Simple Temporal Networks and some its Extensions

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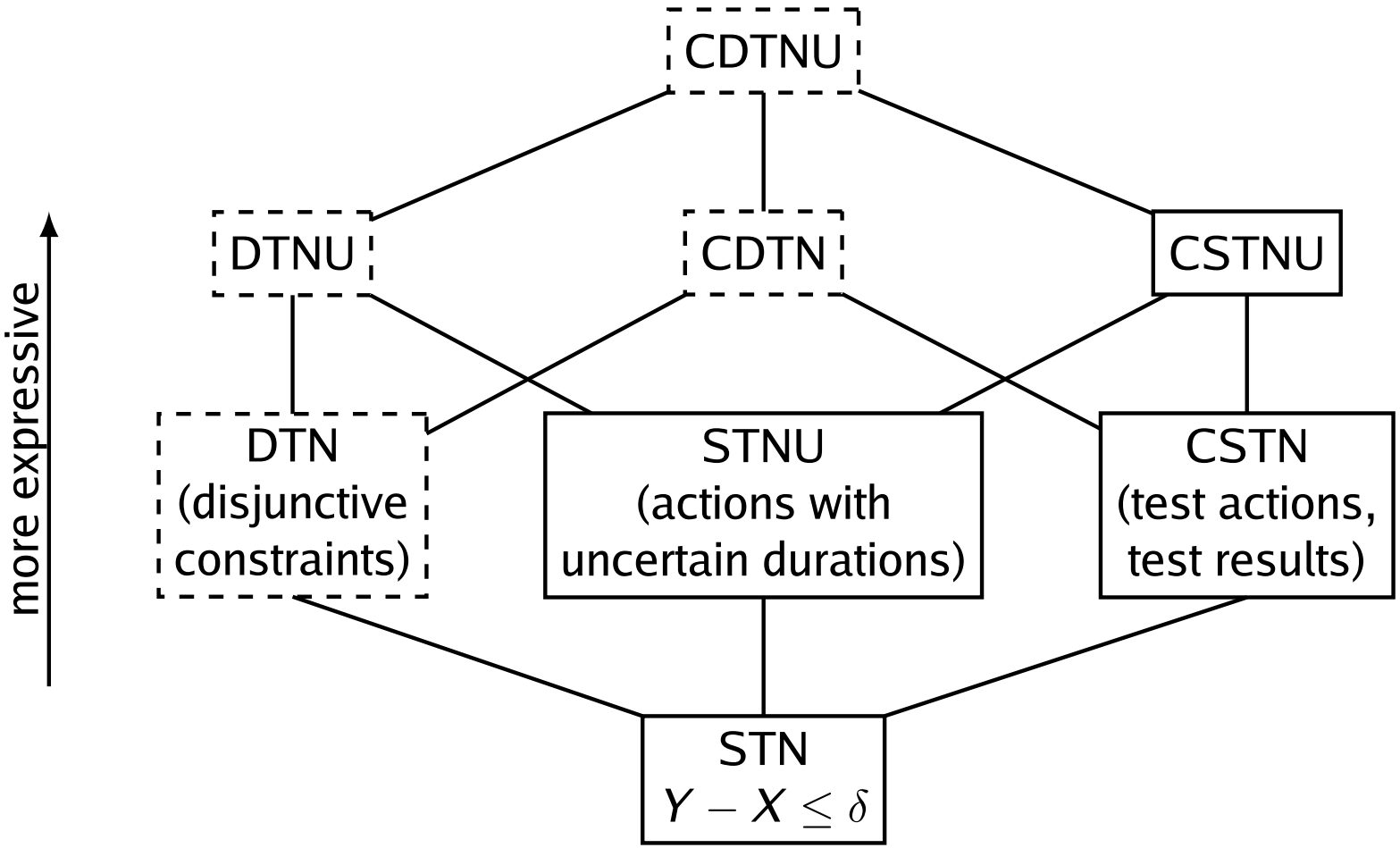
Credits: this presentation is based on Luke Hunsberger and Roberto Posenato slides

Outline



- This lecture does not present dashed types.

Simple Temporal Network
Motivating Example

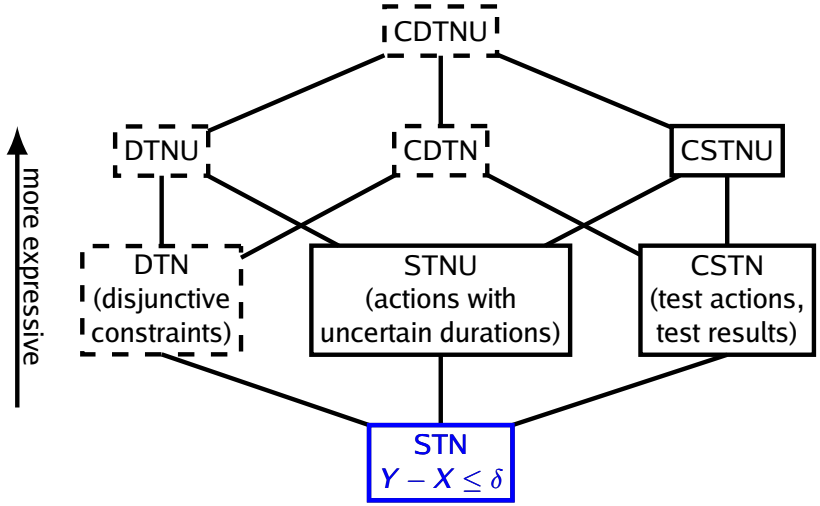
- Given a plan of a fly from New York to Rome,
- Determine if the plan is **consistent**,
- If it is consistent, determine a **temporal schedule**.

Fly from New York to Rome

- Leave New York after 4 p.m., June 8
- Return to New York before 10 p.m., June 18
- Away from New York no more than 7 days
- In Rome at least 5 days
- Return flight lasts no more than 7 hours

Simple Temporal Network (STN)*
Description

- Includes time–points and temporal constraints of only one kind.
- Flexible: Time–points may "float"; not "nailed down" until they are *executed*
- Efficient algorithms for determining consistency, managing real–time execution, and handling new constraints



* [Dechter et al., 1991]

Simple Temporal Network*
Definition

Definition 1 (Simple Temporal Network (STN))

A *Simple Temporal Network (STN)* is a pair, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, where:

- $\mathcal{T}$ is a set of real–valued variables called time–points: $\{X_1, \ldots, X_n \mid X_i \in \mathbb{R}\}$; and
- $\mathcal{C}$ is a set of binary constraints, each of the form:

$$X_j - X_i \leq \delta$$

where $X_j, X_i \in \mathcal{T}$ and $\delta \in \mathbb{R}$.

- Time–points represent events.
- Each binary constraint represents the maximal temporal distance between the first time point and the second one.
- There is only one time unit.

*[Dechter et al., 1991]

Simple Temporal Network
The *Zero* Time–Point, Z

- It is useful to have one time–point, called Z, whose value is fixed at 0.
- Binary constraints involving Z are equivalent to unary constraints.

Example 1

$$X - Z \leq 7 \iff X \leq 7$$

$$Z - X \leq -3 \iff X \geq 3$$

- A *solution* to an STN $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ is a complete set of assignments to the time–points in $\mathcal{T}$:

$$\{X_1 = t_1,\ X_2 = t_2,\ \ldots,\ X_n = t_n\}$$

  that together satisfy all of the constraints in $\mathcal{C}$.
- An STN with at least one solution is *consistent.*
- STNs with identical solution sets are *equivalent*.

Simple Temporal Network
STN for Travel Example

$\mathcal{T} = \{Z, X_1, X_2, X_3, X_4\}$, where Z = Noon, June 8

$$\mathcal{C} = \begin{cases} Z - X_1 \leq -4 & X_1\text{=leave–time NYC (Lv NYC after 4 p.m., June 8)} \\ X_4 - Z \leq 250 & X_4\text{=return–time NYC (Ret NYC by 10 p.m., June 18)} \\ X_4 - X_1 \leq 168 & \text{(Gone no more than 7 days)} \\ X_2 - X_3 \leq -120 & X_2(X_3)\text{=arrive(leave)–time Rome (In Rome at least 5 days)} \\ X_4 - X_3 \leq 8 & \text{(Return flight less than 8 hrs)} \\ X_1 - X_2 \leq -7 & \text{(Flight requires at least 7 hrs)} \\ X_3 - X_4 \leq -7 & \text{(Return flight requires at least 7 hrs)} \end{cases}$$

Simple Temporal Network
Graph for an STN*

The *graph* for an STN, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, is a graph, $\mathcal{G} = (\mathcal{T}, \mathcal{E})$, where:

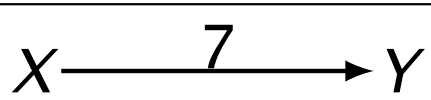
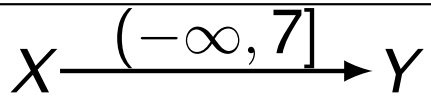
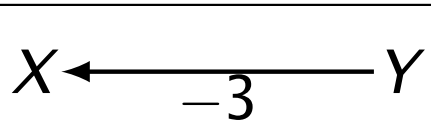
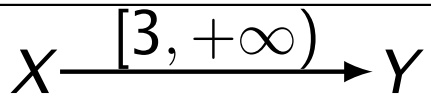
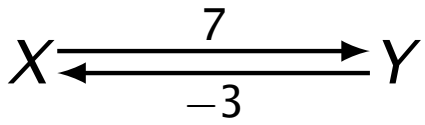
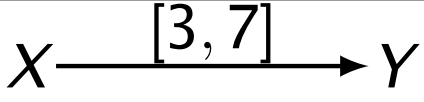
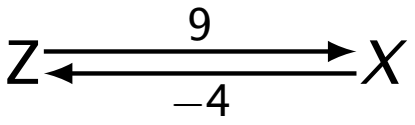
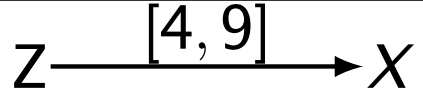
$$\text{Time–points in } \mathcal{S} \iff \text{nodes in } \mathcal{G}$$

$$\text{Constraints in } \mathcal{C} \iff \text{edges in } \mathcal{E}\text{:}$$

$$Y - X \leq \delta \iff X \xrightarrow{\delta} Y$$

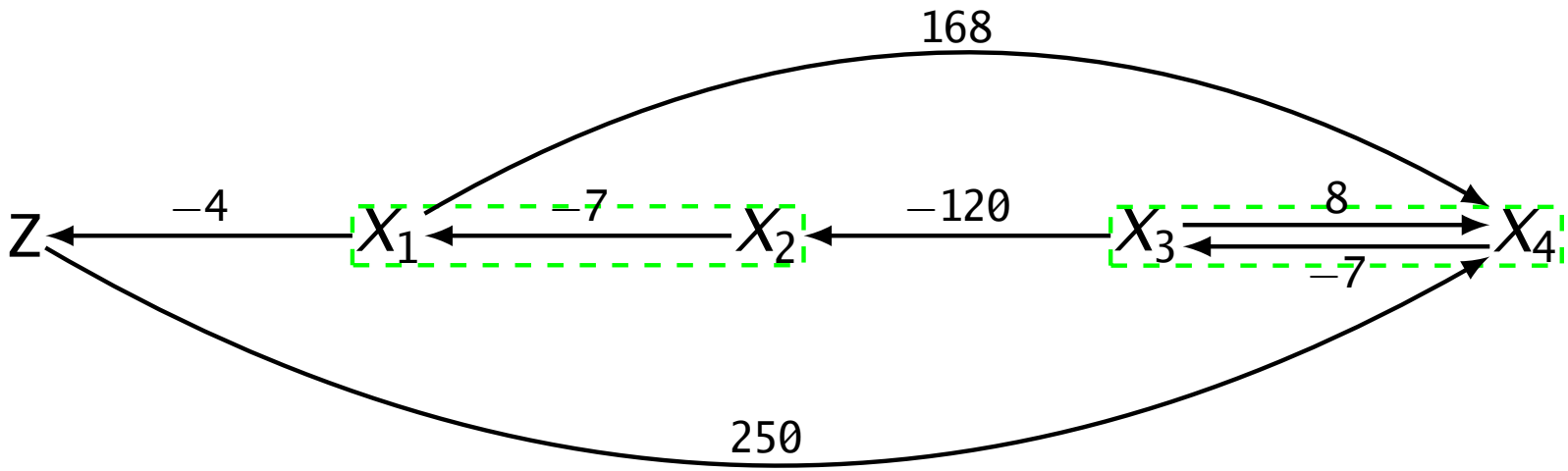
*[Dechter et al., 1991]

Simple Temporal Network
Graphical Representations

Constraint(s)	Edge(s)	Interval Notation
$Y - X \leq 7$		
$X - Y \leq -3$ ($\Leftrightarrow 3 \leq Y - X$)		
$3 \leq Y - X \leq 7$		
$4 \leq X \leq 9$		

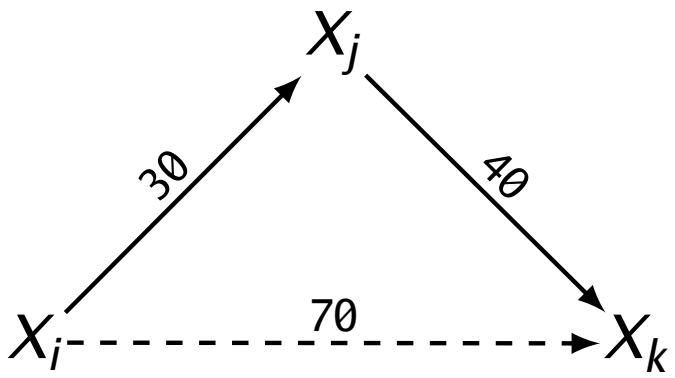
Simple Temporal Network
Graph for Airline Scenario

$$\begin{cases} Z - X_1 \leq -4, & X_4 - Z \leq 250 \\ X_4 - X_1 \leq 168, & X_2 - X_3 \leq -120 \\ X_4 - X_3 \leq 8, & X_1 - X_2 \leq -7 \\ X_3 - X_4 \leq -7 & \end{cases}$$



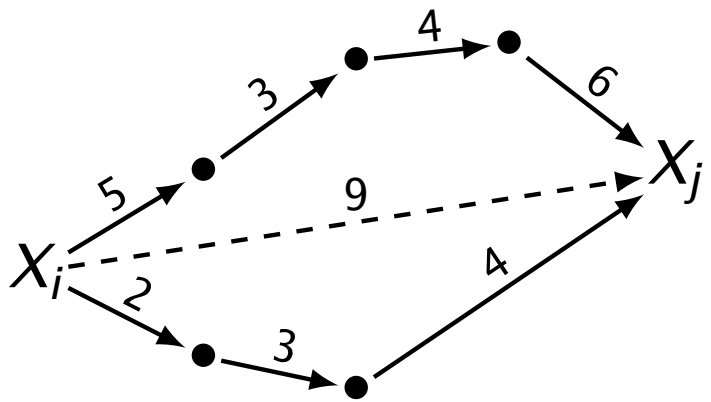
Explicit constraints combine (propagate) to form implicit constraints:

$$\begin{array}{rcl} X_j - X_i & \leq & 30 \\ X_k - X_j & \leq & 40 \\ \hline X_k - X_i & \leq & 70 \end{array}$$



Simple Temporal Network
Chains of Constraints as Paths

- Chains of constraints correspond to **paths** in the graph.
- **Stronger** constraints correspond to **shorter** paths.

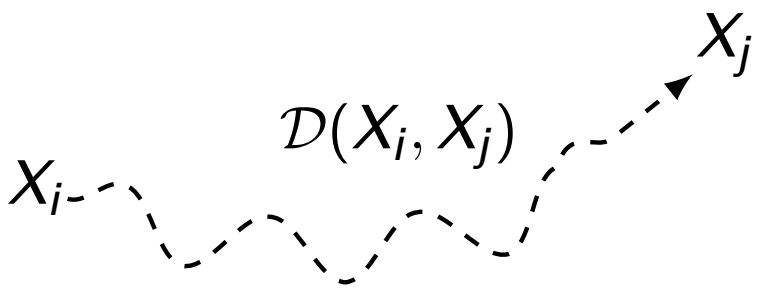


Simple Temporal Network
Distance Matrix*

Definition 2 (Distance Matrix)

The Distance Matrix for an STN, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, is a matrix $\mathcal{D}$ defined by:

$\mathcal{D}(X_i, X_j) =$ Length of Shortest Path from X_i to X_j in the graph for $\mathcal{S}$



- The strongest implicit constraint on X_i and X_j in $\mathcal{S}$ is:

$$X_j - X_i \leq \mathcal{D}(X_i, X_j)$$

*[Dechter et al., 1991]

Simple Temporal Network
Distance Matrix

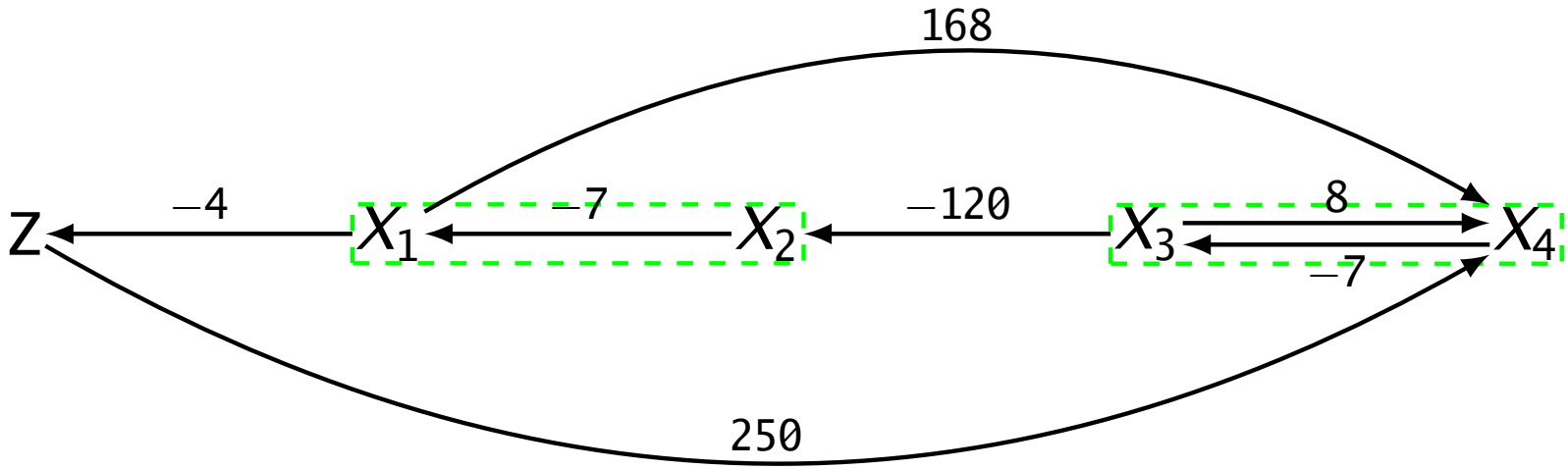
- The strongest implicit constraint on X_i and X_j in $\mathcal{S}$ is:

$$X_j - X_i \leq \mathcal{D}(X_i, X_j)$$

- $\mathcal{D}$ is the *All–Pairs, Shortest–Path* (APSP) Matrix for the STN's graph.*
  - Floyd–Warshall Algorithm
  - Johnson's Algorithm

*[Cormen et al., 2009]

Simple Temporal Network
Travel Scenario's Distance Matrix



$\mathcal{D}$	Z	X_1	X_2	X_3	X_4
Z	0	116	123	243	250
X_1	−4	0	41	161	168
X_2	−11	−7	0	154	161
X_3	−131	−127	−120	0	8
X_4	−138	−134	−127	−7	0

Gray cells contain the original values.

