On logics of abstract illocutionary forces

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1. Semantics and Pragmatics of abstract illocutionary acts.

• *Propositions* can be *true* or *false* (as in Frege).

• *Illocutionary acts* are events that can be *justified* or *unjustified*; In a given social context they are *felicitous* or *infelicitous* and have *perlocutionary effects*. They must have a *propositional content* (as in Austin).

Examples. *Question - answer*:

- *Is Furio there with you?*

- *No, he is in Udine where he is getting married.*

Suppose in the situation where the question is asked, the propositional content of the question is false, and the propositional content of the answer is true.

For the question to be *justified*, it must be possible to find out where Furio is. For the answer to be *justified*, she who answers must have *conclusive evidence* of the truth of the propositional content.

The question may be *infelicitous*, if asked, e.g., to Furio's former partner. Understanding felicity and perlocutionary effects requires a *theory of actions*.

2. Abstract illocutionary acts

Examples. Question:

- *Is it permissible to give shelter to illegal immigrants in distress?*

The propositional content can be spelled out as a 1st order relation *"x gives shelter to y and etc..."*

The question is justified only if a positive or negative answer exists in a given normative system (although it may not be known by anybody we may ask).

• We are concerned only with *the justification value* of sentences resulting from *impersonal illocutionary operators* ($\vdash, \mathcal{O}$-) applied to a proposition.

1. $\vdash$ *there are infinitely many prime numbers.*

Assertion 1 is justified by a proof of Euclid's theorem.

2. $\mathcal{O}$-(*Silvio gives shelter to Muhammar*), because $\vdash$ (*Muhammar is an illegal immigrant in distress*).

Obligation 2 is justified by conclusive evidence of the status of Muhammar and conclusive evidence of the conditional obligation within a normative system.

3. Building a language from expressions of illocutionary acts.

Obligation 2, formally:
$(\vdash I(m) \cap \vdash D(m)) \supset (\mathcal{O}\!\!-\text{-}S(s,m))$.

But wait! *Truth functional connectives cannot be applied to expressions of illocutionary acts.* (Frege)

Use **non truth-functional** *connectives!* (Dalla Pozza). Here the *justification conditions* of complex expressions depend from the justification conditions of the components in a compositional way.

Examples (i) The *generalized Frege-Geach problem* (Dalla Pozza).

p = any king of France is bold.
q = there is exactly one king of France.

- *The king of France is bold.* $(\vdash q) \cap (\vdash p)$.
- *Is the king of France bold?* $(\vdash q) \cap (p?)$.
- *Shave the king of France!* $(\vdash q) \cap (\mathcal{O}\!\!-p)$.

(ii) Assertions, obligations and causal consequence relation (Ranalter 2008).

$$\frac{\vdash p, \vdash q, (\vdash q_i \multimap \vdash q_j), \Rightarrow \vdash r}{\vdash p, \mathcal{O}\!\!-q, (\vdash q_i \multimap \vdash q_j), \Rightarrow \mathcal{O}\!\!-r} \tag{1}$$

A rule for conditional obligations, with *causal implication* $\vdash q_i \multimap \vdash q_j$. Ranalter gives a category-theoretic model for such a logic.

4. The language $\mathcal{L}^{AH}$, assertions and hypotheses

$$A, B \quad := \quad \vdash p \;\mid\; \top \;\mid\; \sim A \;\mid\; A \supset B \;\mid\; A \cap B \;\mid\; -C$$

$$C, D \quad := \quad \mathcal{H} p \;\mid\; \bot \;\mid\; \frown C \;\mid\; C \setminus D \;\mid\; C \curlyvee D \;\mid\; -A$$

$\vdash p$: "$\vdash$" illocutionary force of *assertion*,
"p" an *atomic* proposition;

$\mathcal{H}p$: "$\mathcal{H}$" illoc. force of *hypothesis*, "p" atomic;

$A \supset B, A \cap B, \top$: complex *assertive types*,
implication, conjunction, validity;

$C \setminus D, C \curlyvee D, \bot$: complex *hypothetical types*,
subtraction, disjunction, invalidity;

$C \setminus D$: *perhaps C and not D.*

Four negations:

- *definitely not*: $\sim A =_{df} A \supset \mathbf{inv}$,
inv an invalid assertive expression;

- *perhaps not*: $\frown C =_{df} \mathbf{val} \setminus C$,
val a valid conjectural expression;

- $-A$, $-C$ **dualities**: $- - A \equiv A$, $- - C \equiv C$;

$- \sim A \equiv \frown -A, \qquad - \frown C \equiv \sim -C$;

$-(A \supset B) \equiv (-B \setminus -A) \qquad -(C \setminus D) \equiv (-D \supset C)$;

$-(A \cap B) \equiv (-A \curlyvee -B) \qquad -(C \curlyvee D) \equiv (-C \cap -D)$.

