f(\mathrm{J})) \rightarrow L(f(\mathrm{R}), x)$$

EXERCISE 2.

FORMALIZE THE FOLLOWING SENTENCES BY ABRAHAM LINCOLN:

" YOU CAN FOOL ALL THE PEOPLE SOME OF THE TIME, AND SOME OF THE PEOPLE ALL THE TIME, BUT YOU CANNOT FOOL ALL THE PEOPLE ALL THE TIME."

There is a problem here: here the pronoun *"you"* stands for *a generic name*, i.e., an expression like *"John Smith"* in English or *"Mario Rossi*" in Italian. A generic name is a device for expressing generality without using a universal quantification. Therefore the best formalization is one that retains a generic name – unlike the suggestion in the paper.

Lexicon:

Predicates: $\{$ *_ is a person* $= P(\_)$, *_ is a period ot time* $= T(\_)$ *(unary predicates);*
*_ fools _ at time_* $= F(\_, \_, \_)$ *(ternary predicates)* $\}$.

Names: $= \{a\}$

Formalization:

$$(\exists\, t.\ T(x) \wedge (\forall y.\ P(y) \to (F(a, y, t)\ )\ ) \ \wedge\ (\exists\, y.\ P(y) \wedge (\forall t.\ T(t) \to (F(a, y, t)\ )\ ) \ \wedge$$
$$\neg\ (\forall y.\ P(y) \to (\forall t. T(t) \to F\ (a, y, t\ )\ )\ ).$$

EXERCISE 3.

FORMALIZE THE FOLLOWING SENTENCES IN PROPOSITIONAL LOGIC. INDICATE ALWAYS THE KEY, E.G. J = "JOHN COMES".

(i) JOHN IS NOT ONLY STUPID BUT NASTY TOO.

A = John is not only stupid but nasty too
S = John is stupid
N = John is nasty

Formalization:

$$A := S \wedge N;$$

(ii) IF YOU STAY WITH ME IF I WON'T DRINK ANYMORE, THEN I WILL NOT DRINK ANYMORE.

B = If you stay with me if i won't drink anymore, then i will not drink anymore
D = I drink
S = you stay with me

Formalization:

$$B := (\neg D \to S\ ) \to \neg D$$

(iii) CHARLES COMES IF ELSA AND THE OTHER WAY AROUND.

D = Charles comes if Elsa and the other way around
Charles comes = C
Elsa comes = E

Formalization:

$$D := (E \rightarrow C) \land (E \rightarrow C);$$

(iv) JOHN IS GOING TO SCHOOL AND IF IT IS RAINING SO IS PETER

C = John is going to school and if it is raining so is Peter
J = John is going to school
R = it is raining
P = Peter is going to school

Formalization:

$$C := J \land (R \rightarrow P).$$