Simple Temporal Network
Fundamental Theorem of STNs*

For an STN $\mathcal{S}$, with graph $\mathcal{G}$, and distance matrix $\mathcal{D}$, the following are equivalent:

- $\mathcal{S}$ is consistent
- $\mathcal{D}$ has non–negative values on main diagonal
- $\mathcal{G}$ has no negative–length loops

Moreover, any consistent STN $\mathcal{S}$ is backtrack–free relative to the constraints in its distance matrix.

*[Dechter et al., 1991]

Simple Temporal Network
Finding a solution for an STN

- $\mathcal{D}$ has all necessary information.
- Time window for any X_i: $[-\mathcal{D}(X_i, Z), \mathcal{D}(Z, X_i)]$
- Two solutions are always given by free:
  1. Earlier execution–time solution:
     $X_1 = -\mathcal{D}(X_1, Z), X_2 = -\mathcal{D}(X_2, Z), \ldots, X_n = -\mathcal{D}(X_n, Z)$;
  2. Latest execution–time solution:
     $X_1 = \mathcal{D}(Z, X_1), X_2 = \mathcal{D}(Z, X_2), \ldots, X_n = \mathcal{D}(Z, X_n)$.
- Simple algorithm to find a different solution:
  - Pick any time–point that doesn't yet have a value;
  - Give it a value from its time–window;
  - Update $\mathcal{D}$; $\Leftarrow$ expensive ...
  - Repeat until all time–points have values.

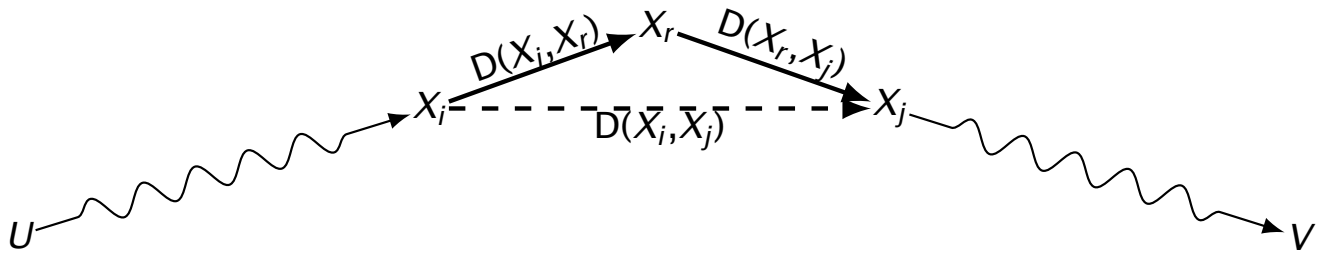
*[Dechter et al., 1991]

Simple Temporal Network
Computing $\mathcal{D}$ from Scratch*

- $\mathcal{D}$ is the *All–Pairs, Shortest–Paths* (APSP) Matrix for $\mathcal{G}$.
- If $\mathcal{S}$ has n time–points and m constraints:
  - Floyd–Warshall Algorithm: $O(n^3)$
  - Johnson's Algorithm: $O(n^2 \log n + mn)$
    - uses Bellman–Ford and Dijkstra

*[Cormen et al., 2009]

Floyd–Warshall Algorithm*



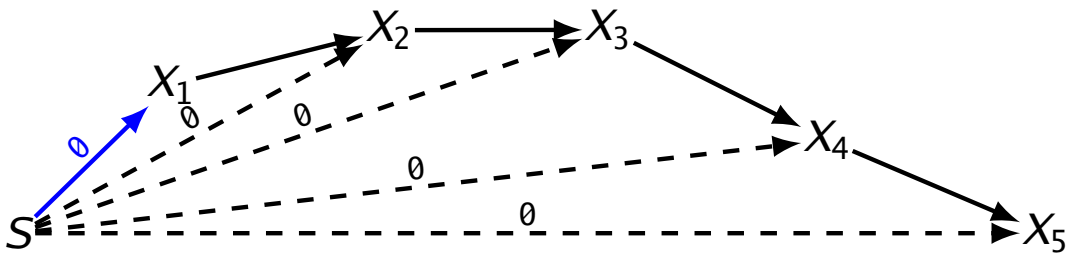
Algorithm 1: Flowd–Warshall(D)

Initialize D(_,_) using edge–weights;
for $r := 1$ *to* n **do**
  for $i := 1$ *to* n **do**
    for $j := 1$ *to* n **do**
      $D(X_i,X_j) := \min\{D(X_i,X_j), D(X_i,X_r) + D(X_r,X_j)\}$;

- The r^{th} cycle finds all minimal paths having r intermediate nodes at most;
- Time complexity $\Theta(n^3)$

*[Cormen et al., 2009]

Bellman–Ford Algorithm



Algorithm 2: Bellman–Ford()

foreach X_i **do**
  $d(X_i) := \infty$
$d(S) := 0$;
for $i := 1$ *to* $n - 1$ **do**
  foreach *edge* (X_i, δ, X_j) **do**
    $d(X_j) := \min\{d(X_j), d(X_i) + \delta\}$;
foreach *edge* (X_i, δ, X_j) **do**
  if $d(X_j) > d(X_i) + \delta$ **then return** *NO*;
return *{D(S,X$_1$),..., D(S,X$_n$)}*;

- Time complexity $O(nm)$

Dijkstra's SSSP Algorithm
Single–Source Shortest–Paths

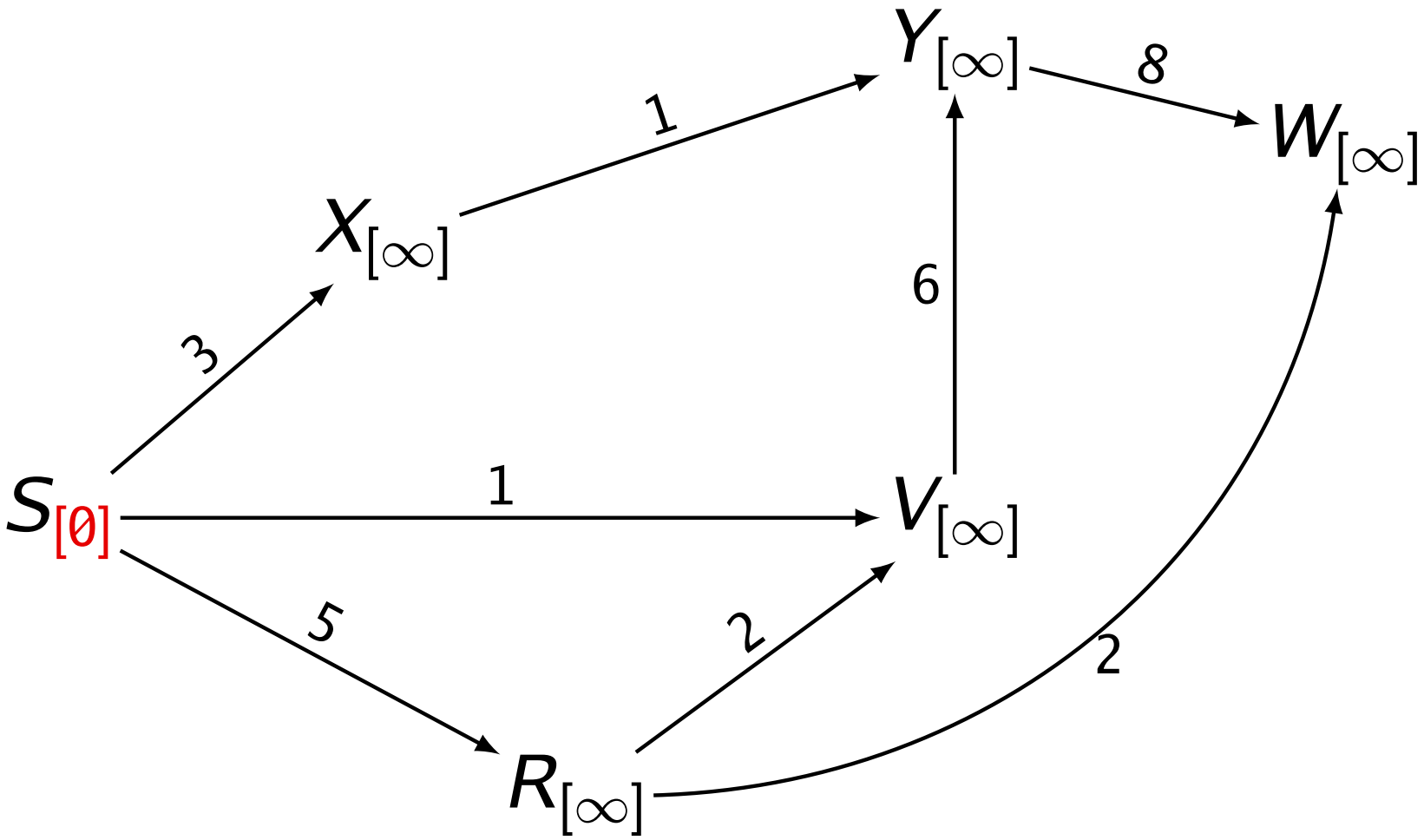
- Works on STN graphs with non–negative edges
- For a given source node S, computes $\mathcal{D}(S, X)$ for all X

Algorithm 3: Dijkstra()

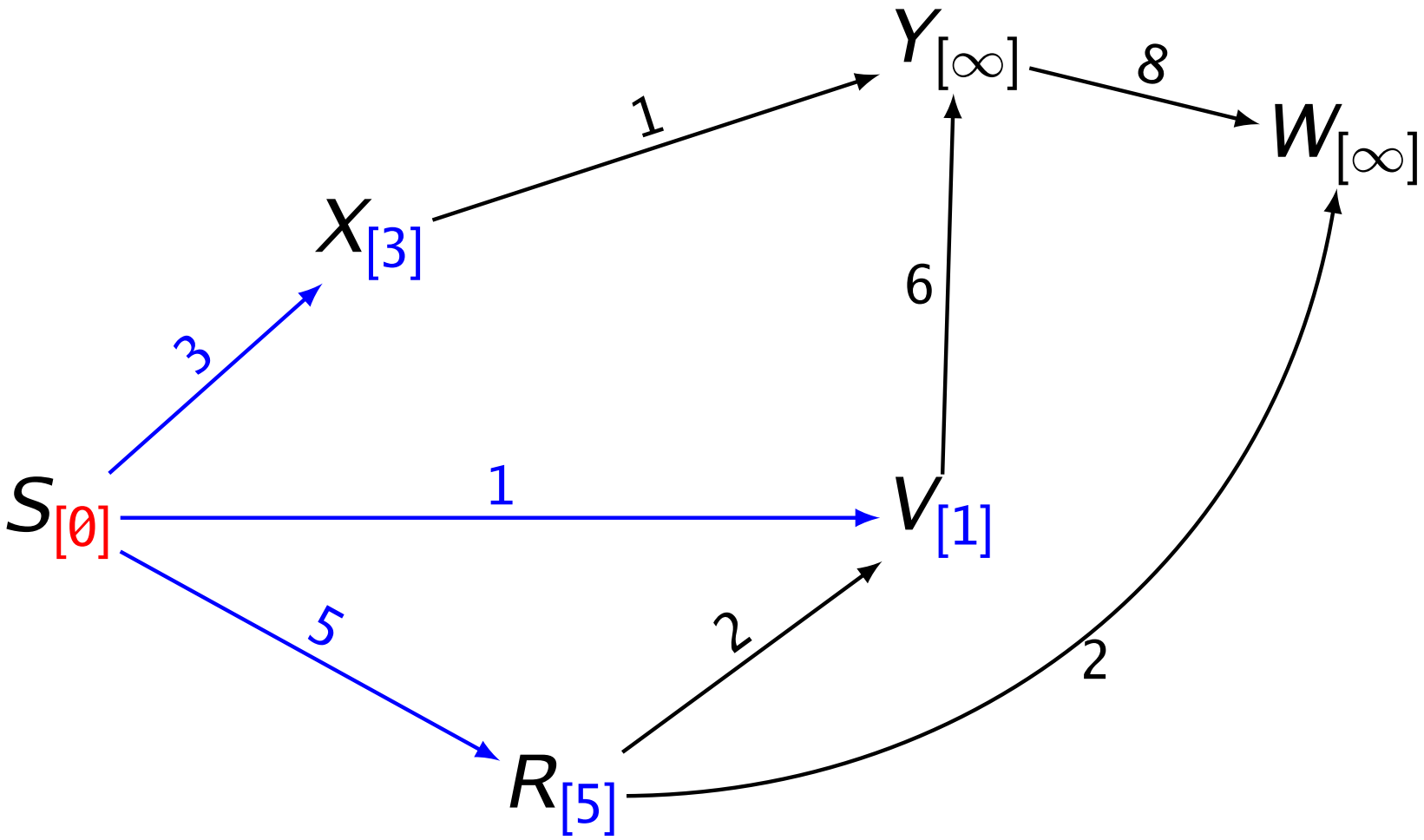
foreach X_i **do**
  $d(X_i) := \infty$
$d(S) := 0$;
$\mathcal{Q}$:= an empty priority queue;
Insert S into $\mathcal{Q}$ with priority 0;
while $\mathcal{Q} \neq \emptyset$ **do**
  X :=ExtractMinFrom($\mathcal{Q}$);
  foreach *edge* (X, δ, Y) **do**
    $d(Y) := \min\{d(Y), d(X) + \delta\}$;
return *{D(S,X$_1$),..., D(S,X$_n$)}*