5. Justification conditions for expressions of $\mathcal{L}^{AH}$

$$A, B \quad := \quad \vdash p \;\mid\; \top \;\mid\; \sim A \;\mid\; A \supset B \;\mid\; A \cap B \;\mid\; -C$$

$$C, D \quad := \quad \mathcal{H} p \;\mid\; \bot \;\mid\; \frown C \;\mid\; C \setminus D \;\mid\; C \curlyvee D \;\mid\; -A$$

• $\vdash p$ is justified by *conclusive evidence* that p is true;
• $\mathcal{H} p$ is justified by a *scintilla of evidence* that p is true;
• $A \supset B$ is justified by a *method* transforming a justification of A into a justification of B
• $C \setminus D$ is justified by a *scintilla of evidence* that there is a justification of C and no justification of D; etc.

Four negations:

• *definitely not*: $\sim A =_{df} A \supset \mathbf{inv}$ is justified by a method to transform a justification of A into an absurdity;
• *perhaps not*: $\frown C =_{df} \mathbf{val} \setminus C$ is justified by a scintilla of evidence that C is unjustified;
• $-A$ is justified by a scintilla of evidence that A is unjustified;
• $-C$ is justified by conclusive evidence that C is unjustified.

Pragmatic validity: An expression X is *pragmatically valid* if for any assignment of evidence to its elementary expressions, X is justified.

Pragmatically valid: $- - A \equiv A$, $- - C \equiv C$; etc.

• The interpretation of *assertive expressions* is *Kolmogorov-Heyting semantics of intuitionistic logic*;
• that of *hypothetical expressions* is a (*constructive*) *pragmatic interpretation* of co-intuitionistic logic.
• *Polarization*: only dualities connect the two sides.

6. Semantic projection of $\mathcal{L}^{AH}$ into classical S4

$$A, B \quad := \quad \vdash p \;\mid\; \top \;\mid\; \sim A \;\mid\; A \supset B \;\mid\; A \cap B \;\mid\; -C$$

$$C, D \quad := \quad \mathcal{H} p \;\mid\; \bot \;\mid\; \frown C \;\mid\; C \setminus D \;\mid\; C \curlyvee D \;\mid\; -A$$

Tarski-Gödel-Kripke translation in S4.

$$\begin{array}{cc}
(\top)^M =_{df} \textbf{true} & (\bot)^M =_{df} \textbf{false} \\
(\vdash p)^M =_{df} \Box p & (\mathcal{H} p)^M =_{df} \Diamond p \\
(A \supset B)^M =_{df} \Box(A^M \rightarrow B^M) & (C \setminus D)^M =_{df} \Diamond(C^M \wedge \neg D^M) \\
(A_1 \cap A_2)^M =_{df} A_1^M \wedge A_2^M & (C_1 \curlyvee C_2)^M =_{df} C_1^M \vee C_2^M \\
(\sim A)^M =_{df} \Box \neg A^M & (\frown C)^M =_{df} \Diamond \neg C^M \\
(-C)^M =_{df} \neg C^M & (-A)^M =_{df} \neg A^M
\end{array}$$

- *Epistemic interpretation* of **S4**: models $(W, R, \Vdash)$ with R a pre-order representing the evolution of states of knowledge $w_i \in W$.
- **AH**: the set of all expressions in $\mathcal{L}^{AH}$ that are valid in the **S4** modal translation.
- **AH-G3**: a sequent calculus for **AH** *sound and complete for* **S4** where sequents are of the form

$$\Theta \;;\; \Rightarrow \; A \;;\; \Upsilon$$

$$\Theta \;;\; C \; \Rightarrow \;\;;\; \Upsilon$$

- Θ is a sequence of *assertive* formulas $A_1, \ldots, A_m$;
- Υ a sequence of *hypothetical* formulas $C_1, \ldots, C_n$.

7. Sequent Calculus AH-G3

identity rules

logical axiom:

$$\Theta \;;\; C \;\Rightarrow\; ;\; C, \Upsilon$$

logical axiom:

$$A, \Theta \;;\; \Rightarrow\; A \;;\; \Upsilon$$

cut$_A$:

$$\frac{\Theta \;;\; \Rightarrow\; A \;;\Upsilon \qquad A, \Theta' \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon'}{\Theta, \Theta' \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, \Upsilon'}$$

cut$_H$:

$$\frac{\Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\Upsilon, C \qquad \Theta' \;;\; C \;\Rightarrow\; ;\; \Upsilon'}{\Theta, \Theta' \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, \Upsilon'}$$

logical rules, implication, subtraction, duality

right $\supset$:

