Homework Assignment 7

Due **Friday 2-04-04**

(Double Assignment)

Part I

1. Define a Non-Deterministic Finite State Automaton N on the alphabet $A = \{0, 1\}$ which accepts exactly the language $\mathcal{L} = \{u \in A^* | u = w0xy\}$, i.e., precisely the words on A where the third letter from the end is 0. Define a Deterministic Finite State Automaton M equivalent to N and find the minimal deterministic automaton equivalent to N.

(*5 points*)

2. Let $f : N \times N \to N$ be a total function. Given an Abacus Machine that computes f, define an Abacus Machine that computes the total function $h(m, n) = \mu y < n.f(m, y) = 0$, i.e., the function which maps the arguments m, n to the least $y < n$ such that $f(m, y) = 0$, if such a y exists, and returns n otherwise.

(*5 points*)

3. Let $A(x, y)$ be a binary predicate, and consider the following formulas

$B = \forall x.\neg A(x, x)$

$C = \forall x.\forall y.\forall z.(A(x, y) \wedge A(y, z) \to A(x, z))$

$D = \forall x.\exists y.A(x, y)$

Thus $(B \wedge C) \wedge D$ says that the universe of discourse is a strict ordering without maximal points. Show that $(B \wedge C) \wedge D$ has an infinite model but no finite model.

(*5 points*)

Part II

4. Apply the "semantic tableaux procedure" to the following formulas. If the formula is not valid, construct an interpretation that falsifies it.

(i) $((A \to B) \to A) \to A$.

(*2.5 points*)

(ii) $(\forall x.\exists y.A(x, y)) \to (\exists y.\forall x.A(x, y))$

(*2.5 points*)

(iii) $(\forall x.A(x)) \to (\forall y.B(y)) \to \exists x.\forall y.(A(x) \to B(y))$

(*2.5 points*)

(5) Show that there are infinitely many prime numbers of the form $6n - 1$.

(*7.5 points*)