- Time complexity $O(m + n \log n)$ if using *Fibonacci Heap* for priority queue

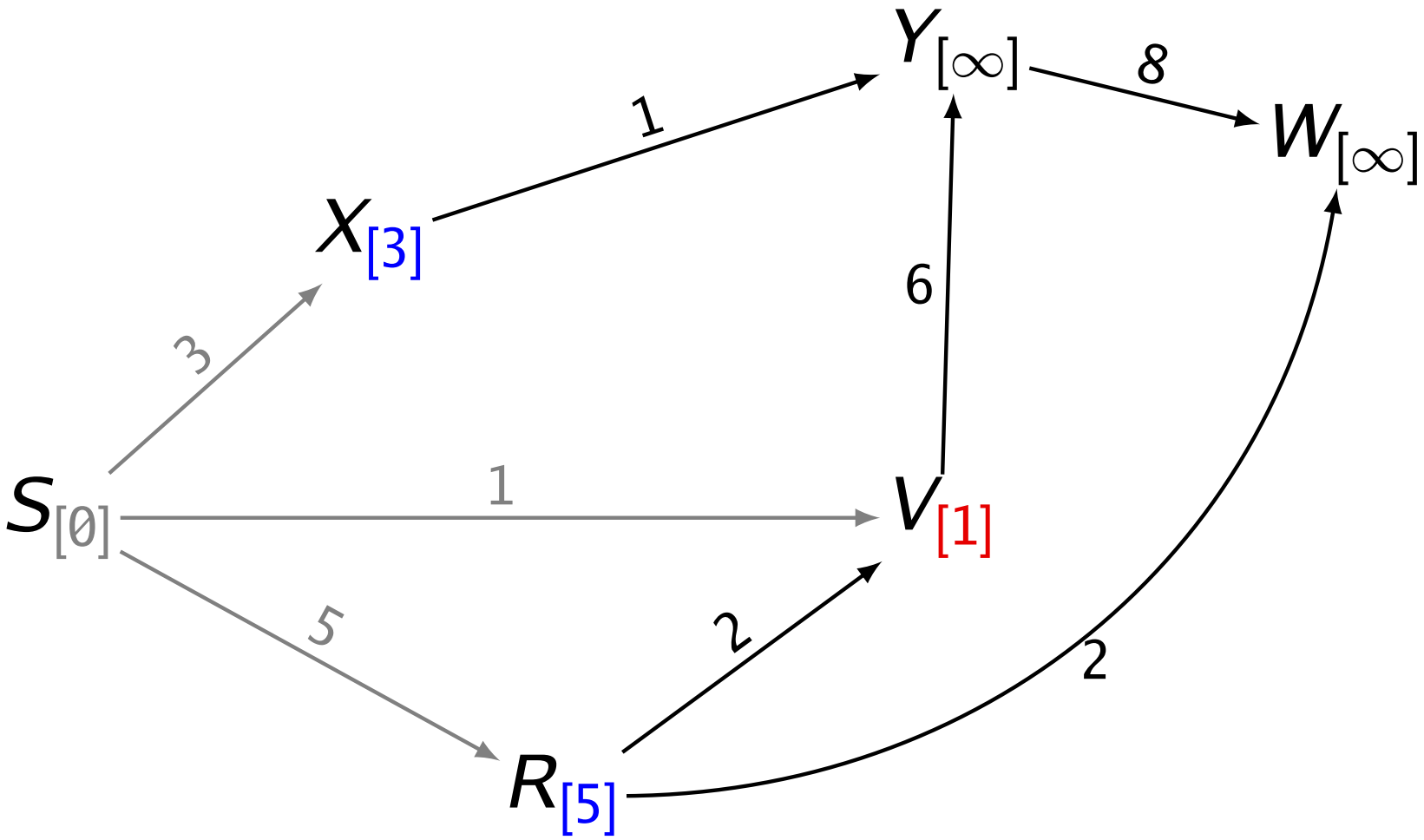
Dijkstra Example



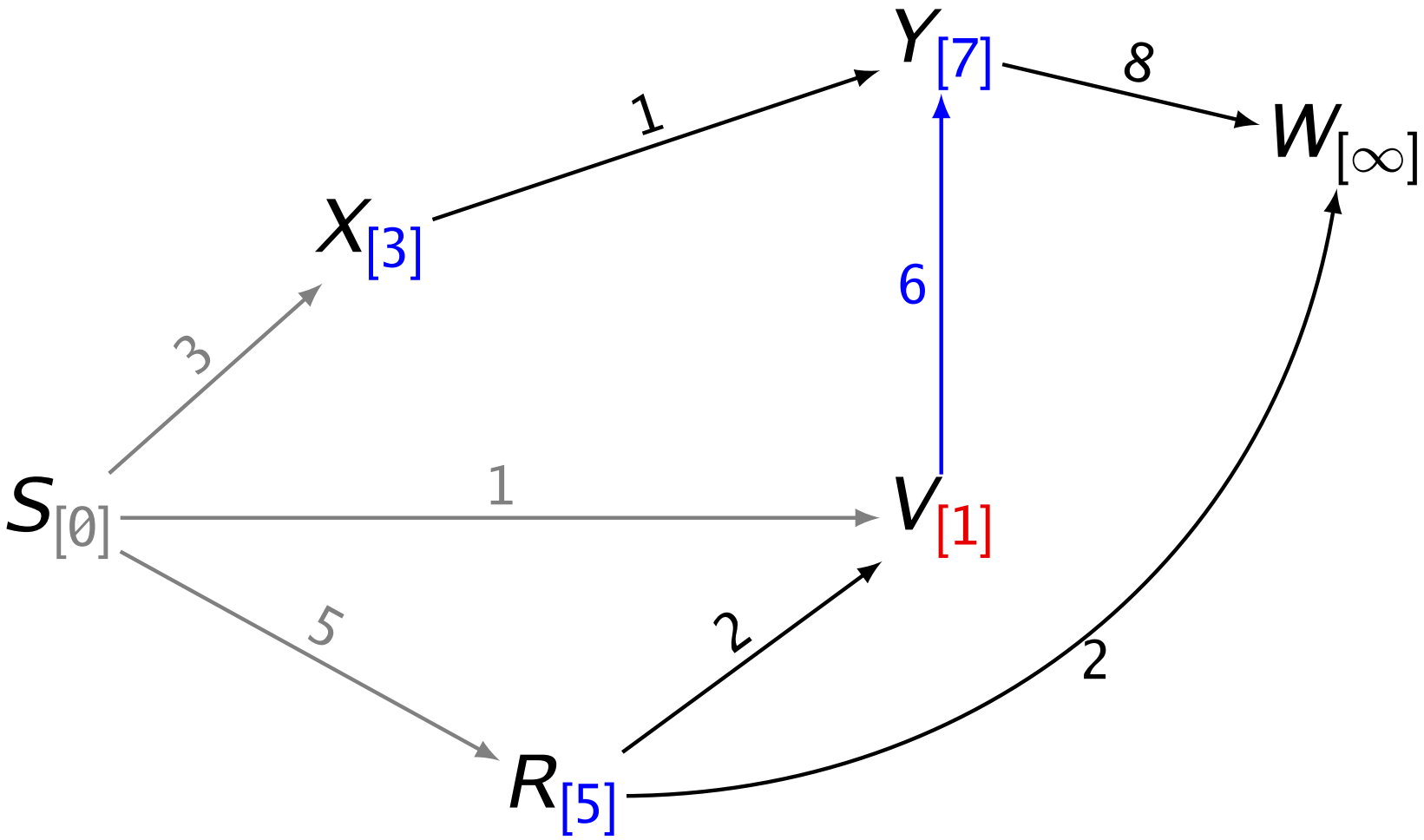
Dijkstra Example



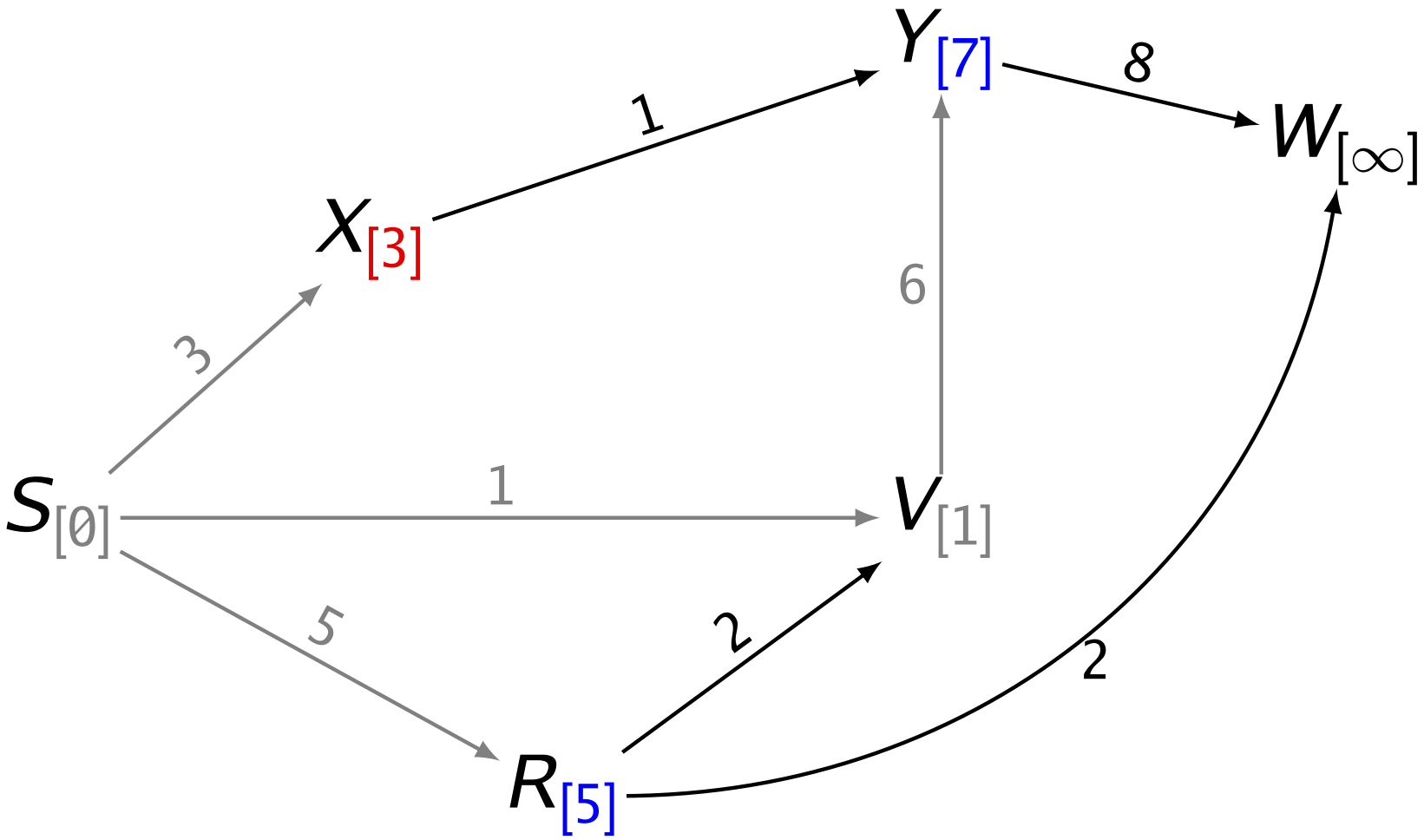
Dijkstra Example



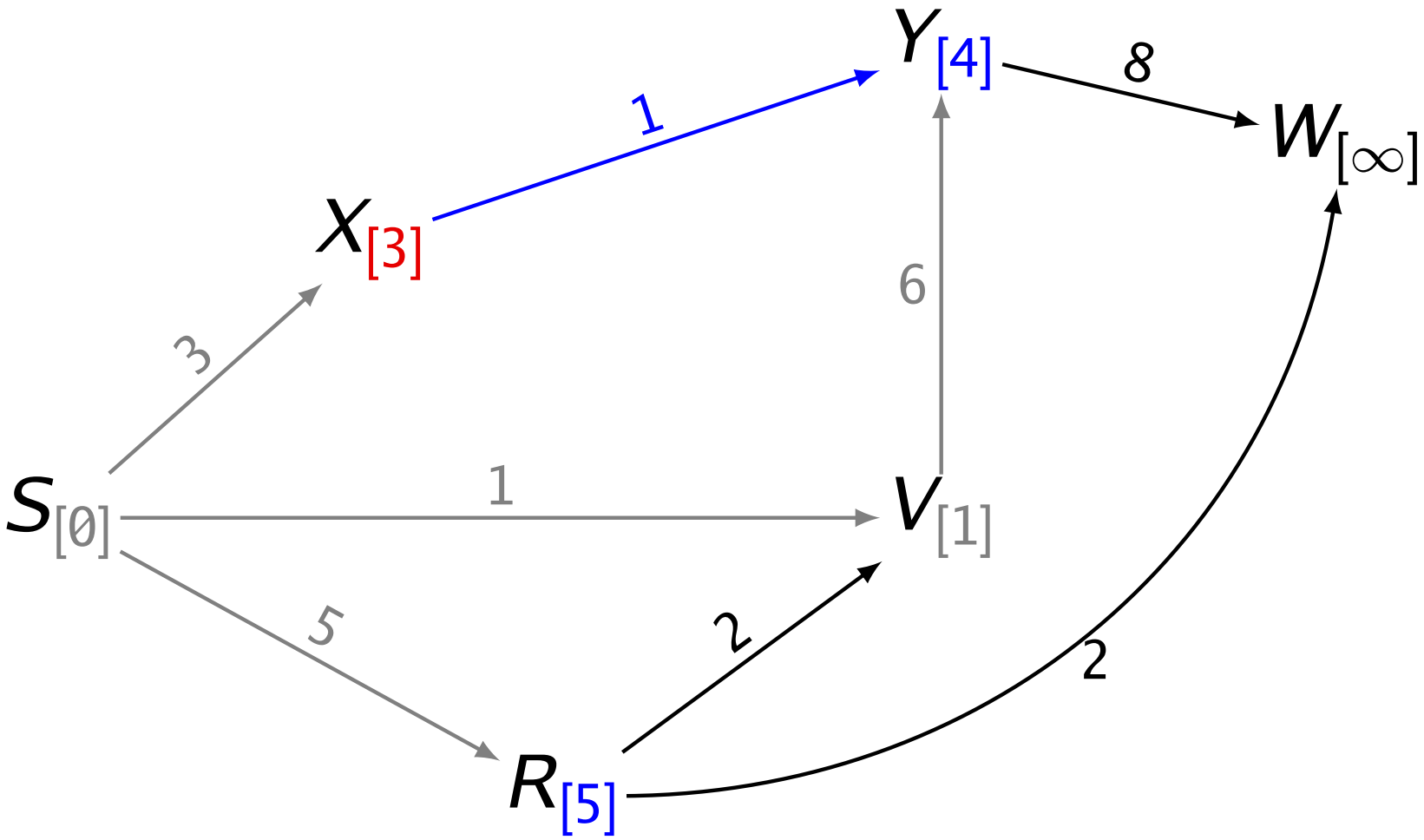
Dijkstra Example



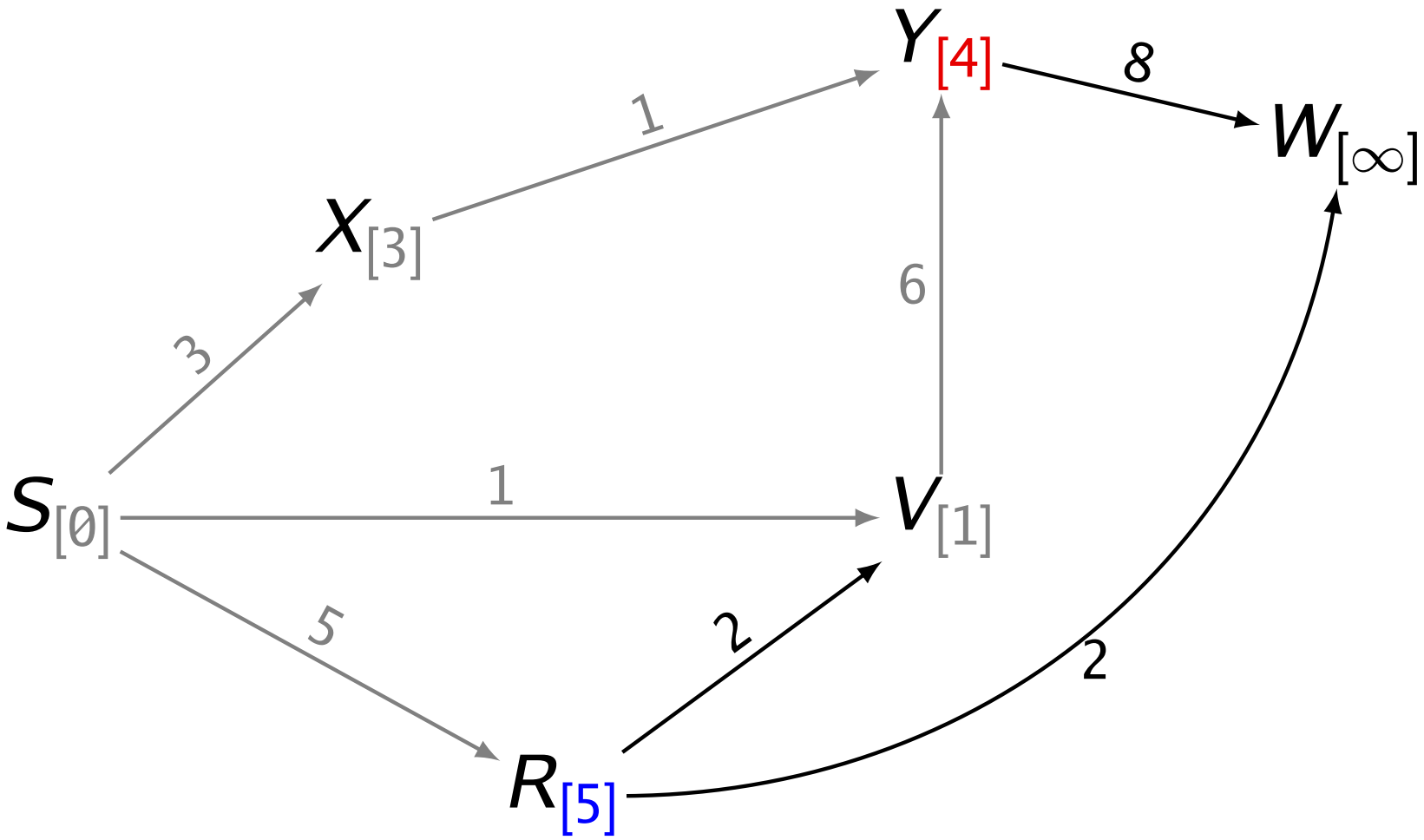
Dijkstra Example



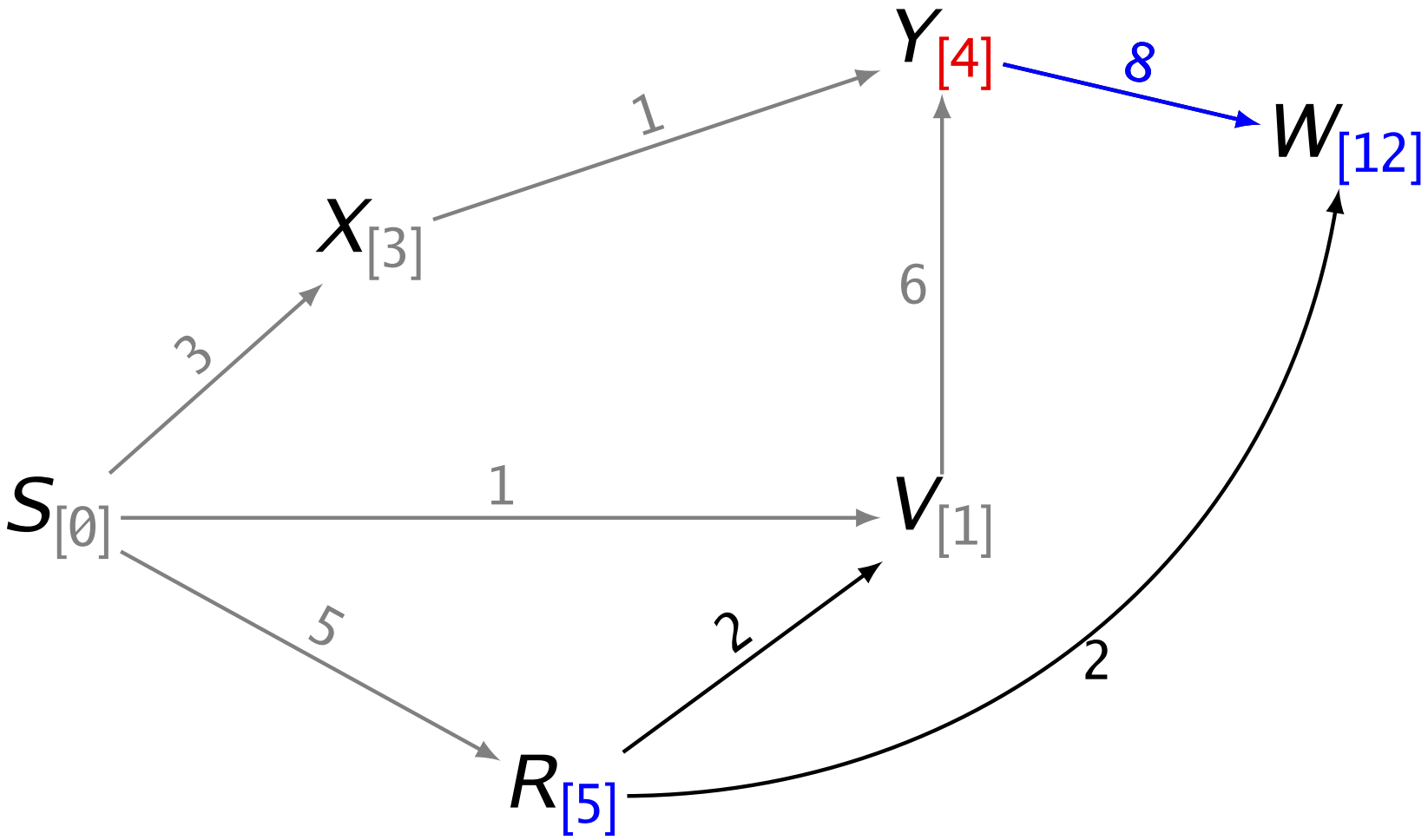
Dijkstra Example



Dijkstra Example



Dijkstra Example



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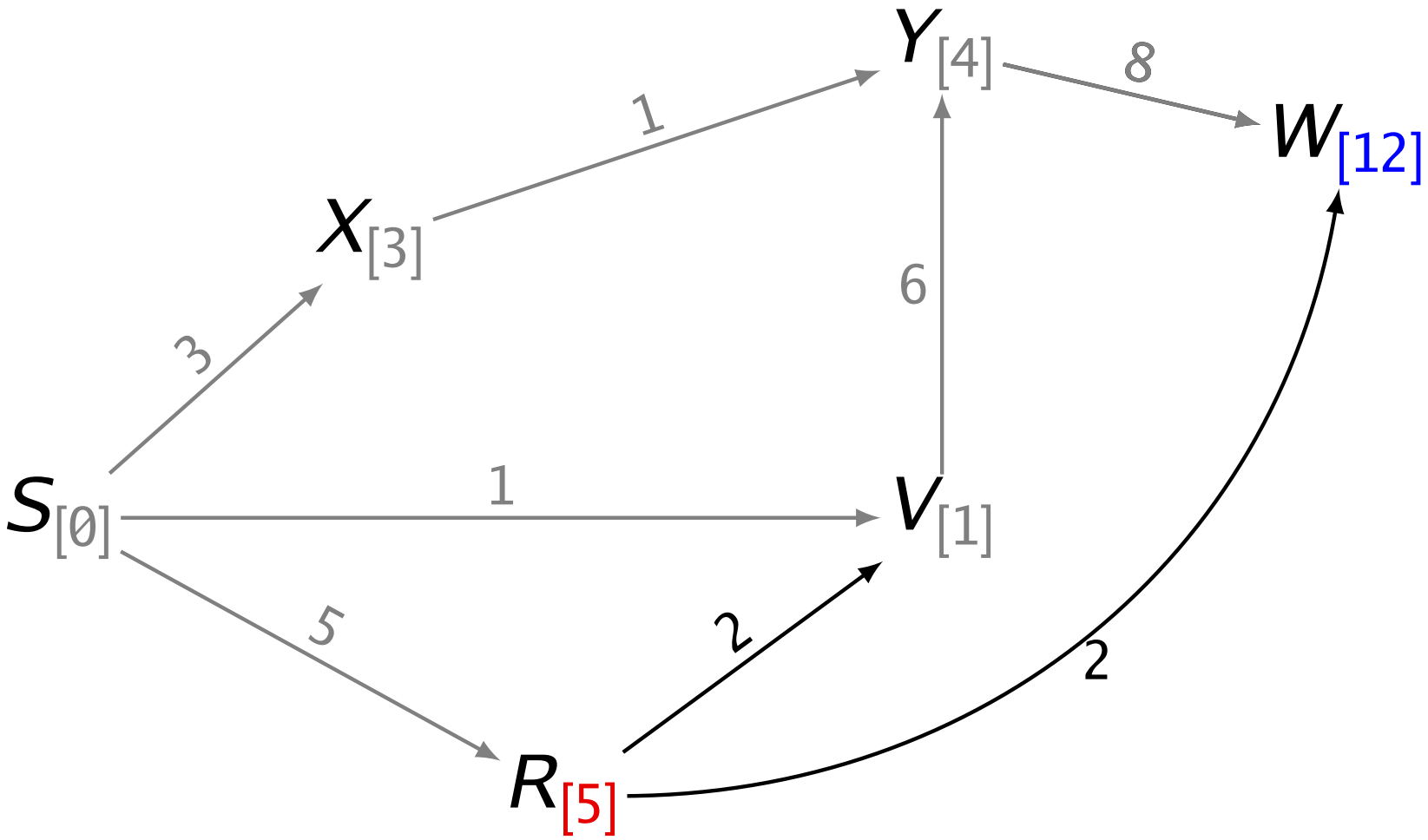

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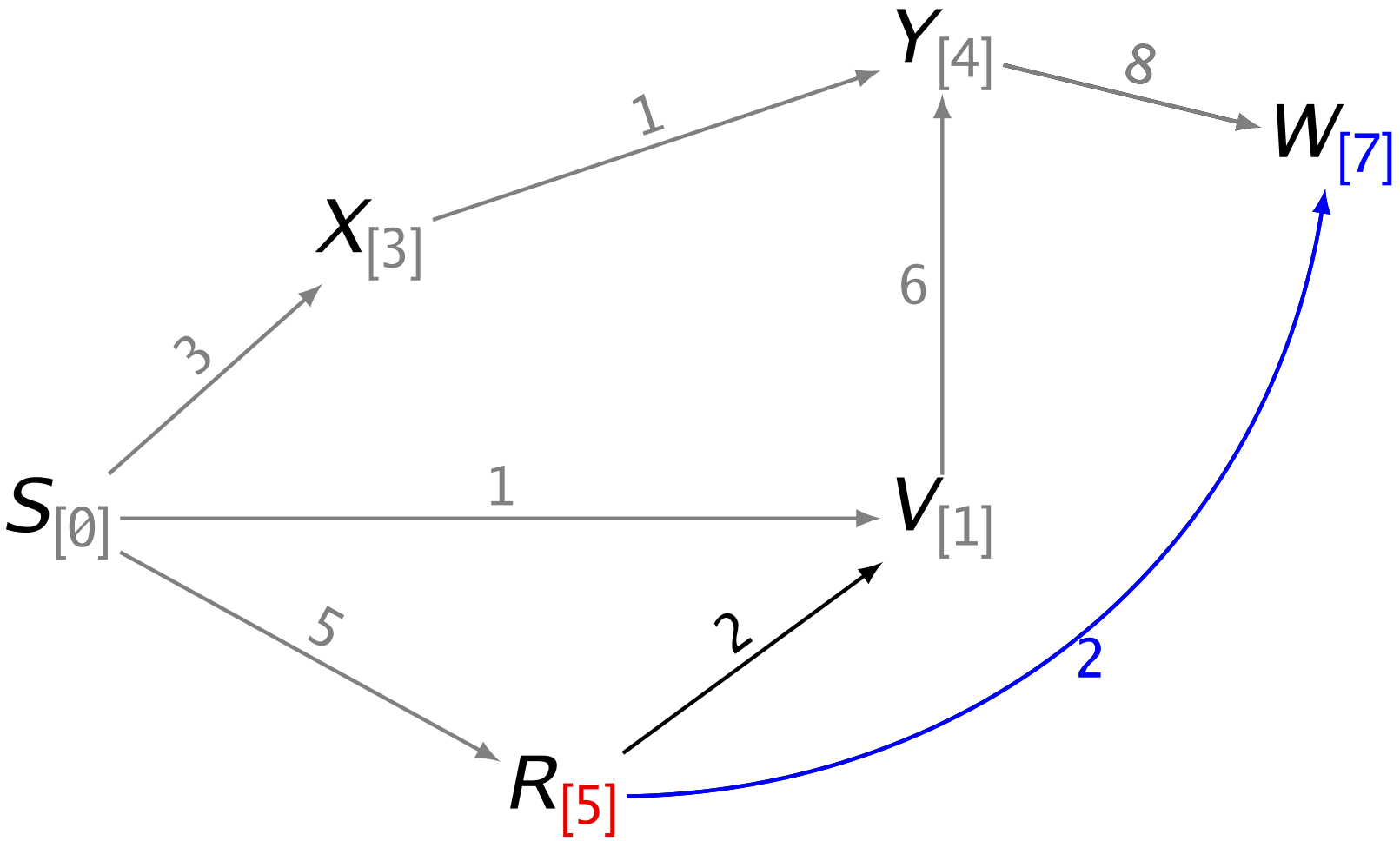
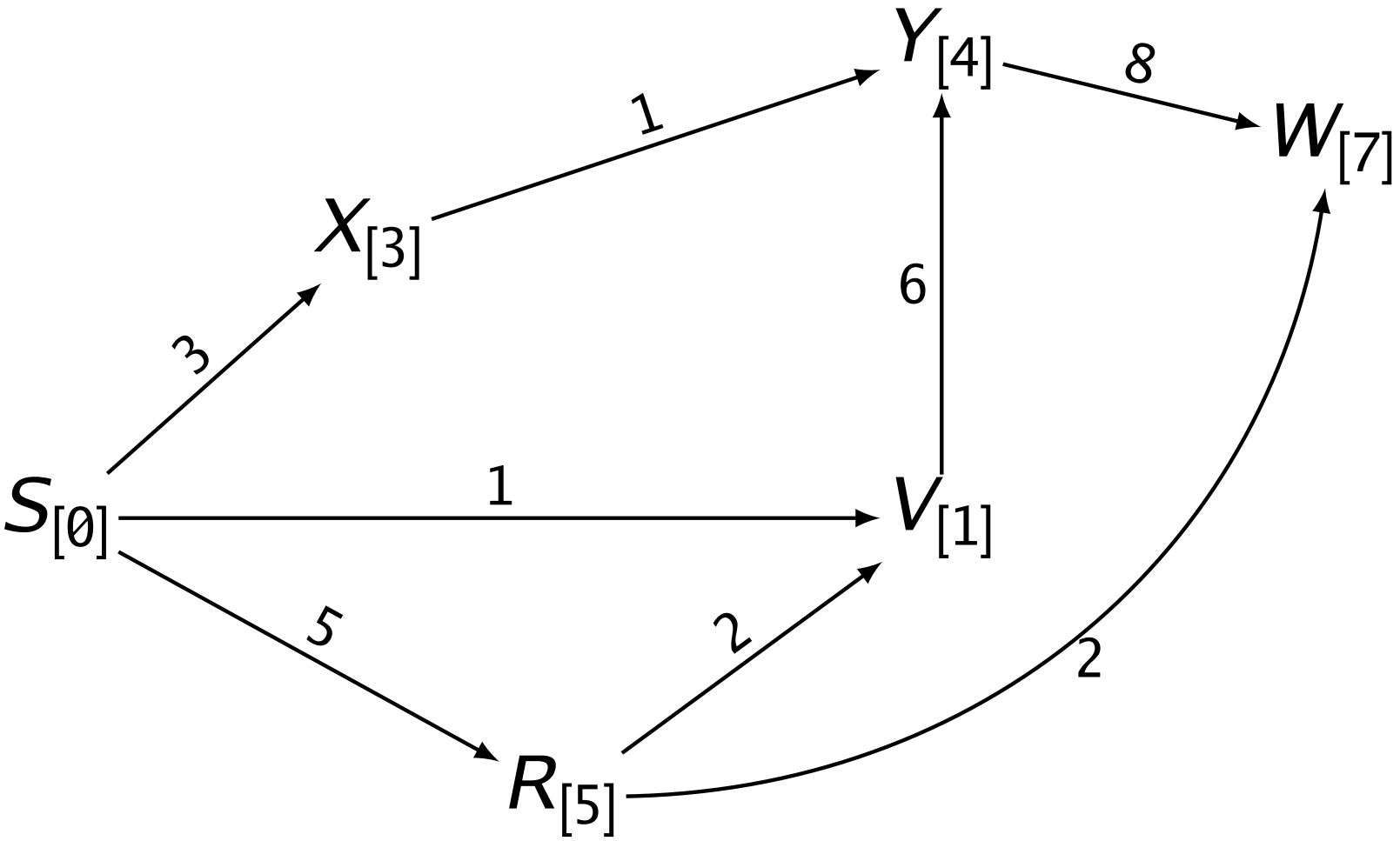


Dijkstra Example



Potential Functions & Re–weighted Graphs

- Let $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ be any consistent STN.
- Let $f : \mathcal{T} \to \mathbb{R}$ be any solution for $\mathcal{S}$.
  - Then $f(Y) - f(X) \leq \delta$ for each constraint $(Y - X \leq \delta) \in \mathcal{C}$
  - In other words, $0 \leq f(X) + \delta - f(Y)$
  - Let $\mathcal{C}' = \{(X, \delta', Y) \mid (X, \delta, Y) \in \mathcal{C}\}$, where $\delta' = f(X) + \delta - f(Y)$
  - Then $\mathcal{S}' = (\mathcal{T}, \mathcal{C}')$ has only non–negative edges.
  - Therefore, can use Dijkstra's SSSP algorithm on $\mathcal{S}'$

Johnson's Algorithm*

Algorithm 4: Johnson()

Input: An STN, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$
Output: $\mathcal{S}$ minimized if consistent, NO otherwise
Run Bellman–Ford SSSP with *new* source node S;
if *Bellman–Ford returns NO* **then return** *NO*;