$$\frac{\Theta, A \;;\; \Rightarrow\; B \;;\; \Upsilon}{\Theta \;;\; \Rightarrow\; A \supset B \;;\; \Upsilon}$$

left $\setminus$:

$$\frac{\Theta \;;\; C \;\Rightarrow\; ;\; \Upsilon, D}{\Theta \;;\; C \setminus D \;\Rightarrow\; ;\; \Upsilon}$$

left $\supset$:

$$\frac{A \supset B, \Theta \;;\; \Rightarrow\; A \;;\; \Upsilon \qquad B, \Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon}{A \supset B,\; \Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon}$$

right $\setminus$:

$$\frac{\Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, C \qquad \Theta \;;\; D \;\Rightarrow\; ;\; \Upsilon, C \setminus D}{\Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, C \setminus D}$$

right assertive $-$:

$$\frac{A, \Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon}{\Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, -A}$$

left assertive $-$:

$$\frac{\Theta \;;\; \Rightarrow\; A \;;\; \Upsilon}{\Theta \;;\; -A \;\Rightarrow\; ;\; \Upsilon}$$

right hypothetical $-$:

$$\frac{\Theta \;;\; C \;\Rightarrow\; ;\; \Upsilon}{\Theta \;;\; \Rightarrow\; -C \;;\; \Upsilon}$$

left hypothetical $-$:

$$\frac{\Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon, C}{-C, \Theta \;;\; \epsilon \;\Rightarrow\; \epsilon' \;;\; \Upsilon}$$

8. AH-G1 Sequent calculus

- **G3** sequent calculi are useful for completeness results.
- **G1** calculi with explicit structural rules are 'more basic':

identity rules

logical axiom:

$$A\ ;\ \Rightarrow\ A\ ;$$

logical axiom:

$$;\ C\ \Rightarrow\ ;\ C$$

exchange left:

$$\frac{\Theta, A, B, \Theta'\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}{\Theta, B, A, \Theta'\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}$$

exchange right:

$$\frac{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon, C, D, \Upsilon'}{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon, D, C, \Upsilon'}$$

weakening left:

$$\frac{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}{A, \Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}$$

weakening right:

$$\frac{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon, C}$$

contraction left:

$$\frac{A, A, \Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon}{A, \Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ;\ \Upsilon}$$

contraction right:

$$\frac{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon, C, C}{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ;\ \Upsilon, C}$$

logical rules, implication, subtraction

right $\supset$:

$$\frac{\Theta, A\ ;\ \Rightarrow\ B\ ;\ \Upsilon}{\Theta\ ;\ \Rightarrow\ A \supset B\ ;\ \Upsilon}$$

left $\smallsetminus$:

$$\frac{\Theta\ ;\ C\ \Rightarrow\ ;\ \Upsilon, D}{\Theta\ ;\ C \smallsetminus D\ \Rightarrow\ ;\ \Upsilon}$$

left $\supset$:

$$\frac{\Theta\ ;\ \Rightarrow\ A\ ;\ \Upsilon \qquad B, \Theta'\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ;\ \Upsilon'}{A \supset B, \Theta, \Theta'\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ;\ \Upsilon, \Upsilon'}$$

right $\smallsetminus$:

$$\frac{\Theta\ ;\ \epsilon\ \Rightarrow\ \epsilon'\ ;\ \Upsilon, C \qquad \Theta'\ ;\ D\ \Rightarrow\ ;\ \Upsilon'}{\Theta, \Theta';\ \epsilon\ \Rightarrow\ \epsilon'\ ; \Upsilon, \Upsilon', C \smallsetminus D}$$

9. A question by Grigori Mints

What's the meaning of the sequent symbols:

$$\_\_, \ldots, \_\_; \_\_ \Rightarrow \_\_; \_\_, \ldots, \_\_? \tag{2}$$

In Ranalter's conditional obligation rule (1) *causal implication* requires the *relevant consequence relation*. Take *linear consequence relation* as basic

$$(\_\_ \otimes \ldots \otimes \_\_) \multimap (\_\_ \wp \ldots \wp \_\_) \tag{3}$$

