OPTIMAL POLYNOMIAL ADMISSIBLE MESHES FOR REAL COMPACT SETS WITH MILDLY SMOOTH BOUNDARY

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Abstract. Admissible Meshes (AM) [1] are sequences \( \{A_n\}_{n \in \mathbb{N}} \) of finite subsets of a given compact \( K \subset \mathbb{R}^d \) (or \( \mathbb{C}^d \)) such that the following inequality holds for any polynomial \( p \in \mathcal{P}_n(\mathbb{R}^d) \) of degree at most \( n \)

\[ \|p\|_{C(K)} \leq C \max_{A_n} |p| \]

and moreover \( \text{Card}(A_n) \) grows polynomially w.r.t. \( n \).

AM was introduced in [1] as suitable sets where perform the sampling for the discrete least square polynomial approximation. In [2] A. Kroó defines Optimal AMs as AMs which cardinality grows at optimal rate \( O(n^d) \).

In this talk we present our recent result [4][Th. 3.7]. If \( \Omega \subset \mathbb{R}^d \) is a bounded \( C^1 \) domain, then \( \Omega \) has an Optimal Admissible Mesh. The proof is fully constructive and relays on the regularity property of the distance function w.r.t. \( \overline{\Omega} \).

The main features of AMs are also discussed, both as theoretical motivations and applications. Namely, starting from an AM, one can extract by standard Linear Algebra quasi-optimal interpolation arrays, say Approximate Fekete and Leja points [3]. Moreover the sequence of discrete probability measures \( \{\mu_n\} \) canonically associated to an AM can be used to compute the Pluripotential Equilibrium Measure of the given compact and the Siciak Extremal Function.

References


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