$f(X) := D(S, X)$ for each $X \in \mathcal{T}$; // It is a solution for $\mathcal{S}$
foreach $(X, \delta, Y \in \mathcal{C})$ **do**
  $\delta' := f(X) + \delta - f(Y)$; // $\delta' \geq 0$
$\mathcal{S}' := (\mathcal{T}, \mathcal{C}')$; // Re–weighted graph based on δ'
foreach $X \in \mathcal{T}$ **do**
  Run Dijkstra on $\mathcal{S}'$ with X as source node; // Computes $\mathcal{D}'(X, Y)$ for all $Y \in \mathcal{T}$.
foreach $X, Y \in \mathcal{T}$ **do**
  $\mathcal{D}(X, Y) = -f(X) + \mathcal{D}'(X, Y) + f(Y)$; // Reverse the re–weighting to obtain $\mathcal{D}$ for $\mathcal{S}$

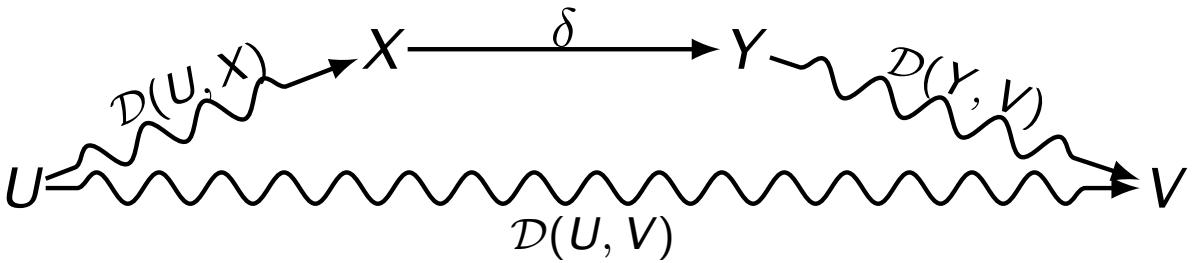
Time Complexity: $O(mn) + n * O(m + n \log n) = O(mn + n^2 \log n)$

*[Cormen et al., 2009]

Simple Temporal Network
Incrementally updating $\mathcal{D}$

- Given a consistent STN $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ with distance matrix $\mathcal{D}$.
- Insert new constraint $(Y - X \leq \delta)$.
- How to update $\mathcal{D}$?
  - Re–compute from scratch: $O(mn + n^2 \log n)$.
  - "Naïve" algorithm: $O(n^2)$: For each $U, V \in \mathcal{T}$,

$$\mathcal{D}(U, V) := \min\{\mathcal{D}(U, V),\ \mathcal{D}(U, X) + \delta + \mathcal{D}(Y, V)\}$$



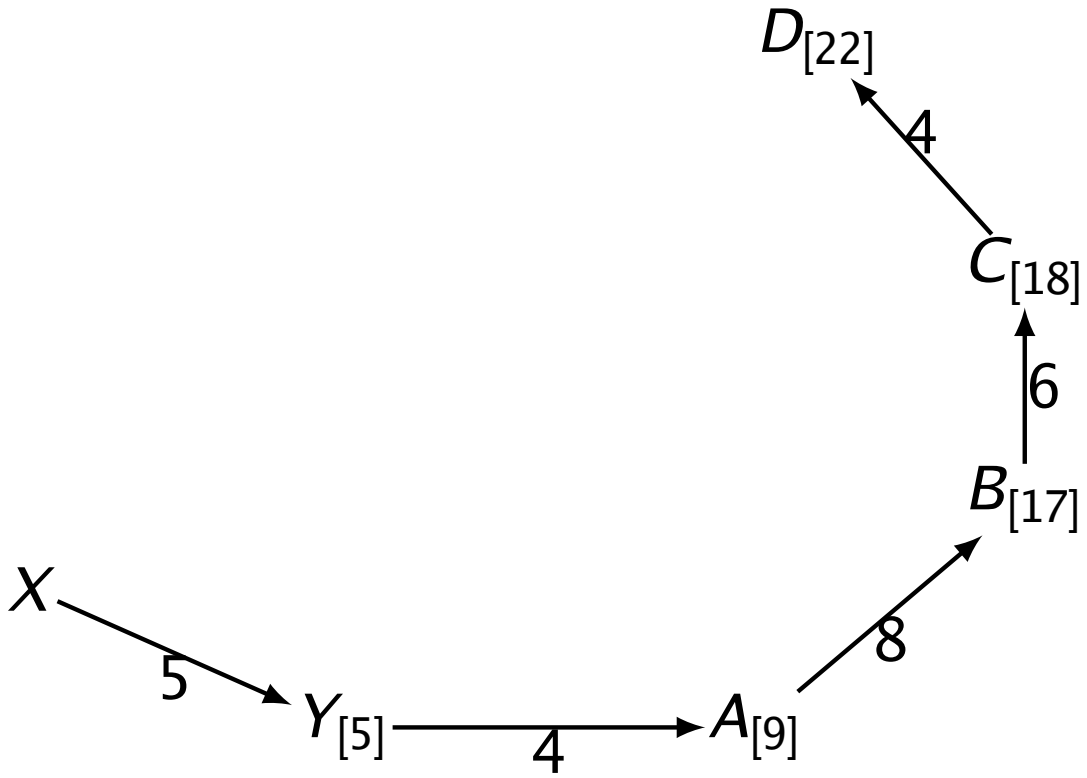
Simple Temporal Networks
Incremental Update Algorithm

- Propagate updates to $\mathcal{D}$ along edges in graph
- Only propagate along *tight* edges; // $(Y - X \leq \delta)$ is tight iff $\mathcal{D}(X, Y) = \delta$
- Phase I: propagate forward
- Phase II: propagate backward
- Checks no more than $b \cdot \Delta$ cells of $\mathcal{D}$, where:
  - $\Delta =$ number of cells needing updating;
  - $b =$ max number of edges incident to any node.

