FLAT MITTAG-LEFFLER MODULES AND DRINFELD VECTOR BUNDLES

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Drinfeld [2] suggested to use flat Mittag–Leffler modules instead of finitely generated projective modules in the definition of an infinite dimensional vector bundle on a scheme X. We call such bundles the *Drinfeld vector bundles*. Flat Mittag–Leffler modules over a general ring R were studied in [9] and [1], but only recently [7], they were proved to coincide with the $\aleph_1$–projective modules in the sense of Eklof and Mekler [3].

Classic work of Quillen [10] made it possible to compute morphisms between two objects A and B of the derived category of the category $\mathcal{Q}(X)$ of all quasi–coherent sheaves on X. First, one introduces a model category structure on $\mathcal{U}(X)$ (= the category of unbounded chain complexes on $\mathcal{Q}(X)$). Morphisms between A and B can then be computed as the $\mathcal{U}(X)$–morphisms between cofibrant and fibrant replacements of A and B, respectively, modulo chain homotopy.

Hovey [8] showed that model category structures naturally arise from small cotorsion pairs on $\mathcal{U}(X)$. Thus Gillespie [6] produced a model category structure on $\mathcal{U}(X)$ using flat quasi–coherent sheaves. By a different approach, a model category structure was produced on $\mathcal{U}(X)$ when X is the projective line, using quasi–coherent sheaves all of whose sections in open affine sets are projective, [4].

In [5] a general method of constructing model category structures was presented that includes the results of [4] and [6]. The method also works for all bounded flat Mittag–Leffler quasi–coherent sheaves. But it does not apply to the unbounded (i.e., Drinfeld vector bundle) case, because by [7], the class of all $\aleph_1$–projective modules is deconstructible only if R is perfect. This shows a remarkable difference between Drinfeld vector bundles and flat (or projective) quasi–coherent sheaves.

In my talk I will present the main results of [5] and [7], and some related open problems.

References

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