(possibly *non-commutative*, Sambin: *non-associative*) and modify the consequence relation depeding on the illocutionary force. Here:

$$\begin{array}{ll} (\mathbf{!}A_1 \otimes \ldots \otimes \mathbf{!}A_m) \multimap (A \wp (\mathbf{?}C_1 \wp \ldots \wp \mathbf{?}C_n)) \\ \equiv \quad (A_1 \cap \ldots \cap A_m) \multimap (A \wp (C_1 \curlyvee \ldots \curlyvee C_n)) \\ \equiv \quad (A_1 \cap \ldots \cap A_m) \supset A, \quad \text{if } n = 0. \end{array}$$

$$\begin{array}{ll} (\mathbf{!}A_1 \otimes \ldots \otimes \mathbf{!}A_m) \otimes C \multimap (\mathbf{?}C_1 \wp \ldots \wp \mathbf{?}C_n) \\ \equiv \quad (A_1 \cap \ldots \cap A_m) \otimes C \multimap (C_1 \curlyvee \ldots \curlyvee C_n) \\ \equiv \quad \text{if } m = 0 \; (C \searrow (C_1 \curlyvee \ldots \curlyvee C_n)) \multimap \bot \\ \equiv \quad -(C \searrow (C_1 \curlyvee \ldots \curlyvee C_n)). \end{array}$$

The valid sequent $\Rightarrow A \; ; \; -A$ is *not* equivalent to any assertive or hypothetical formula.

10. The domain of pragmatics.

Meaning as use: Principle of *harmony* between *intro* ed *elim* rules in Natural Deduction. An *intro* rule gives an *operational meaning* of a connective.

Example: $$\cap\ \mathrm{I}\,\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \cap B}$$

The elimination rules are *justified* by the meaning given by the introduction rule.

$$\cap_1\ \mathsf{E}\,\frac{\Gamma \vdash A \cap B}{\Gamma \vdash A} \qquad \cap_2\ \mathsf{E}\,\frac{\Gamma \vdash A \cap B}{\Gamma \vdash B}$$

Then an *intro* followed by an *elim* should yield the same information than was already in (one of) the premises (*inversion principle*, Prawitz 1965).

$$\cap_1\ \mathsf{E}\,\frac{\cap\ \mathrm{I}\,\dfrac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \cap B}}{\Gamma \vdash A} \quad \text{reduces to} \quad \Gamma \vdash A$$

Reverse the procedure: take the elim rules as defining the meaning of the connective and define the intro rule in such a way that the inversion principle holds. (Dummett *The logical Foundations of Metaphysics*).

Example:

The $\otimes$ I rule is *not* in harmony with $\cap_1$ E and $\cap_2$ E:

$$\otimes\ \mathrm{I}\,\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}$$

11. The domain of pragmatics (cont)

Indeed

$$\otimes\,\mathrm{E}^*\frac{\otimes\,\mathrm{I}\dfrac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}}{\Gamma, \Delta \vdash A}$$

The elimination rule in harmony with $\otimes$-I is

$$\otimes\,\mathrm{E}\,\frac{A \otimes B \vdash A \otimes B \qquad A, B, \Pi \vdash E}{A \otimes B, \Pi \vdash E}$$

Indeed

$$\otimes\,\mathrm{E}\,\frac{\otimes\,\mathrm{I}\dfrac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B} \qquad A, B, \Pi \vdash E}{\Gamma, \Delta, \Pi \vdash E}$$

reduces to

$$\frac{\Gamma \vdash A \qquad \dfrac{\Delta \vdash B \qquad A, B, \Pi \vdash E}{A, \Delta \Pi \vdash E}\,\mathit{subst}}{\Gamma, \Delta, \Pi \vdash E}\,\mathit{subst}$$

The principle of harmony limits the correct use us expressions on the basis of an operational definition of meaning, *in contrast with classical semantics*.

Slogan: *General Proof Theory is about pragmatics.*

Let us extend the language $\mathbf{L}^{AH}$ with *intuitionistic assertive disjunction* $A \cup B$ which is justified if either A or B is justified. The formula $A \cup \sim A$ is *justified* iff A is *decidable*.

12. Conjectures and expectations.

• *What's the difference between a conjecture and a hypothesis?*
I am justified in *asserting* p iff I'm not justified in *making the hypothesis that* $\neg p$.
But I may not be justified in *conjecturing* $\neg p$ without being justified in *asserting* p - simply, I may not know enough about p.
For a *conjecture* p ($\mathcal{C}p$) to be justified there must be *some* situation in which p could be justifiably asserted.

Semantic projection in **S4**:

$$(\mathcal{H}p)^M = \Diamond p \qquad\qquad (\mathcal{C}p)^M = \Diamond\Box p.$$

• *What illocutionary act is the dual of conjecturing?*
We may call it (hopeful or fearful) *safe expectation* ($\mathcal{E}p$): in *every* situation the hypothesis p is justified.

