Computational logic: exercise sheet 6

19/11/2008

EXERCISE 1 Consider the fragment of propositional intuitionistic sequent calculus LJ with the logical constant $\top$ (truth) and the binary connectives $\wedge$ (conjunction) and $\to$ (implication). Construct $\mathit{Seq}(\mathrm{LJ})$ in such a way that it has formulae as objects and sequents $A \Longrightarrow B$ as morphism $\mathit{Hom}_{\mathit{Seq}(\mathrm{LJ})}(A, B)$. Show that $\mathit{Seq}(\mathrm{LJ})$ is a category.

EXERCISE 2 Show that the category $\mathit{Seq}(\mathrm{LJ})$ from exercise 1 is cartesian.

EXERCISE 3 Consider the simply typed λ-calculus associated with the fragment of propositional intuitionistic natural deduction NJ with the logical constant $\top$ (truth) and the binary connectives $\wedge$ (conjunction) and $\to$ (implication). Construct $\mathcal{C}(\lambda)$ in such a way that it has types A, B, C, ... as objects and the class of λ-terms $x\colon A \vdash t(x)\colon B$ where

$$x\colon A \vdash t(x)\colon B \quad \simeq \quad y\colon A \vdash t(y)\colon B \qquad \text{if} \quad t(y)[x/y] = t(x)$$

as morphisms $\mathit{Hom}_{\mathcal{C}(\lambda)}(A, B)$. Show that $\mathcal{C}(\lambda)$ is a category.

EXERCISE 4 Show that the category $\mathcal{C}(\lambda)$ from exercise 3 is cartesian.

EXERCISE 5 Show that, in any cartesian category $\mathbb{C}$, the object $A \times 1$ is isomorphic to the object A, i.e. that there exist morphisms $f\colon A \times 1 \longrightarrow A$ and $g\colon A \longrightarrow A \times 1$ of $\mathbb{C}$ such that $g \circ f = \mathrm{id}_{A\times 1}\colon A \times 1 \longrightarrow A \times 1$ and $f \circ g = \mathrm{id}_A\colon A \longrightarrow A$.

EXERCISE 6 Does *Pset*, the category with sets as objects and partial functions as morphisms, have a terminal object and binary products?