* [Even and Gazit, 1985, Ramalingam and Reps, 1996, Rohnert, 1985]

Incremental Update Algorithm

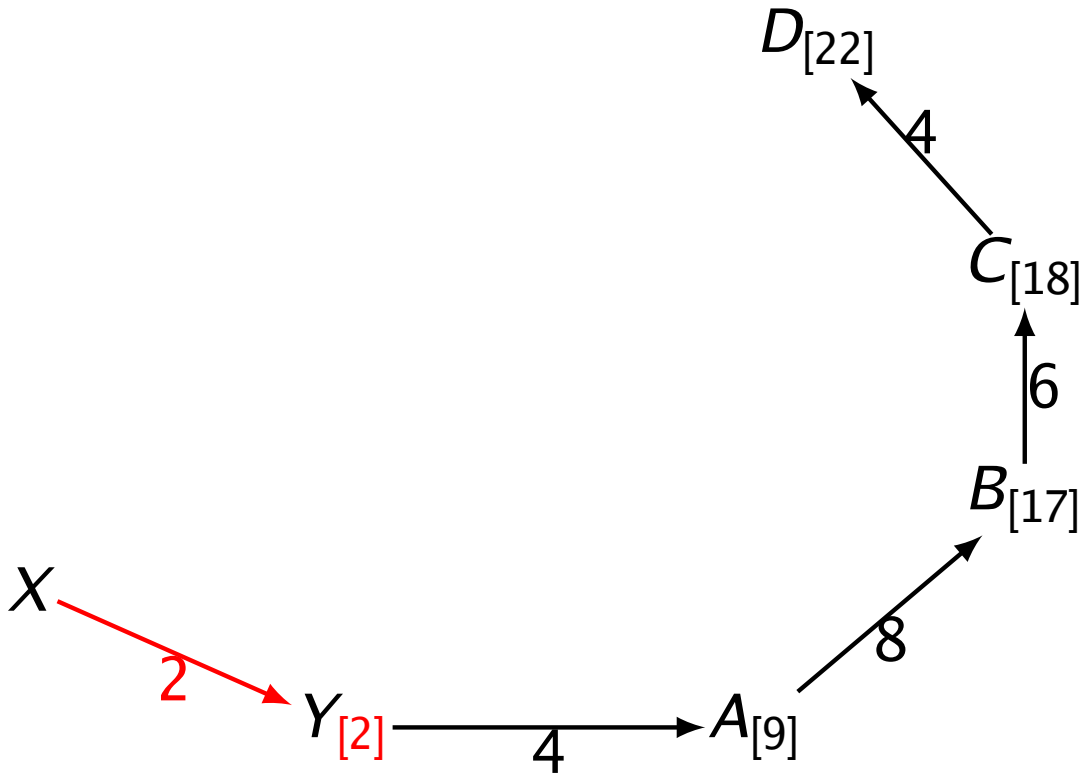
Propagate forward from XY as long as new values obtained



Numbers in brackets are current values of $\mathcal{D}(X, \_)$.

Incremental Update Algorithm

Propagate forward from XY as long as new values obtained



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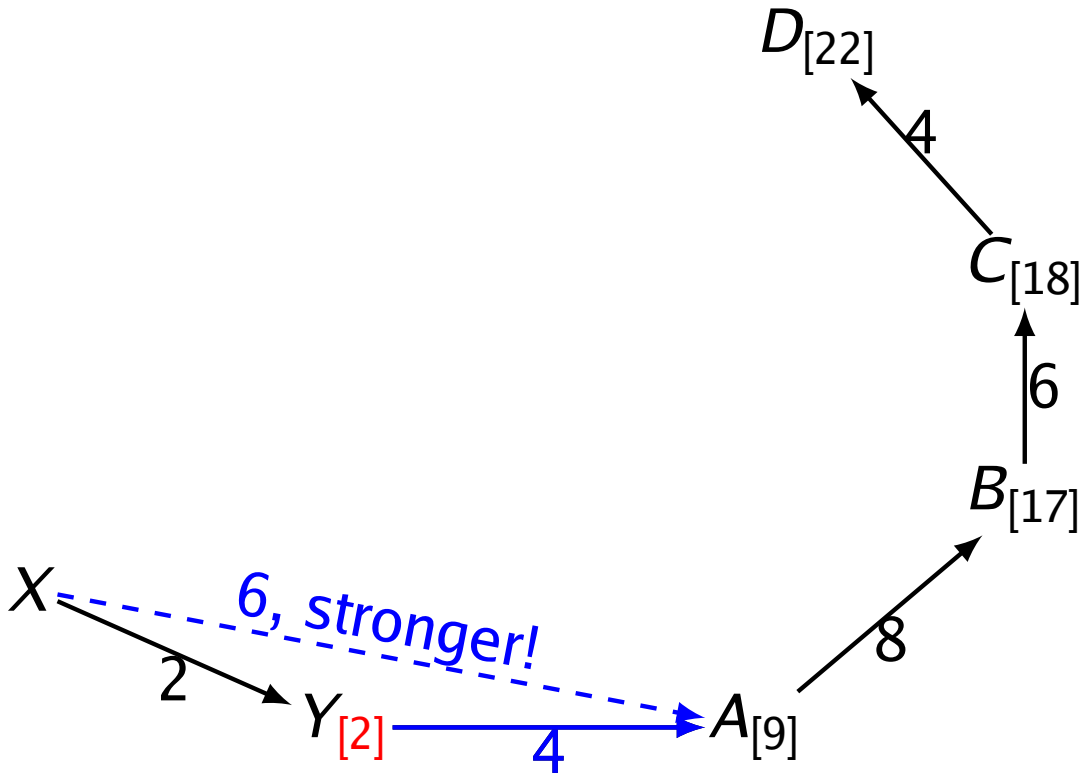
Incremental Update Algorithm

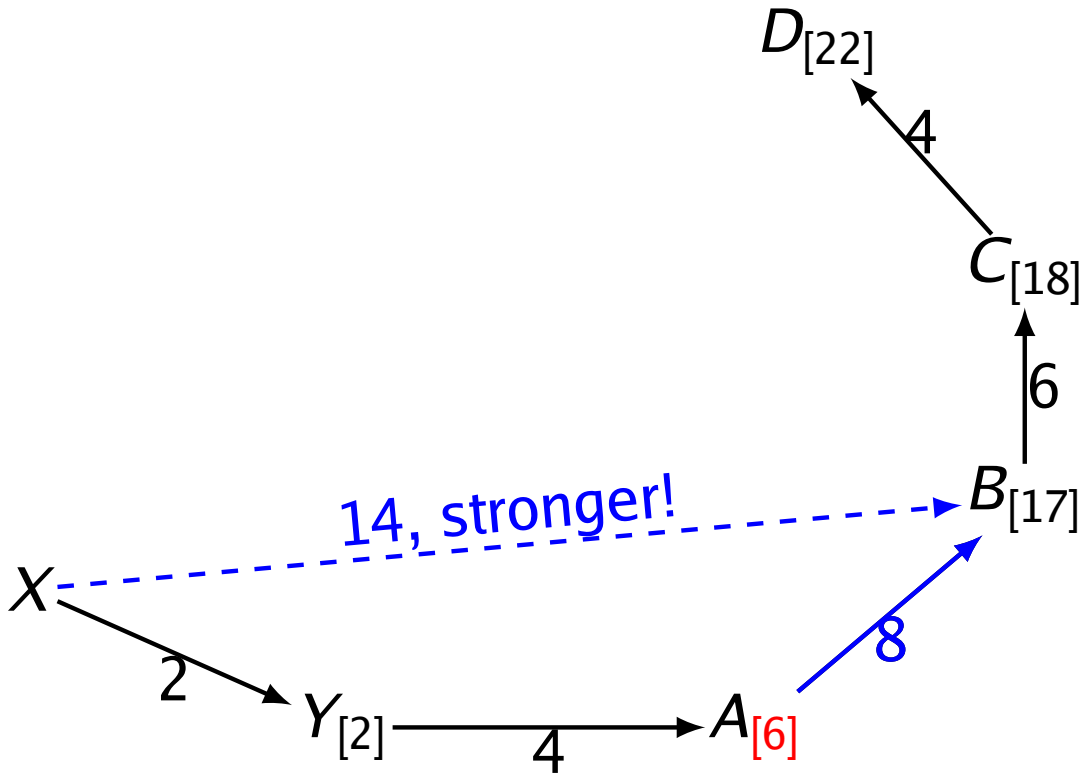
Propagate forward from XY as long as new values obtained



Numbers in brackets are current values of $\mathcal{D}(X, \_)$.

Incremental Update Algorithm

Propagate forward from XY as long as new values obtained



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Incremental Update Algorithm

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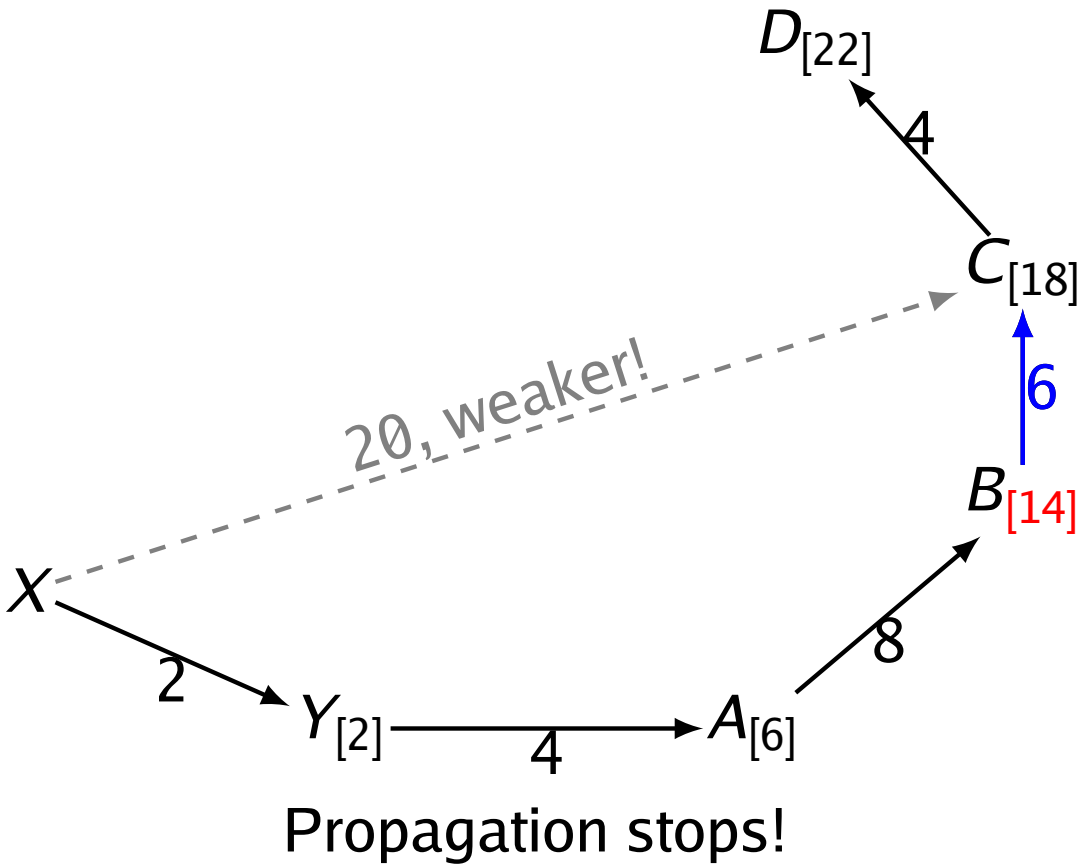
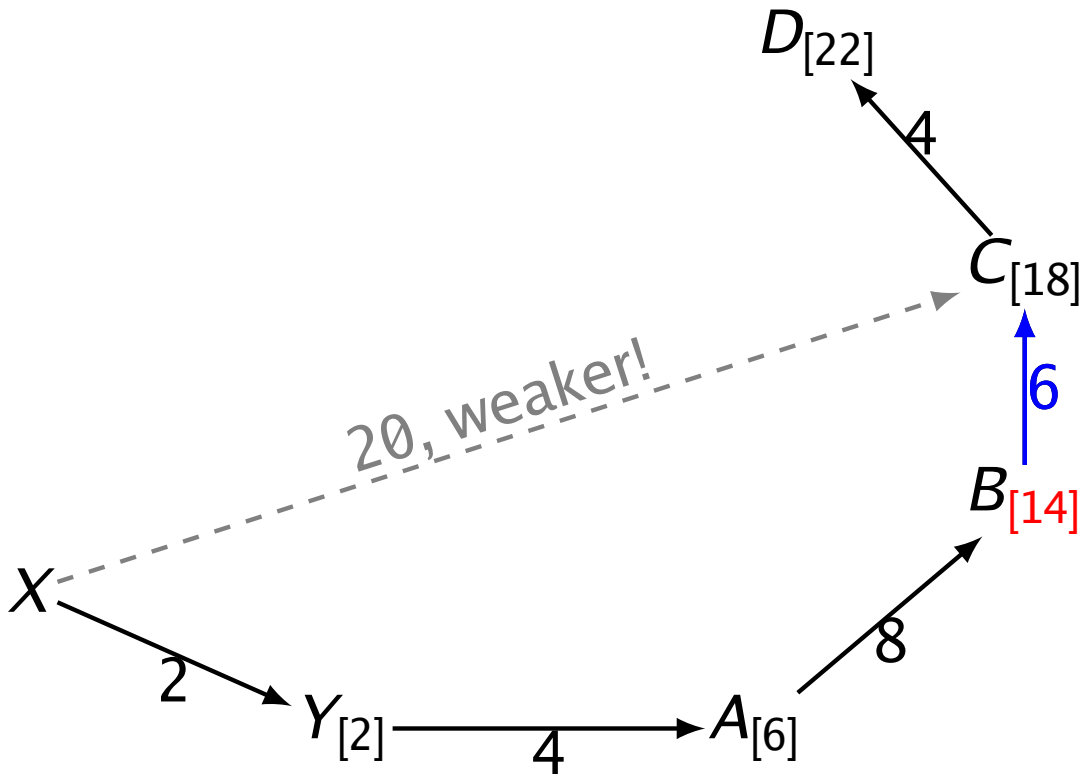


Incremental Update Algorithm

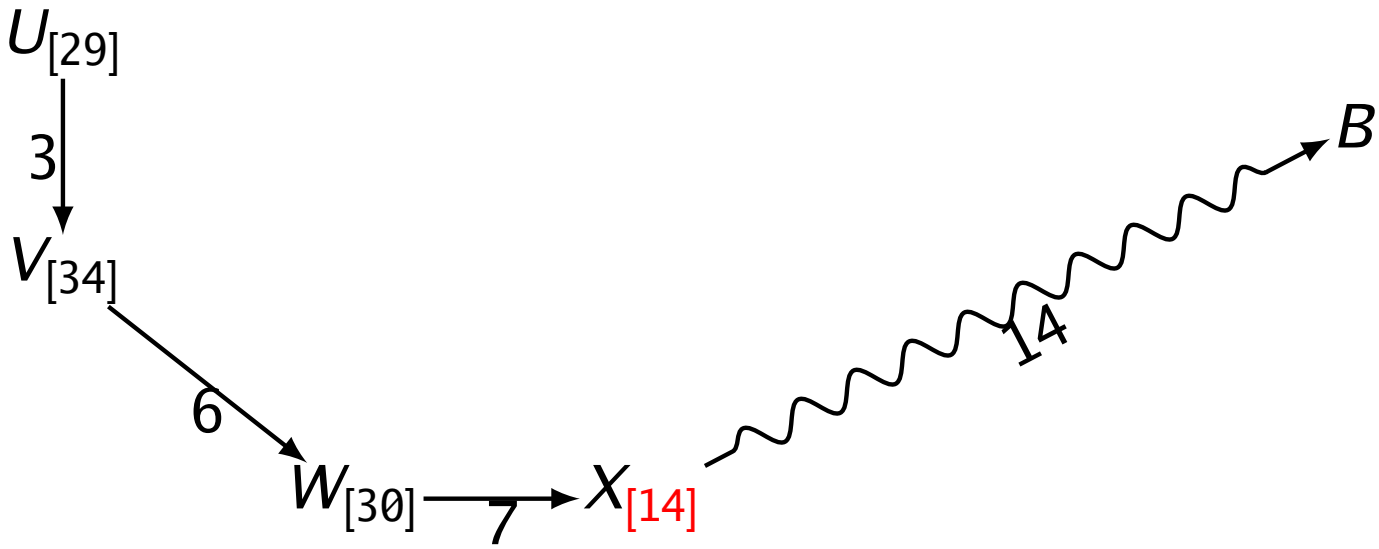
Propagate forward from XY as long as new values obtained



Forward propagation done along this path.