Semantic projection in **S4**:

$$(\mathcal{C}p)^M = \Diamond\Box p \qquad\qquad (\mathcal{E}p)^M = \Box\Diamond p.$$

A remarkable property:

$$\sim\sim \vdash p \not\equiv \vdash p \qquad \text{but} \qquad \sim\sim \mathcal{E}p \equiv \mathcal{E}p.$$

Indeed $(\sim\sim \vdash p)^M = \Box\Diamond\Box p \ \not\Rightarrow\ \Box p = (\vdash p)^M$,
but $(\sim\sim \mathcal{E}p)^M = \Box\Diamond\Box\Diamond p \equiv \Box\Diamond p = (\mathcal{E})^M$. Similarly,

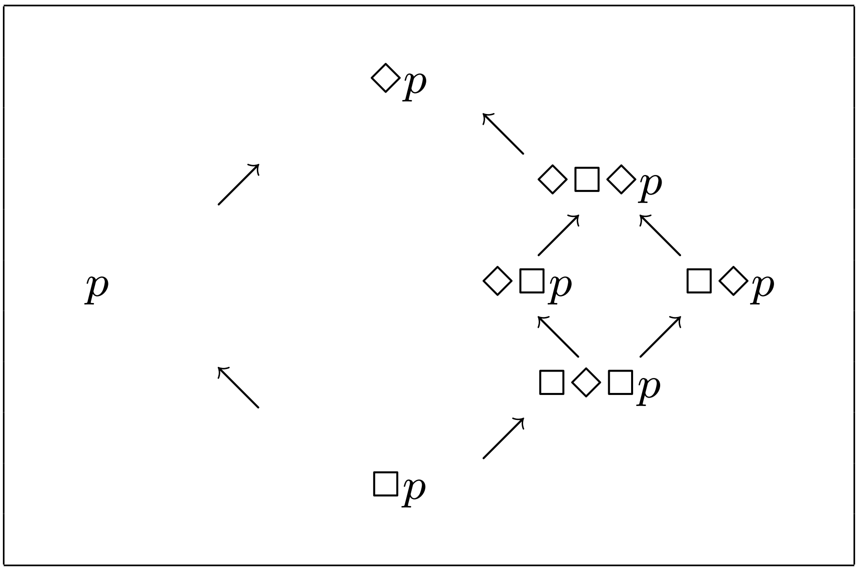
$$\frown\frown \mathcal{H}p \not\equiv \mathcal{H}p \qquad \text{but} \qquad \frown\frown \mathcal{C}p \equiv \mathcal{C}p.$$

Indeed $(\mathcal{H}p)^M = \Diamond p \ \not\Rightarrow\ \Diamond\Box\Diamond p = (\frown\frown \mathcal{H}p)^M$,
but $(\frown\frown \mathcal{C}p)^M = \Diamond\Box\Diamond\Box p \equiv \Diamond\Box p = (\mathcal{C})^M$.

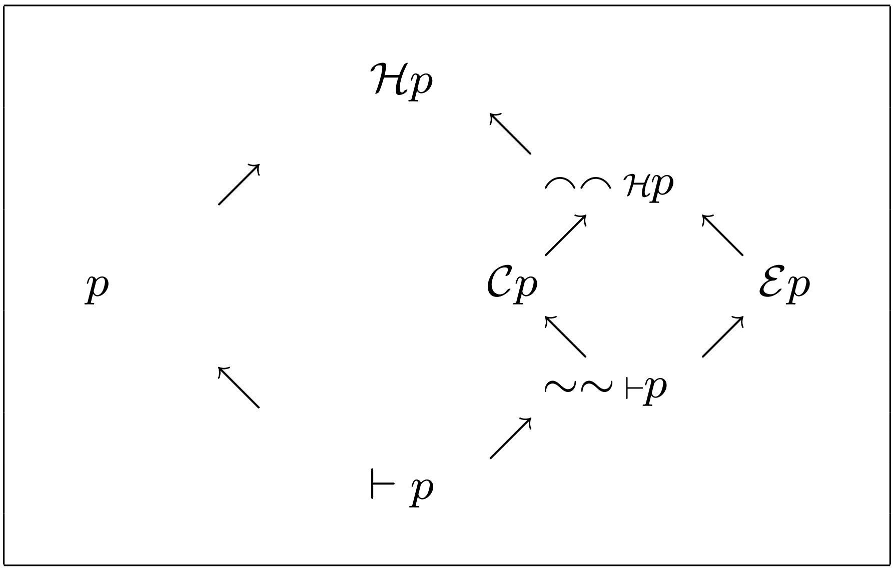
However $\mathcal{E}p \cup \sim \mathcal{E}p$ remains invalid.

Thus the logics of expectations and of conjectures share some properties of classical logic.

