1 Computational Logic 2008 - Dr G.Bellin

Coursework 5

1. Let $\perp$ be a propositional constant interpreted as a logically absurd sentence. The principle "*double negation is assertion*" is formalized by the rule of inference

$$\dfrac{\begin{array}{c}(1)\\ [\neg A]\\ \vdots\\ \perp\end{array}}{A}\ \perp\text{-C}$$

with the deduction rule

$$\dfrac{\neg A, \Gamma \vdash \perp}{\Gamma \vdash A}\ \perp\text{-C}$$

Let $\mathbf{NK}^{\rightarrow\wedge\perp}$ be the Natural Deduction system having the $\perp$-C rule in addition to the introduction and elimination rules for implication and conjunction.

(a) **Write a derivation of** $((A \rightarrow B) \rightarrow A) \rightarrow A$ in $\mathbf{NK}^{\rightarrow\wedge\perp}$.

(b) **Show**: *every derivation d in $\mathbf{NK}^{\rightarrow\wedge\perp}$ can be transformed into a derivation d' where all conclusions A of a $\perp$-C rule are atomic.*

2. The *leftmost reduction strategy* is the rule that prescribes to reduce always the leftmost redex, i.e., the redex $(\lambda x.u)t$ whose indicated parentheses occurs leftmost in the term.
The set Λ_I of the λ_I terms is defined as follows:

- every variable x belongs to Λ_I;
- if $u, t \in \Lambda_I$, then $(u)i \in \Lambda_I$;
- if $u \in \Lambda_I$ and x actually occurs in u, then $\lambda x.u \in \Lambda_I$.

Show that every λ_I-term t is normalizable by leftmost reduction if and only if t is strongly normalizable.

Hint: Use induction on the lexicographical ordering of the pairs $(l(v), c(v))$, where $l(v)$ is the length of the leftmost reduction of v and $c(v)$ is the number of symbols of v. Distinguish the cases

- $v = \lambda x_1 \ldots \lambda x_m.((\ldots(x)t_1 \ldots t_{n-1})t_n$;
- $v = \lambda x_1 \ldots \lambda x_m.((\ldots((\lambda x.u)t)t_1 \ldots t_{n-1})t_n$, and notice that here x occurs in u, hence if $u[t/x]$ is strongly normalizable, then so is t.