Incremental Update Algorithm

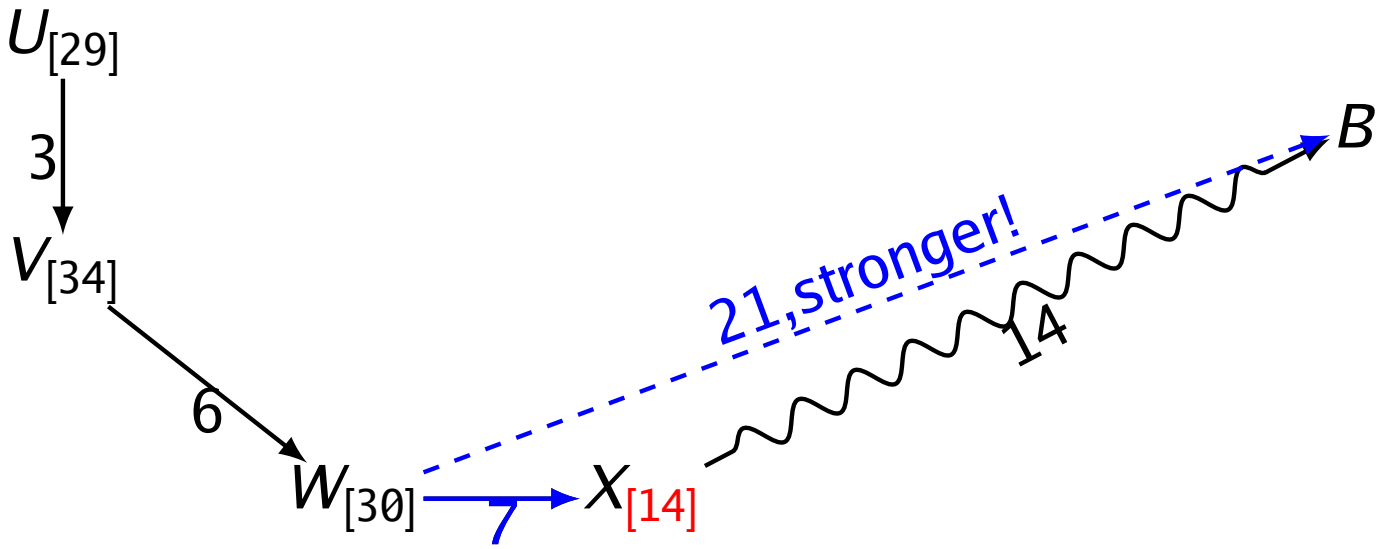
For each entry $\mathcal{D}(X, \_)$ that was updated, propagate backward from X



Numbers in brackets are $\mathcal{D}(\_, B)$ values.

Incremental Update Algorithm

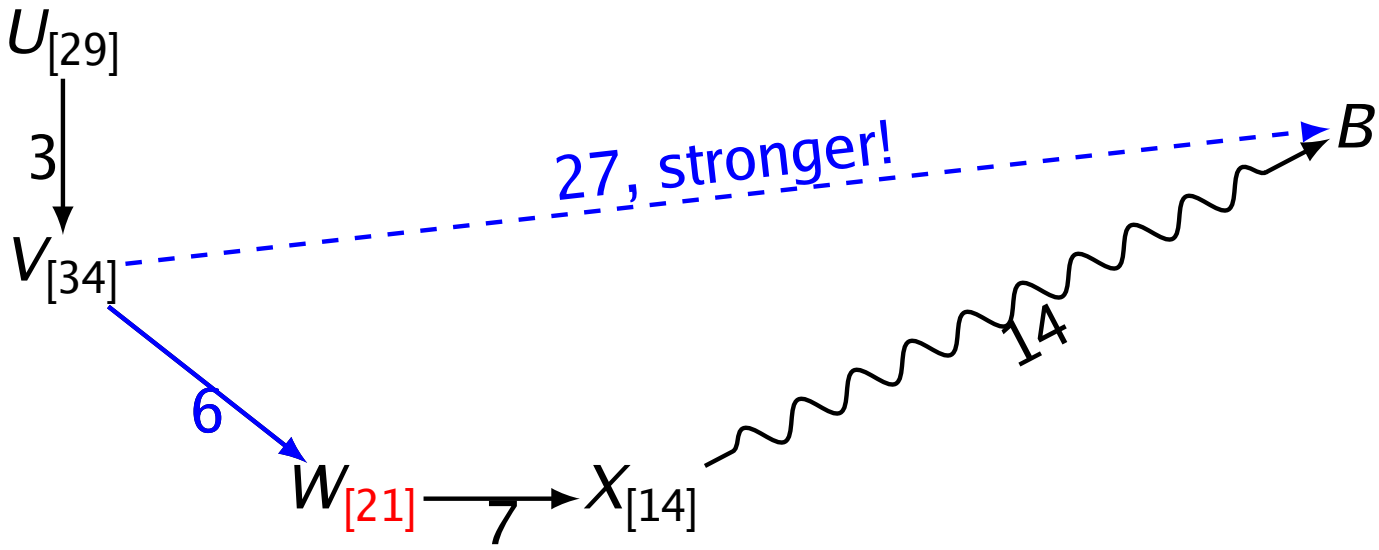
For each entry $\mathcal{D}(X, \_)$ that was updated, propagate backward from X



Numbers in brackets are $\mathcal{D}(\_, B)$ values.

Incremental Update Algorithm

For each entry $\mathcal{D}(X, \_)$ that was updated, propagate backward from X



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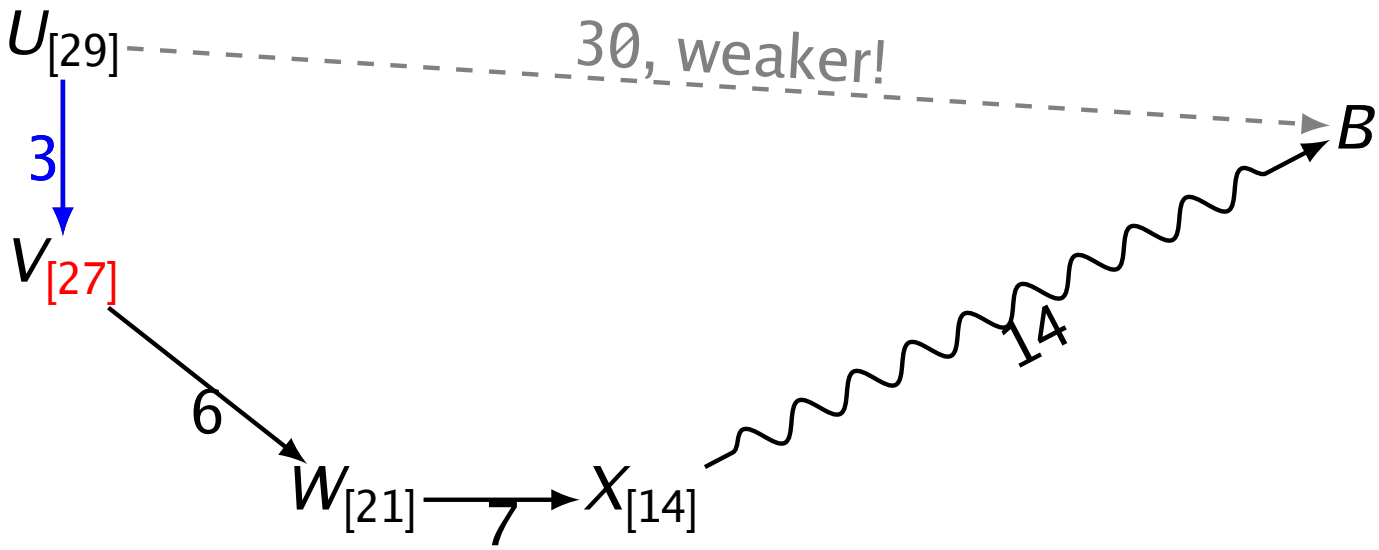
Incremental Update Algorithm

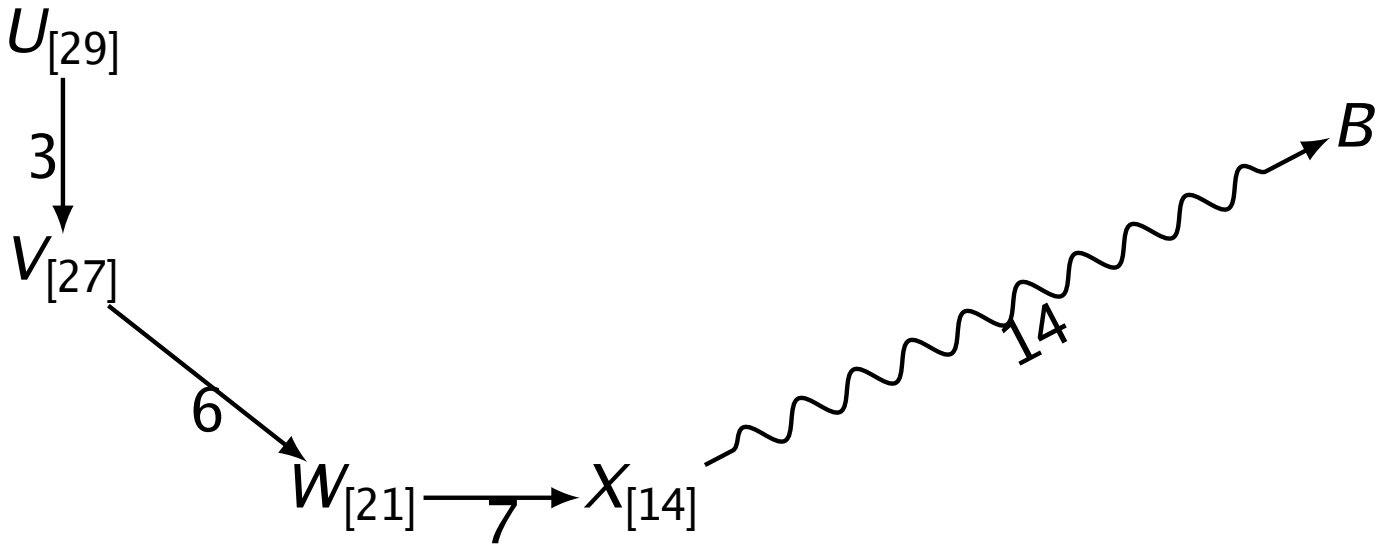
For each entry $\mathcal{D}(X, \_)$ that was updated, propagate backward from X



No further updates along this path.

Incremental Update Algorithm

For each entry $\mathcal{D}(X, \_)$ that was updated, propagate backward from X



No further updates along this path.

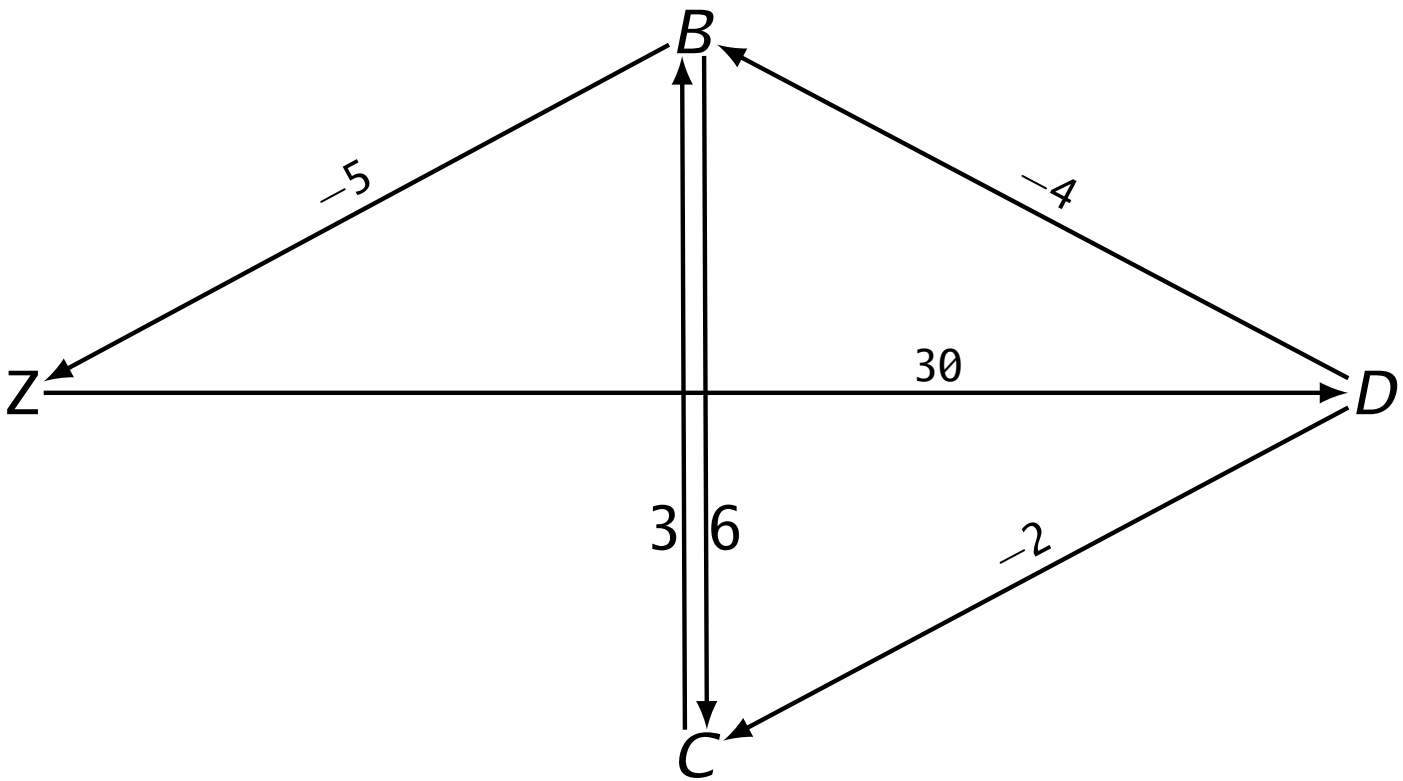
Simple Temporal Network
Incremental Consistency

Verifying consistency of an STN after inserting, weakening or deleting a constraint is less expensive than fully updating the distance matrix.*

- Algorithm maintains/updates a solution to the STN.
- After inserting a new constraint (or strengthening an existing one), can verify consistency in $O(m + n \log n)$ time.
- After deleting or weakening a constraint, only need constant time, because the same solution will work for the modified STN.

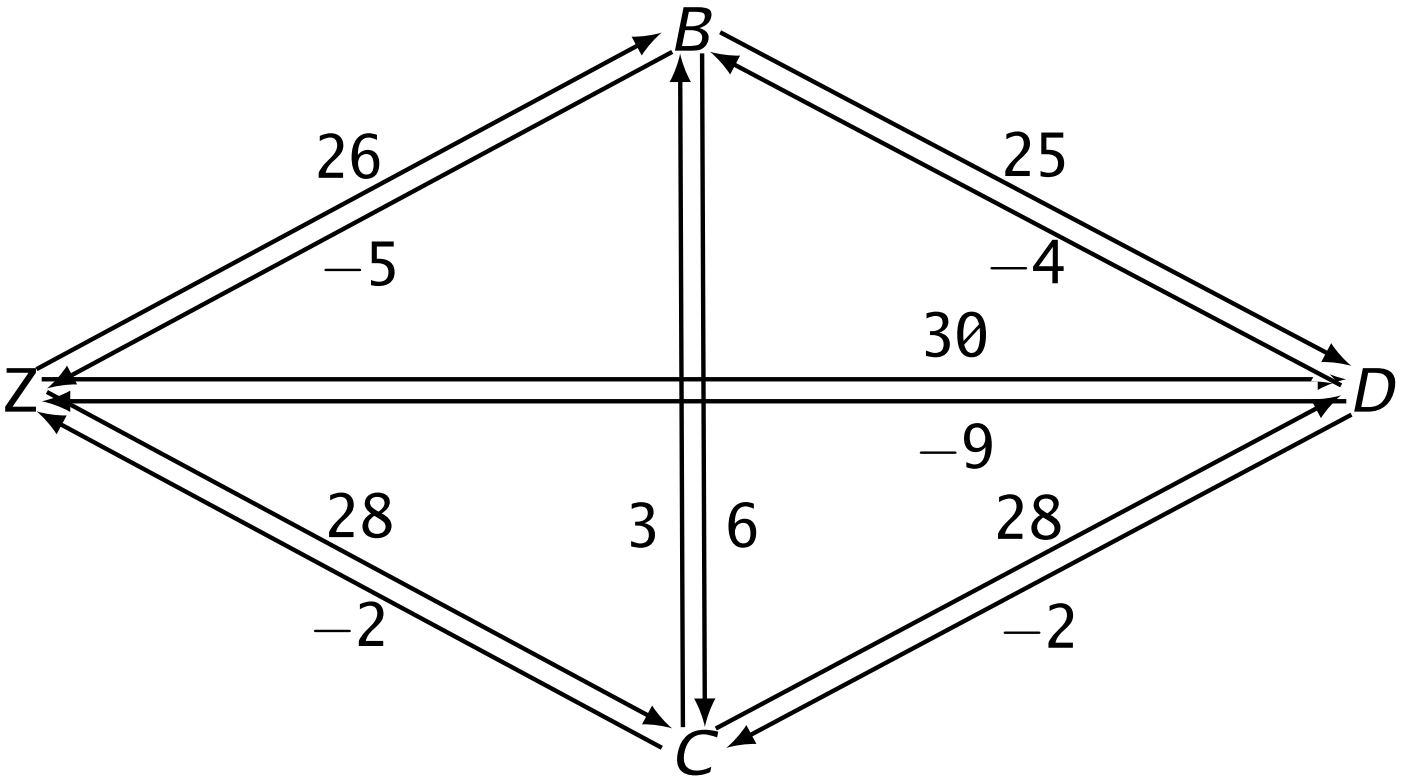
* [Ramalingam et al., 1999]

