

## Direct limits in the heart of the t-structure of Happel-Reiten-Smalø

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T-structures on triangulated categories were introduced in the early eighties by Beilinson, Bernstein and Deligne in their study of the perverse sheaves on an algebraic or an analytic variety (see [BBD]). The main discovery of this concept was the existence of an abelian category, called the heart of the t-structure, which allowed the development of a homology theory that is intrinsic to the triangulated category.

In [HRS], Happel, Reiten and Smalø, associated to each torsion pair in an abelian category  $\mathcal{A}$ , a t-structure in the bounded derived category  $D^b(\mathcal{A})$ . This t-structure is the restriction of a t-structure in  $D(\mathcal{A})$ . We study the behavior of direct limits in the heart of this t-structure. For a given Grothendieck category  $\mathcal{G}$  and a torsion pair  $\mathfrak{t} = (\mathcal{T}, \mathcal{F})$  in  $\mathcal{G}$ , we show that if the heart of the associated t-structure in the derived category  $D(\mathcal{G})$  is AB5, then  $\mathcal{F}$  is closed under taking direct limits. The reverse implication is true, even implying that the heart is a Grothendieck category, for a wide class of torsion pairs which include the hereditary ones, those for which  $\mathcal{T}$  is a cogenerating class and those for which  $\mathcal{F}$  is a generating class. The results allow to extend well-known results by Buan-Krause, Bazzoni and Colpi-Gregorio to the general context of Grothendieck categories and to improve some results of (co)tilting theory of modules.

## References

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