Peter Schuster (University of Verona): Dynamical Algebra with Jacobson Radicals

Joint work with Daniel Wessel and partly with Giulio Fellin (University of Verona)

The dynamical proof method was designed to make sense in constructive mathematics of the invocations of the axiom of choice abstract algebra abounds with. While originally designed for the Stellensätze of real and commutative algebra, the dynamical method has recently proved to reach into general algebra, and thus to substitute ever more universal transfinite proof principles.

The key role is played by a generalisation, for a Scott-open filter in a complete lattice, of the Jacobson radical as the sum of all small submodules. While semantically the radical remains the intersection of all maximal extensions, syntactically this can be expressed by an inductive generation. Membership of the radical then amounts to termination of a finite binary tree in all paths. This makes possible a constructive interpretation of the Teichmüller-Tukey lemma, including its applications to linear and algebraic dependence.