Simple Temporal Network
Sample STN



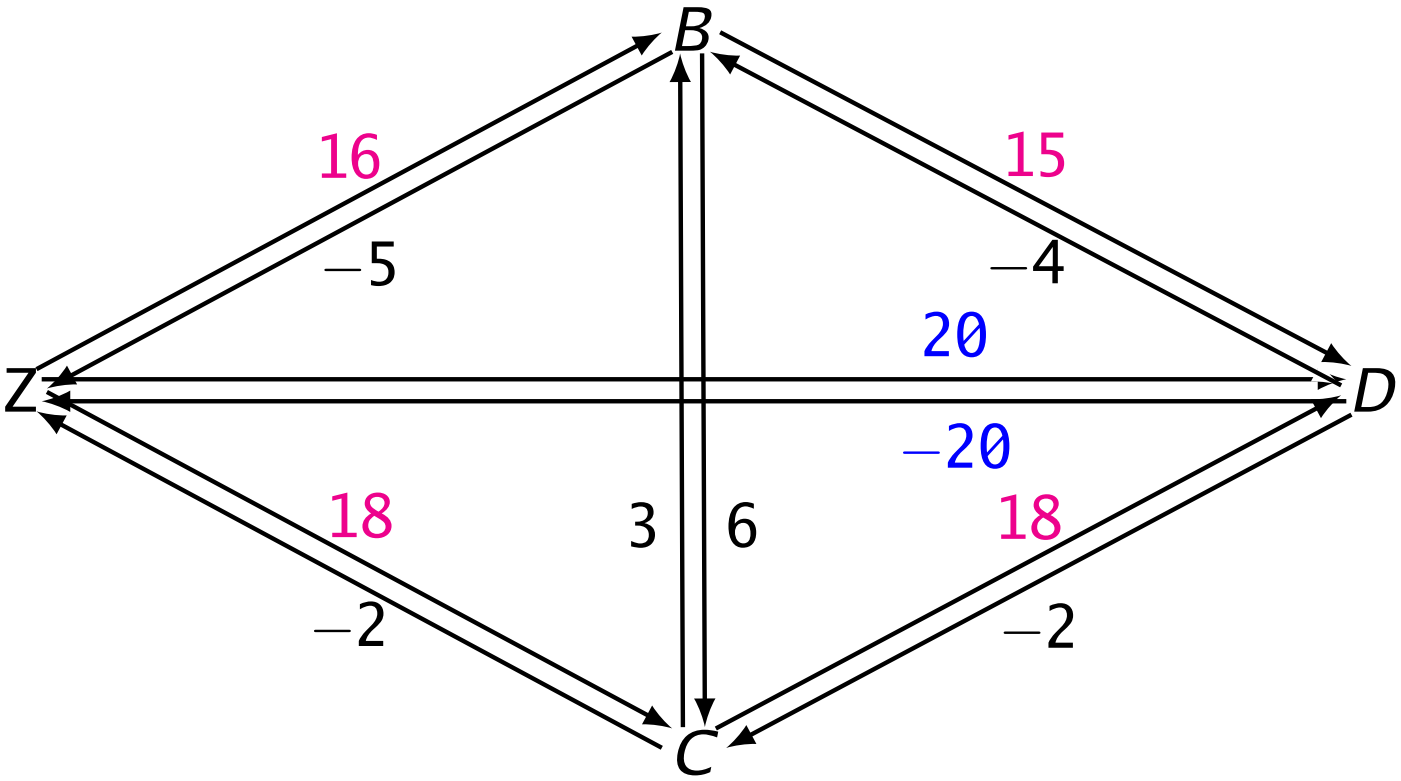
Simple Temporal Network
"Solving" Sample STN

First, form APSP graph (equiv. compute $\mathcal{D}$).



Time Windows: $B \in [5, 26]$, $C \in [2, 28]$, $D \in [9, 30]$

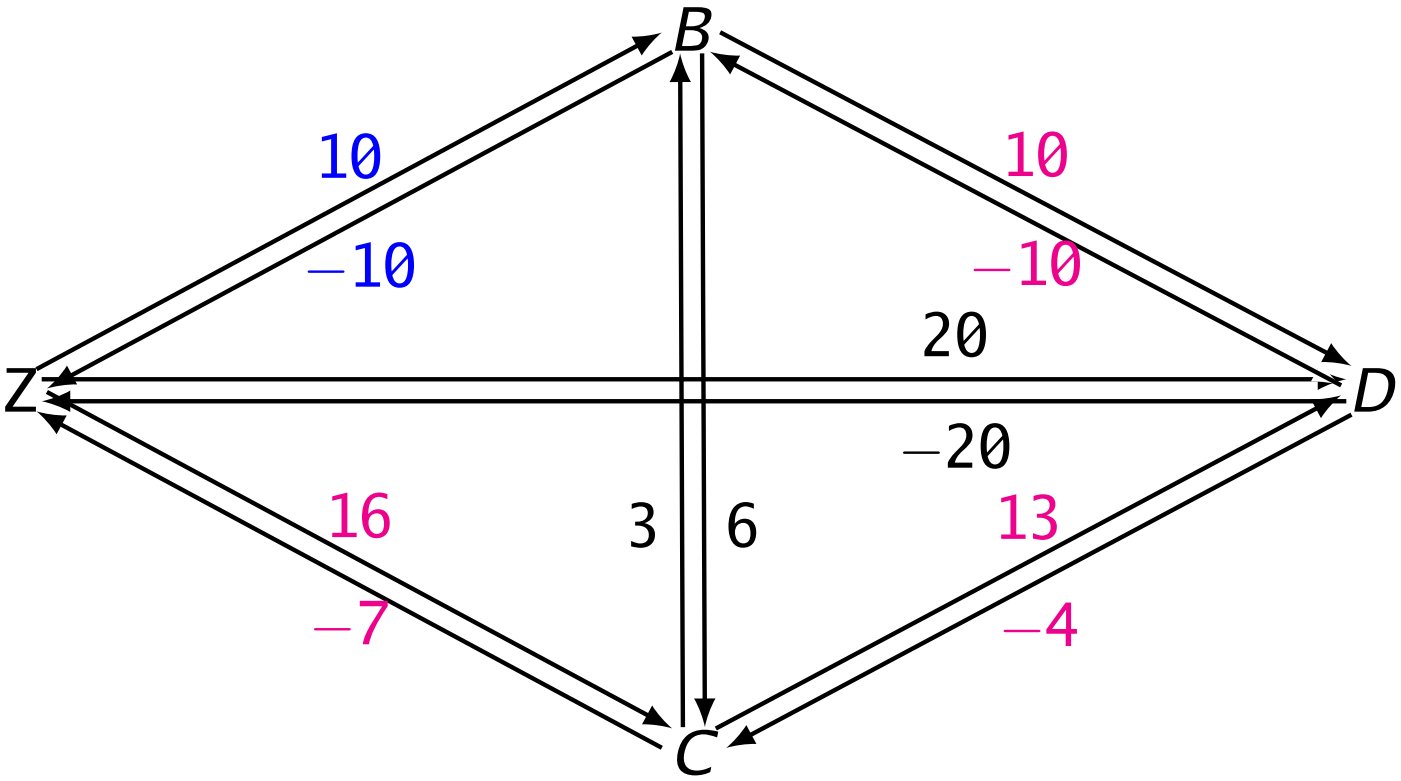
Next, select $D = 20$; and update APSP graph:



Remaining Time Windows: $B \in [5, 16]$, $C \in [2, 18]$

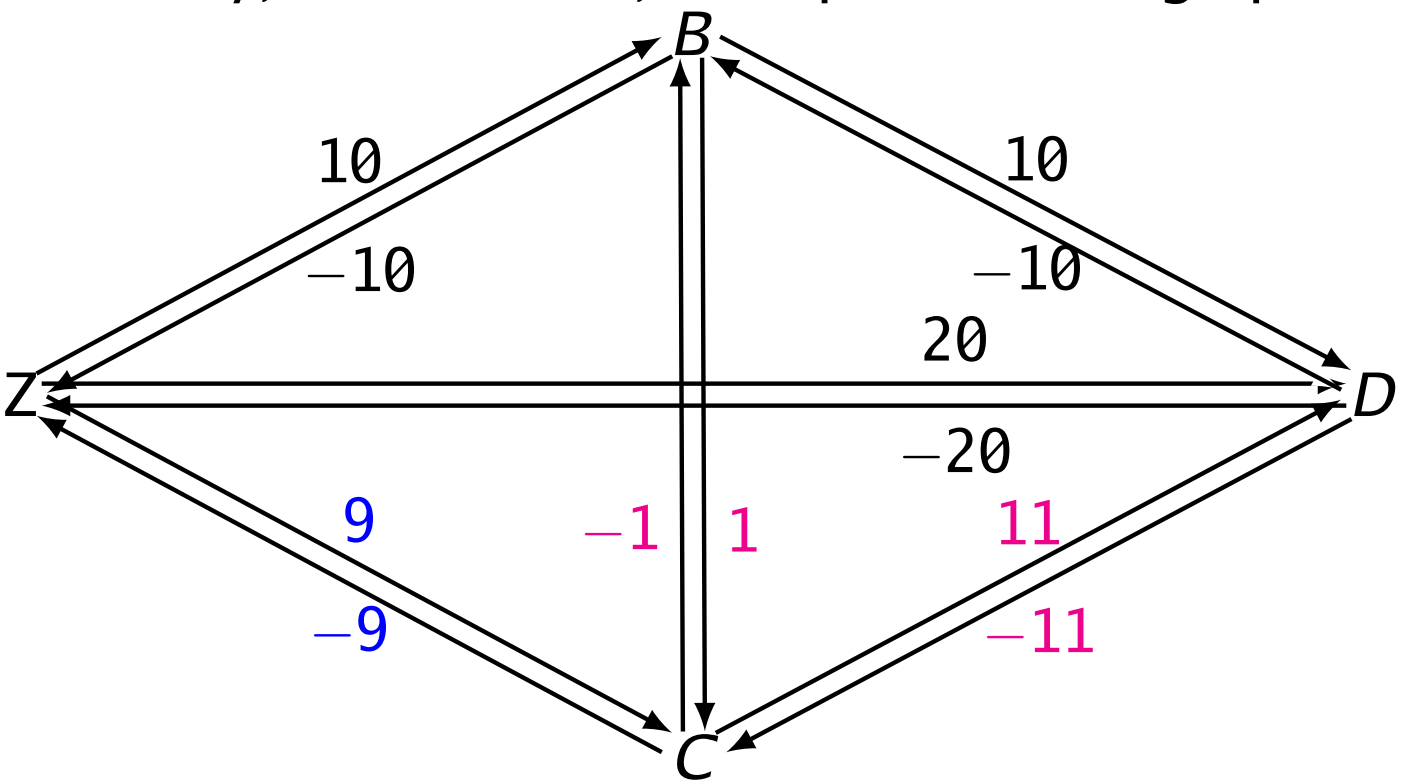
Simple Temporal Network
"Solving" Sample STN

Next, select $B = 10$; and update APSP graph:



Remaining Time Windows: $C \in [7, 16]$

Finally, select $C = 9$; and update APSP graph:



Easy to verify that this is a solution.

Simple Temporal Network
Problems with "Solving" an STN

- May need to go back in time:
  Pick $D = 20$, then after updating, pick $B = 10$
  (i.e., no relationship to real–time execution)
- Expensive to update $\mathcal{D}$

Simple Temporal Network
Executing an STN in real time

- Only executed enabled time–points: those having no negative edges to unexecuted time–points.
- Focus updating on entries involving Z: reduces cost to linear time per update, $O(n^2)$ overall.*
- Alternatively, prior to execution, transform STN into dispatchable form in $O(n^2 \log n + nm)$ time; then during execution, only need to propagate bounds to *neighboring* time–points.†

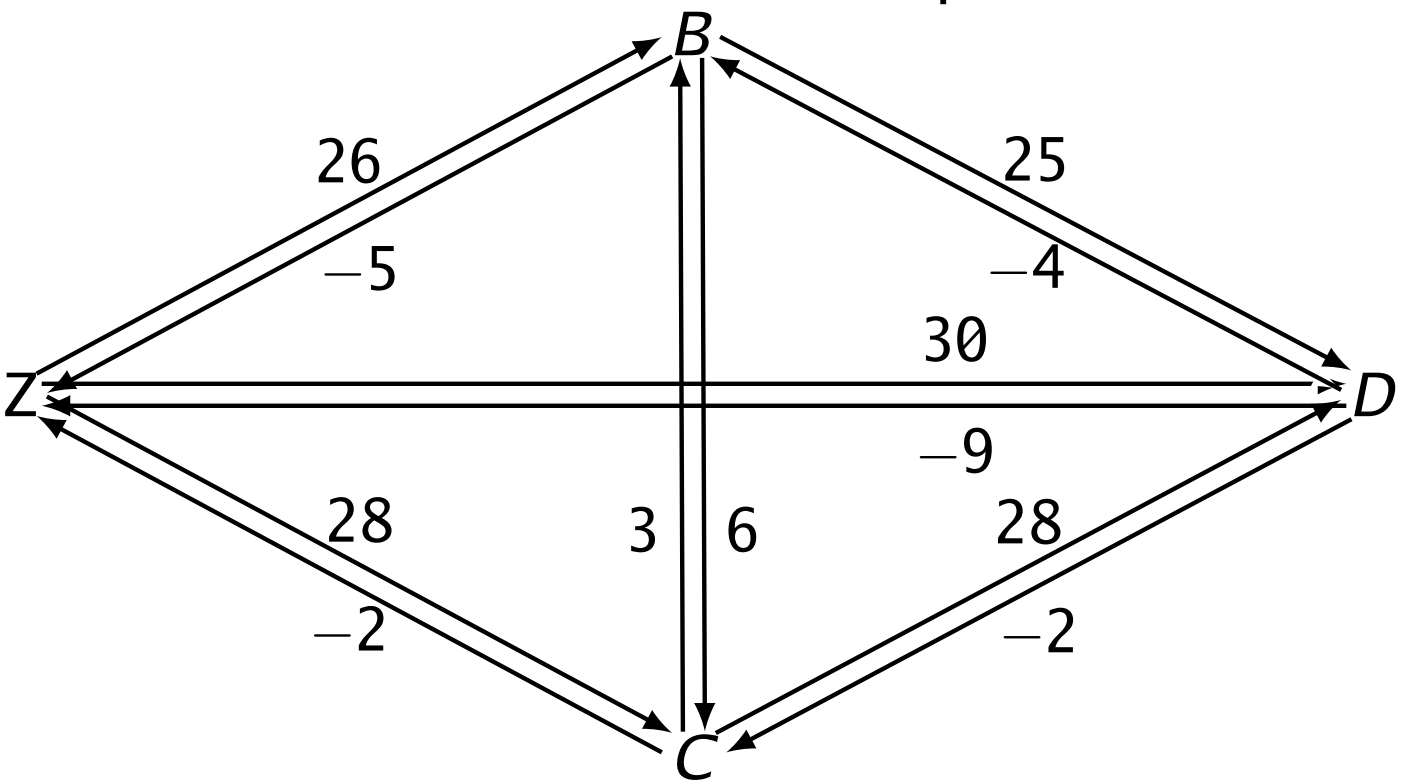
*[Hunsberger, 2008]; †[Muscettola et al., 1998], †[Tsamardinos et al., 1998]

An STN $\mathcal{S}$ is dispatchable if the following algorithm necessarily successfully executes $\mathcal{S}$:

Algorithm 5: ExecuteDispatchableNetwork($\mathcal{G} = (\mathcal{T}, \mathcal{E})$)

$t := 0$; // current time
$\mathcal{X} := \{\}$; // executed nodes
$\mathbf{E} := \{Z\}$; // currently enabled nodes
while $\mathcal{X} \neq \mathcal{T}$ **do**
  Pick any $X \in \mathbf{E}$ such that t is in X's time window;
  $X := t$, $\mathcal{X} := \mathcal{X} \cup \{X\}$;
  Propagate $t \leq X \leq t$ to X's immediate neighbors;
  $\mathbf{E} := \mathbf{E} \cup \{$all time–points Y s.t. all non–positive edges emanating from Y have a destination in $\mathcal{X}\}$;
  Wait until t has advanced to some time in $[\min\{lb(W) \mid W \in \mathbf{E}\}, \min\{ub(W) \mid W \in \mathbf{E}\}]$;

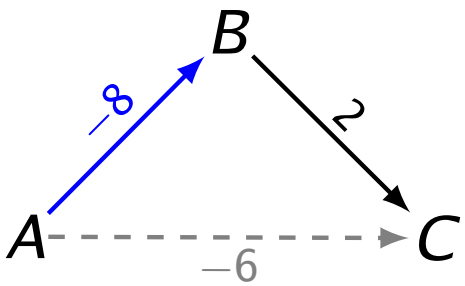
Start with APSP Graph:



Then remove *dominated* edges . . .

Simple Temporal Network
Remove Dominated Edges

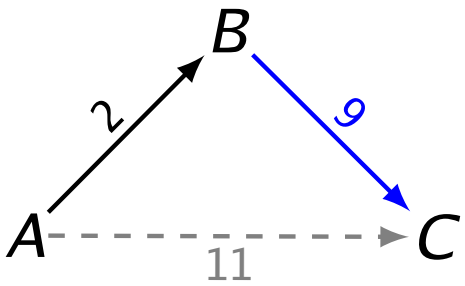
A negative edge AC is dominated by a negative edge AB if $\mathcal{D}(A, B) + \mathcal{D}(B, C) = \mathcal{D}(A, C)$:



- AB and AC have the same source node: A.
- During the execution, it is not necessary to propagate along dominated edges like AC.

Simple Temporal Network
Remove Dominated Edges (ctd.)