13. Assertions, hypotheses, conjectures, expectations.



The modalities of **S4**



Assertions, hypotheses, conjectures, expectations

14. Sequent calculus for logic of conjectures and expectations.

To the calculus **AH-G3** we add the following rules:

$$\mathcal{C}\text{-R}\,\frac{\Theta\ ;\ \Rightarrow\ \vdash p\ ;\ \Upsilon}{\Theta\ ;\ \Rightarrow\ ;\ \mathcal{C}p, \Upsilon} \qquad \mathcal{C}\text{-L}\,\frac{\Theta, \vdash p\ ;\ \Rightarrow\ ;\ \Upsilon}{\Theta\ ;\ \mathcal{C}p\ \Rightarrow\ ;\ \Upsilon}$$

$$\mathcal{E}\text{-R}\,\frac{\Theta\ ;\ \Rightarrow\ ;\ \mathcal{H}p,\ \Upsilon}{\Theta\ ;\ \Rightarrow\ \mathcal{E}p\ ;\ \Upsilon} \qquad \mathcal{E}\text{-L}\,\frac{\Theta\ ;\ \mathcal{H}p\ \Rightarrow\ ;\ \Upsilon}{\Theta, \mathcal{E}p\ ;\ \Rightarrow\ ;\ \Upsilon}$$

Here is a derivation of $\sim\sim \mathcal{E}p :\Rightarrow; \mathcal{E}p$.

$$\mathcal{E}\text{-R}\,\dfrac{\supset\text{-L}\,\dfrac{\supset\text{-R}\,\dfrac{\mathcal{E}\text{-L}\,\dfrac{;\ \mathcal{H}p \Rightarrow\ ;\ \mathcal{H}p}{\mathcal{E}p\ ;\ \Rightarrow\ ;\ \mathcal{H}p}}{;\ \Rightarrow\ \sim \mathcal{E}p\ ;\ \mathcal{H}p}}{\sim\sim \mathcal{E}p\ ;\ \Rightarrow\ ;\ \mathcal{H}p}}{\sim\sim \mathcal{E}p\ ;\ \Rightarrow \mathcal{E}p\ ;}$$

15. A pragmatic interpretation of classical logic.

Classical logic can be axiomatized in Natural Deduction $\mathbf{NJ}^{\supset\cap}$ (intuitionistic) with the addition of the *double negation rule* $\bot_C$:

$$\frac{\dfrac{\sim A, \Gamma \Rightarrow \bot}{\Gamma \Rightarrow \sim\sim A}}{\Gamma \Rightarrow A} \quad \text{or simply} \quad \frac{\sim A, \Gamma \Rightarrow \bot}{\Gamma \Rightarrow A}\,\bot_C$$

Is $\bot_C$ an intro? an elim? neither?

Prawitz 1965: *We need the* $\bot_C$ *rule only with the conclusion* A atomic. This "minimises the disturbance" caused by $\bot_C$ to the pattern intro/elim rules.
Prawitz-Dummett: no *meaning-as-use interpretation* for classical logic: a revision of use in favour of intuitionistic logic is in order.

What does this become in a logic of assertions, hypotheses and expectations?

Multiple conclusions intro and elim rules for $\mathcal{E}$:

$$\frac{\Theta \Rightarrow \bot;\, \mathcal{H}p, \Upsilon}{\Theta \Rightarrow \mathcal{E}p; \Upsilon}\,\mathcal{E}\ \mathrm{I} \qquad \frac{\Theta \Rightarrow \mathcal{E}p; \Upsilon \qquad \mathcal{H}p \Rightarrow;\ \mathcal{H}p}{\Theta \Rightarrow \bot;\, \mathcal{H}p, \Upsilon}\,\mathcal{E}\ \mathrm{E}$$

Inversion principle:

$$\frac{\dfrac{\Theta \Rightarrow \bot;\, \mathcal{H}p, \Upsilon}{\Theta \Rightarrow \mathcal{E}p; \Upsilon}\,\mathcal{E}\ \mathrm{I} \qquad \mathcal{H}p \Rightarrow;\ \mathcal{H}p}{\Theta \Rightarrow \bot;\, \mathcal{H}p, \Upsilon}\,\mathcal{E}\ \mathrm{E} \quad \rightsquigarrow \quad \Theta \Rightarrow \bot;\, \mathcal{H}p, \Upsilon$$

This suffices to formalize classical logic, respecting the into/elim pattern.

16. The lambda mu calculus.

terms:	M, N	$:=$	$x \mid \lambda x.M \mid MN \mid \mu\alpha.Q$	
commands:	Q	$:=$	$[\alpha]M$	(α abstraction)

Substitutions:

ordinary: $M[N/x]$ (capture avoiding);
renaming: $Q[\alpha/\beta]$;
structural: $T[\alpha \Leftarrow L]$: $[\alpha]N$ replaced by $[\alpha]NL$ in T.

