Abstract

Direct-sum relations among finitely generated torsion-free modules

Let R be a local domain of dimension one. For example, R might be the local ring of a singular point on an algebraic curve, or a localization of an order in an algebraic number field. We are particularly interested in the situation where the completion of the ring is *not* an integral domain. Examples include nodal curves and rings such as $\mathbb{Z}[5\sqrt{-1}]$ (suitably localized).

Let $\mathcal{C}(R)$ be the set of isomorphism classes of finitely generated torsion-free R-modules. We make $\mathcal{C}(R)$ an additive semigroup, using the direct-sum relation: $[M]+[N]=[M\oplus N]$, where "[ ]" denotes the isomorphism class of a module. This semigroup encodes all direct-sum relations among finitely generated torsion-free modules.

The structure of this semigroup has been worked out recently in two antipodal cases: (1) when all branches of the completion have infinite representation type, and (2) when R has finite representation type. The intermediate case, where R has infinite representation type but at least one branch has finite representation type seems to be much more difficult, but some progress has been made.

In this talk I will survey some of these results and give concrete examples to show spectacular failure of uniqueness of direct-sum decompositions. For example, given any integer $n \geq 2$ on can find a ring R (as above) and indecomposable modules M, N, V such that $M \oplus N$ is isomorphic to the direct sum of n copies of V.