Homework Assignment 1

Due **Friday 23-01-04**

These exercises are related to Chapter 1 of the book by Boolos, Burgess and Jeffrey *Computability and Logic*, Cambridge UP, (Fourth Edition).

1. Let ϕ be a partial function from the set $\mathbf{N}$ of natural numbers *onto* the set A. Show that either there is a bijection $f : n \to A$ where $n \in \mathbf{N}$ or there is a bijection $f : \mathbf{N} \to A$. In other words, either A is finite or A has the same cardinality as $\mathbf{N}$.

Hint: First show that there is a *total* function $g : \mathbf{N} \to A$ which is onto (surjective), then define a function f which is 1-1 (injective) and onto (surjective).

2. Prove that for all k the sum of the first k positive integers $1 + 2 + \ldots + k = \dfrac{k(k+1)}{2}$.

Hint: One way to prove this is by induction on k: show that the equation holds for $k = 1$; then assuming that it holds for $k = n$, prove that it holds also for $k = n + 1$.

3. Let $\mathbf{N}$ be the set of the natural numbers (including 0). Show that the function $J : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ given by

$$J(m, n) = \frac{(m+n)(m+n+1)}{2} + m$$

is a bijection.

Hint: See Example 1.2 pages 7-9 of the book, which gives a similar function J in the case of the natural numbers without 0.