Reductions:

(β)	$(\lambda x.M)N$	$\rightsquigarrow$	$M[N/x]$;
(μ)	$(\mu\beta.Q)N$	$\rightsquigarrow$	$\mu\beta.Q[\beta \Leftarrow N]$;
(*ren*)	$[\alpha]\mu\beta.Q$	$\rightsquigarrow$	$Q[\alpha/\beta]$;
($\mu\eta$)	$\mu\alpha.[\alpha]M$	$\rightsquigarrow$	M.

Typed reductions:

$$\dfrac{\dfrac{\begin{array}{c}[x:A]\\ \vdots\\ M:B\end{array}}{\lambda x.M : A \supset B} \quad \begin{array}{c}\vdots\\ N:A\end{array}}{(\lambda x.M)N : B} \quad \rightsquigarrow_\beta \quad \begin{array}{c}\vdots\\ [N:A]\\ \vdots\\ M[N/x]:B\end{array}$$

$$\dfrac{\perp_C \dfrac{Q:\perp}{\mu\beta^B.Q : B}}{[\alpha^A]\mu\beta^B.Q : \perp} \quad \rightsquigarrow_{ren} \quad Q[\alpha^A/\beta^B] : \perp$$

$$\perp_C \dfrac{\dfrac{M:A}{[\alpha^A]M : \perp}}{\mu\alpha^A.[\alpha^A]M : A} \quad \rightsquigarrow_{\mu\eta} \quad M : A$$

17. Typed mu reduction. Typing lambda mu with expectations and hypotheses

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{M : A \supset B}}{[\beta^{A\supset B}]M : \bot}\quad \vdots}{Q : \bot}\,\bot_C}{\dfrac{\mu\beta^{A\supset B}.Q : A \supset B \qquad N : A}{(\mu\beta^{A\supset B}.Q : A \supset B)N : B}} \quad \rightsquigarrow_\mu \quad \dfrac{\dfrac{\dfrac{\dfrac{\vdots \quad \vdots}{M : A \supset B \quad N : A}}{MN : B}}{[\beta^B]MN : \bot}\quad \vdots}{\dfrac{Q[\beta^B \Leftarrow N] : \bot}{\mu\beta.Q[\beta^B \Leftarrow N] : B}\,\bot_C}$$

The (μ) reduction commutes an elimination rule above a $\mu\alpha{-}\bot_C$ and up to each α abstraction. It also *reduces the logical complexity* of the conclusion of $\bot_C$.

Let us type the $\lambda\mu$ calculus within our language $\mathcal{L}^{AHCE}$:

$$A, B \quad := \quad \vdash p \mid \mathcal{E}p \mid \top \mid \sim A \mid A \supset B \mid -C$$

$$C, D \quad := \quad \mathcal{H}p \mid \mathcal{C}p \mid \bot \mid \frown C \mid C \smallsetminus D \mid -A$$

Proposition. (i) For all E built from $\mathcal{E}p_i$s,
$(E)^M = \Box\Diamond E^M$;
(ii) For all C built from elements $\mathcal{C}p_j$s, $(C)^M = \Diamond\Box C^M$;
(iii)$\sim\sim E \equiv \sim - \frown -E$ and $\frown\frown C \equiv \frown - \sim -C$.
(iv) $(\sim -C)^M = \Box(C^M)$ $\quad (\frown -E)^M = \Diamond E^M$.

Write $!C$ for $\sim -C$ and $?A$ for $\frown -A$.

18. Lambda mu with $\mathcal{E}$ and $\mathcal{H}$

$$\frac{\overline{x} : \Theta \Rightarrow Q : \bot; \alpha : C, \overline{\beta} : \Upsilon}{\overline{x} : \Theta \Rightarrow \mu\alpha.Q : !C; \overline{\beta} : \Upsilon}\ \mathcal{E}\ \mathrm{I} \qquad \frac{\overline{x} : \Theta \Rightarrow M : !C; \overline{\beta} : \Upsilon}{\Theta \Rightarrow [\alpha]M : \bot; \alpha : C, \overline{\beta} : \Upsilon}\ \mathcal{E}$$

Renaming

$$\frac{\dfrac{\overline{x} : \Theta \Rightarrow Q : \bot; \alpha : C, \overline{\beta} : \Upsilon}{\overline{x} : \Theta \Rightarrow \mu\alpha.Q : !C; \overline{\beta} : \Upsilon}\ \mathcal{E}\ \mathrm{I}}{\overline{x} : \Theta \Rightarrow [\beta]\mu\alpha.Q : \bot; \beta : C, \overline{\beta} : \Upsilon}\ \mathcal{E}\ \mathrm{E}$$

$$\rightsquigarrow \qquad \overline{x} : \Theta \Rightarrow Q[\beta/\alpha] : \bot; \beta : C, \overline{\beta} : \Upsilon$$

