

Esercizi di analisi 1

◊ Calcolare i seguenti integrali:

$$\begin{array}{ccccc}
 \int \frac{2x+3}{2x+1} dx & \int \frac{x^2+1}{x-1} dx & \int \frac{x}{(x+1)^2} dx & \int \frac{3x+1}{\sqrt{5x^2+1}} dx & \int \frac{5\sqrt{x}}{\sqrt{x}} dx \\
 \int \frac{1}{3 \cos(5x - \pi/4)} dx & \int \frac{x}{\cos^2 x^2} dx & \int \frac{3^{\tanh x}}{\cosh^2 x} dx & \int \left(2 + \frac{x}{2x^2+1}\right) \frac{dx}{2x^2+1} & \\
 \int \frac{\sqrt[3]{1+\ln x}}{x} dx & \int \frac{e^{\arctan x} + x \ln(1+x^2) + 1}{1+x^2} dx & \int \frac{(1+x)^2}{x(1+x^3)} dx & \int \frac{dx}{x \ln^2 x} & \\
 \int e^{\sin^2 x} \sin 2x dx & \int \frac{dx}{\sin^2 x \cos^2 x} & \int \frac{\cos 2x}{4 + \cos^2 2x} dx & \int \frac{dx}{x(4 - \ln^2 x)} & \\
 \int \sqrt{\frac{\ln(x + \sqrt{x^2+1})}{1+x^2}} dx & & \int \frac{\sin x \cos x}{\sqrt{2 - \sin^4 x}} dx & &
 \end{array}$$

◊ Dire per quali $x \in \mathbb{R}$ le seguenti serie convergono:

$$\begin{array}{ccccc}
 \sum_{n=1}^{\infty} \frac{x^n}{1+x^{2n}} & \sum_{n=1}^{\infty} x^n \ln x^n & \sum_{n=1}^{\infty} \ln(1+n|x|^n) & \sum_{n=0}^{\infty} \frac{n \sin x^n}{n+x^{2n}} & \\
 \sum_{n=0}^{\infty} \frac{\sqrt{1+x^n}}{x^n} & \sum_{n=0}^{\infty} n x^{n!} & \sum_{n=0}^{\infty} x^{x^n} & \sum_{n=0}^{\infty} \frac{x^{n^2}}{n} & \sum_{n=1}^{\infty} \frac{1}{(\ln x)^{\ln n}} \\
 \sum_{n=0}^{\infty} x^{n-\sqrt{n}} & \sum_{n=1}^{\infty} \left(\frac{x^{2n}}{n} + \frac{n^{2x}}{x} \right) & & \sum_{n=1}^{\infty} \frac{\arctan \frac{1}{n} - \frac{1}{n}}{n^x (1 - \cosh \frac{1}{\sqrt{n}})} & \\
 \sum_{n=1}^{\infty} \left[\tan \left(a + \frac{1}{n} \right) \right]^n & \text{al variare del parametro } a \in] -\frac{\pi}{2}, \frac{\pi}{2} [& & & \\
 \sum_{n=1}^{\infty} \frac{9^n}{2n+1} x^{2n+1} & \text{(stabilirne convergenza semplice e assoluta al variare di } x \in \mathbb{R}) & & &
 \end{array}$$