$\mu\eta$:

$$\frac{\dfrac{\overline{x} : \Theta \Rightarrow M : !C; \overline{\beta} : \Upsilon}{\overline{x} : \Theta \Rightarrow [\beta]M : \bot; \beta : C\overline{\beta} : \Upsilon}\ \mathcal{E}\ \mathrm{E}}{\overline{x} : \Theta \Rightarrow \mu\beta.[\beta]M : !C; \overline{\beta} : \Upsilon}\ \mathcal{E}\ \mathrm{I}$$

$$\rightsquigarrow \qquad \overline{x} : \Theta \Rightarrow M : !C; \overline{\beta} : \Upsilon$$

With elementary types $C = \mathcal{H}p$ and $!C = \mathcal{E}p$ **renaming** is just the inversion principle for $\mathcal{E}$ and $\mu\eta$ is just the η contraction for $\mathcal{E}$.

For non-elementary types, e.g., $A = A_1 \supset A_2$ we have $C = ?A = \frown (-A_2 \setminus -A_1)$ and $A \equiv !?A$ and the rules are derivable.

19. Lambda mu with $\mathcal{E}$ and $\mathcal{H}$ (cont.)

Similarly we can treat the μ reduction. In a more compact notation we have

μ reduction typed with $\mathcal{E}$ and $\mathcal{H}$:

$$\dfrac{\perp_C \dfrac{\dfrac{\dfrac{\vdots}{M : A \supset B}}{[\beta^{\frown(-B\setminus -A)}]M : \perp} \quad \vdots \quad Q : \perp}{\mu\beta^{\frown(-B\setminus -A)}.Q : A \supset B} \quad \dfrac{\vdots}{N : A}}{(\mu\beta^{\frown(-B\setminus -A)}.Q : A \supset B)N : B}$$

reduces to

$$\perp_C \dfrac{\dfrac{\dfrac{\dfrac{\vdots}{M : A \supset B} \quad \dfrac{\vdots}{N : A}}{MN : B}}{[\beta^{\frown -B}]MN : \perp} \quad \vdots \quad Q[\beta^{\frown -B} \Leftarrow N] : \perp}{\mu\beta.Q[\beta^{\frown -B} \Leftarrow N] : B}$$

20. Work in progress: a naif question.

• Dalla Pozza (manuscript):

A question p? *is justified iff* $\vdash p \cup \vdash \neg p$,

i.e., iff p is *decidable*

i.e., iff there is conclusive evidence either for p or for $\neg p$.

Is CH provable in ZF set theory? **unjustified.**

What's wrong with the following exchange?

- *Is it going to rain tomorrow?*
- *Perhaps not.*

• Why not the following *liberal proposal*?

A question p? *is justified iff* $\Rightarrow \vdash p \; ; \; - \vdash p$

i.e., **always**, provided that $\vdash p$ is interpretable, i.e., provided we know what constitutes conclusive evidence for p.

Dalla Pozza: too restrictive?

Liberal proposal: too liberal?

21. Extensions: semantic - pragmatic invariance

Here are some principles relating the semantics and the pragmtics.

- $\vdash(A \wedge B) \quad \equiv \quad \vdash A \cap \vdash B.$

- $\vdash \neg p \quad \equiv \quad - \mathcal{H}p \qquad \mathcal{H}\neg p \quad \equiv \quad - \vdash p$, etc.

- *Illocutionary obligation and deontic logic*

- **KD** classical normal modal logic satisfying

$$\circ A \to \neg \circ \neg A.$$

- $\mathcal{O}$ illocutionary operator of obligtion.

Kurt Ranalter: $\mathcal{O}p \quad \equiv \vdash \circ p.$

A principle of rationality of obligations?

Significant non-invariants.

$\vdash (x)P(x)$ classical first order logic in the semantics.

$\forall x. \vdash P(x)$ intuitionistic first order logic in the pragmatics.

$Ax. \vdash P(x)$ intermediate first order logic of *constant domains* in the pragmatics.

22. Possible individuals

If a classical first order formula is falsifiable, then it can be falsified over a constant domain, e.g., **N**.

If an intuitionistic first order formula is falsifiable, then it is falsifiable over a Kripke model with increasing domain.

Consider *Kuroda's formula*: $\forall x. \sim\sim P(x) \supset \sim\sim \forall x.P(x)$. It is valid in classical logic and in the logic of constant domains, but it is falsifiable in intuitionistic logic.

Thus if a formula is falsifiable in classical logic and in the logic of constant domains, it is falsifiable over an *intended domain*.
To falsify an intuitionistic formula we need *possible individuals*.

Naif question: Has this anything to do with the distinction *quantification de re - quantification de dicto*? Can pragmatic considerations share some light on the issue?

Thanks!

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