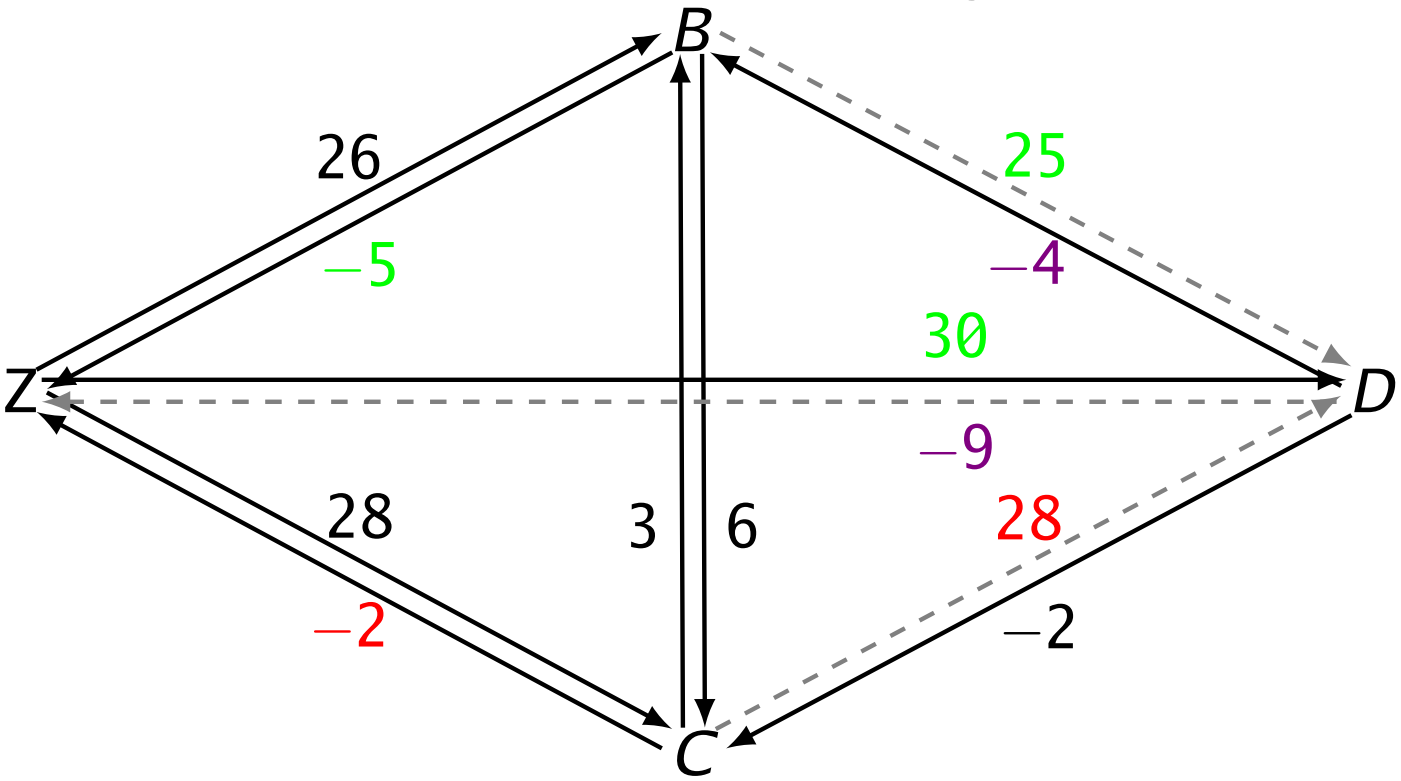
A non–negative edge AC is dominated by a non–negative edge BC if $\mathcal{D}(A, B) + \mathcal{D}(B, C) = \mathcal{D}(A, C)$:



- BC and AC have the same destination node: C.
- During the execution, it is not necessary to propagate along dominated edges like AC.

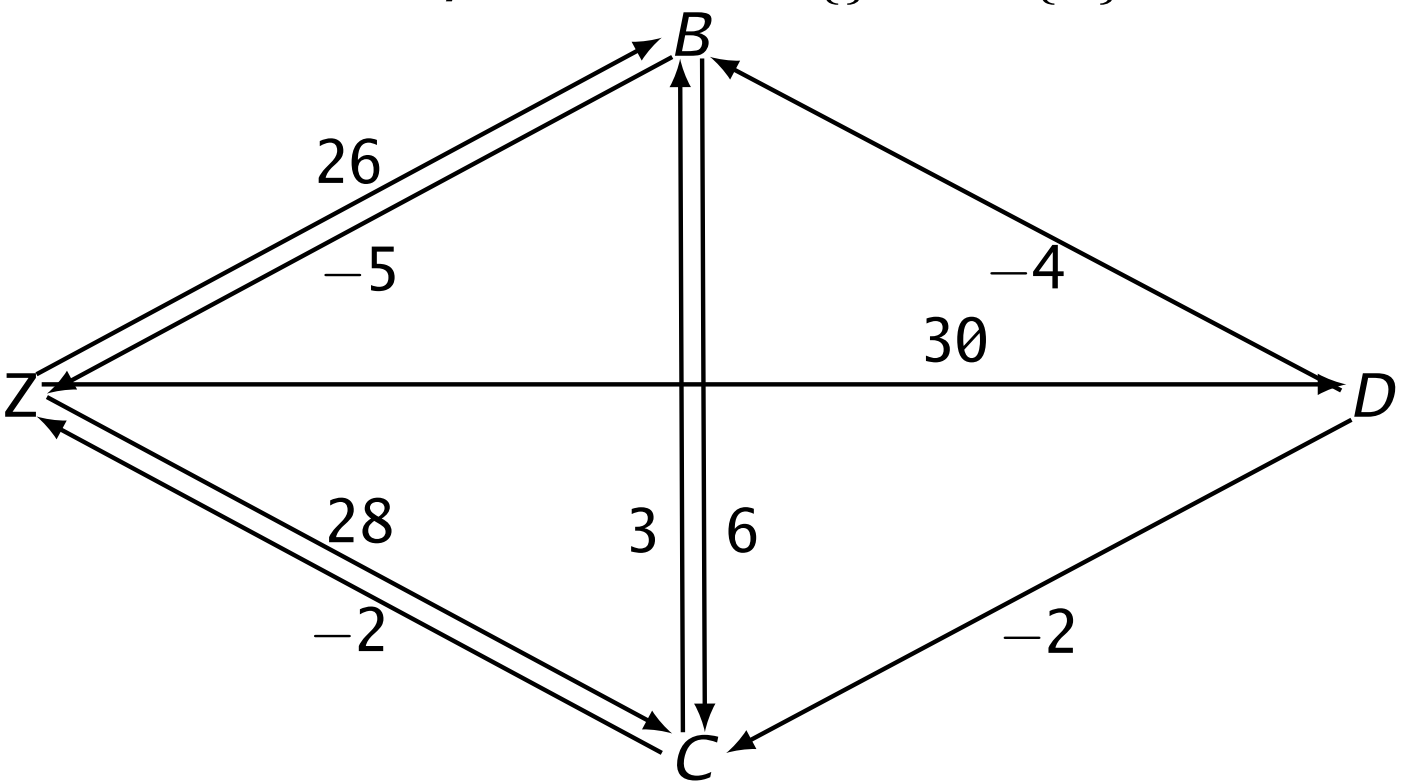
Simple Temporal Network
Making STN Dispatchable (ctd.)

Remove “dominated” edges:*



*[Muscettola et al., 1998]

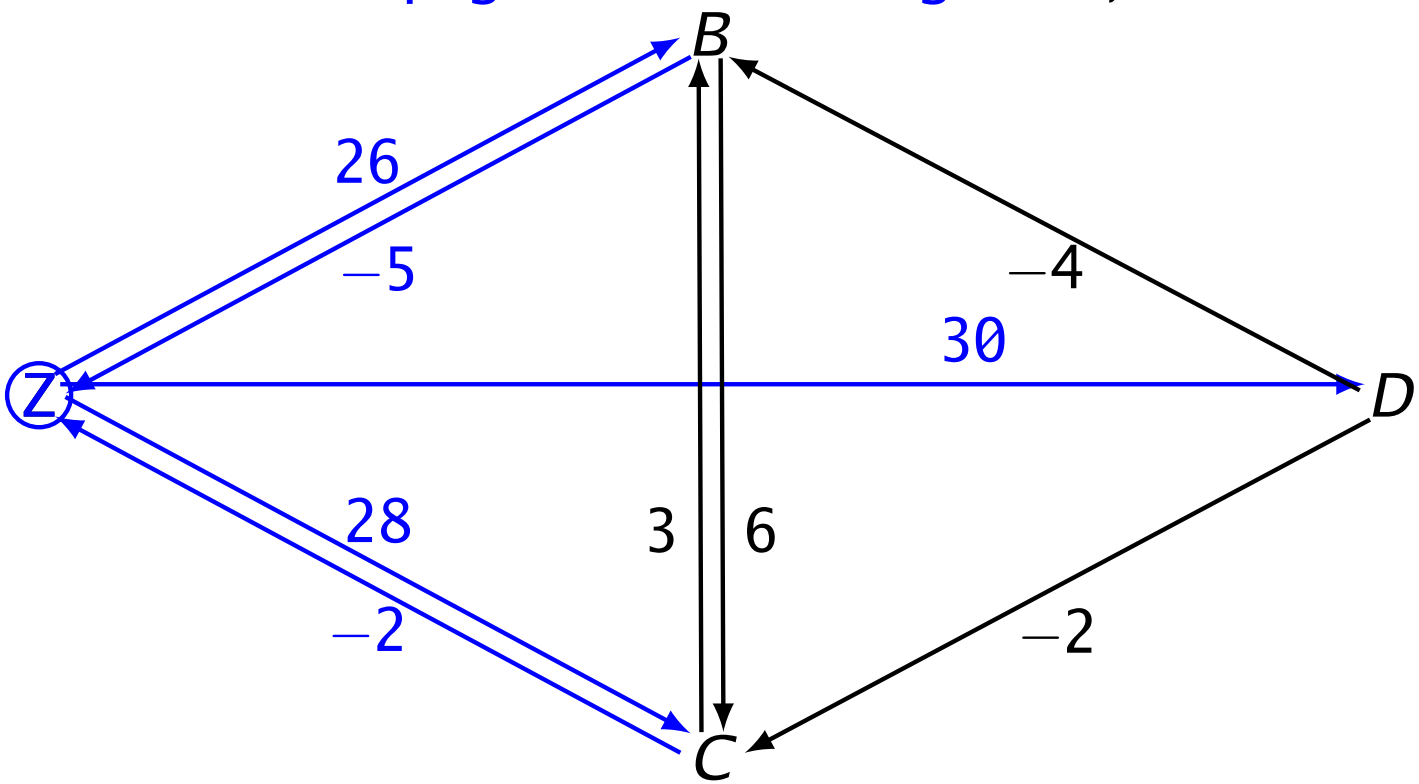
Initially: $t = 0$, $\mathcal{X} = \{\}$, $\mathbf{E} = \{\mathsf{Z}\}$.



Pick Z from $\mathbf{E}$. Set $\mathsf{Z} = 0$.

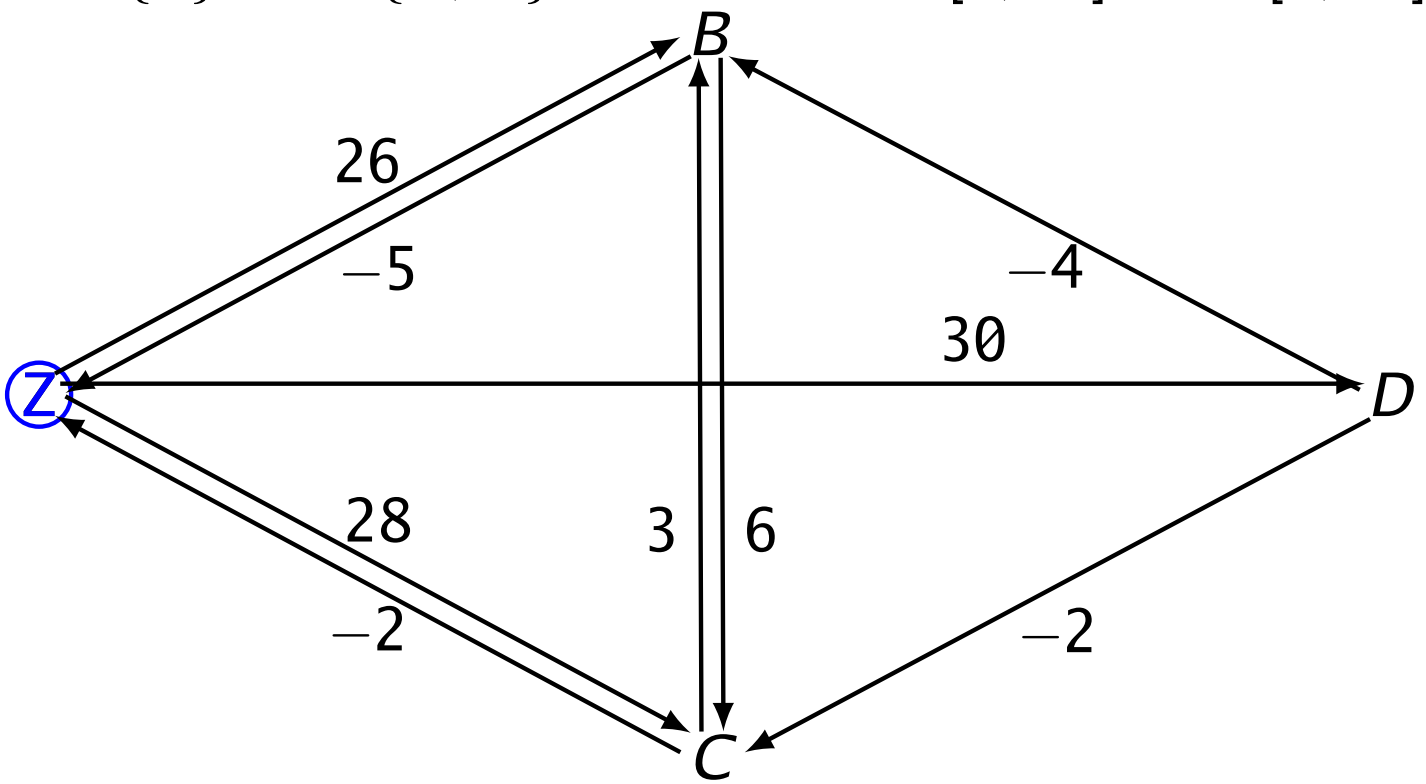
Simple Temporal Network
Dispatching the STN (ctd.)

Propagate $Z = 0$ to neighbors;



$\mathcal{X} = \{Z\}$, $\mathbf{E} = \{B, C\}$; $B \in [5, 26]$, $C \in [2, 28]$, $D \in [0, 30]$.

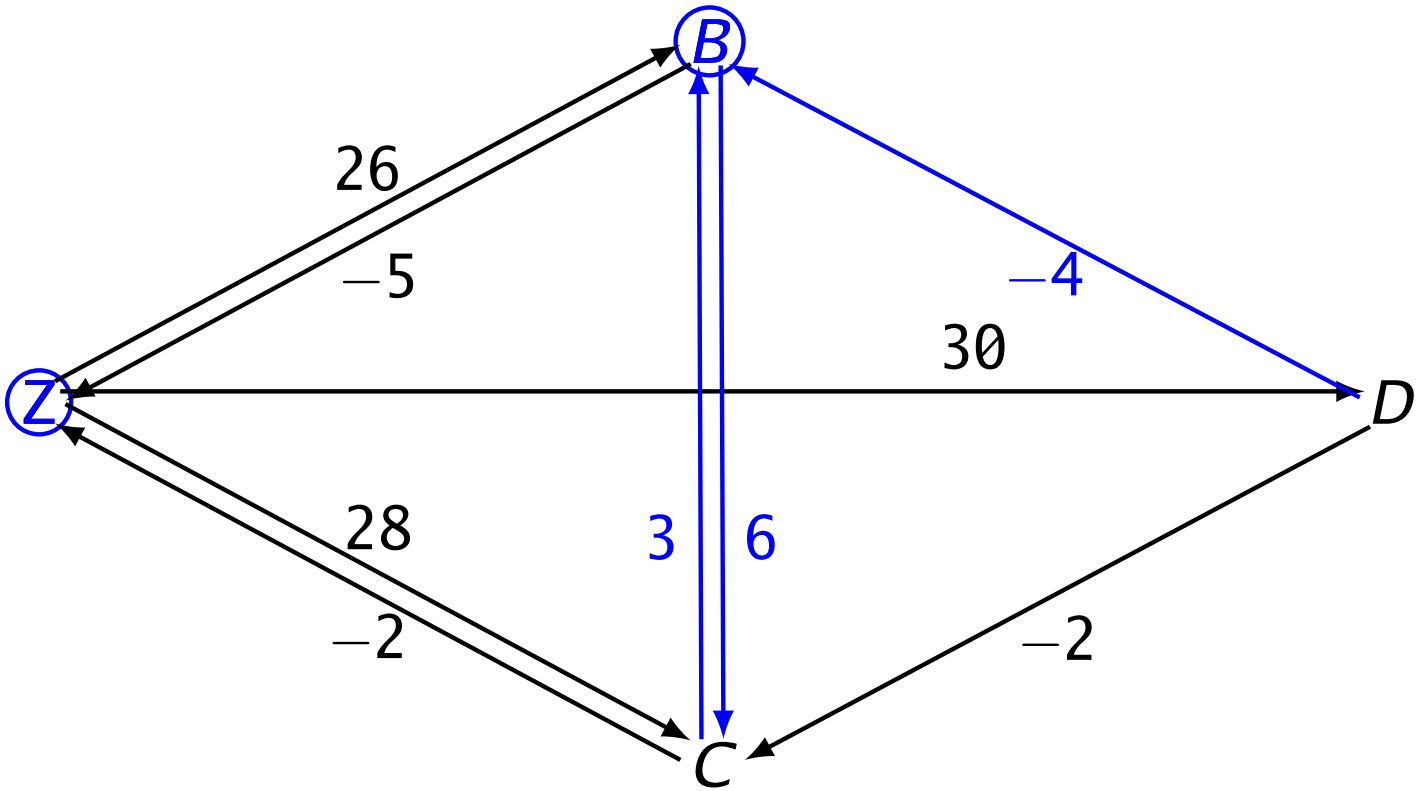
$\mathcal{X} = \{Z\}$, $\mathbf{E} = \{B, C\}$; Bounds: $B \in [5, 26]$, $C \in [2, 28]$.



Let t advance to 12; Pick B from $\mathbf{E}$; Set $B = 12$.

Simple Temporal Network
Dispatching the STN (ctd.)

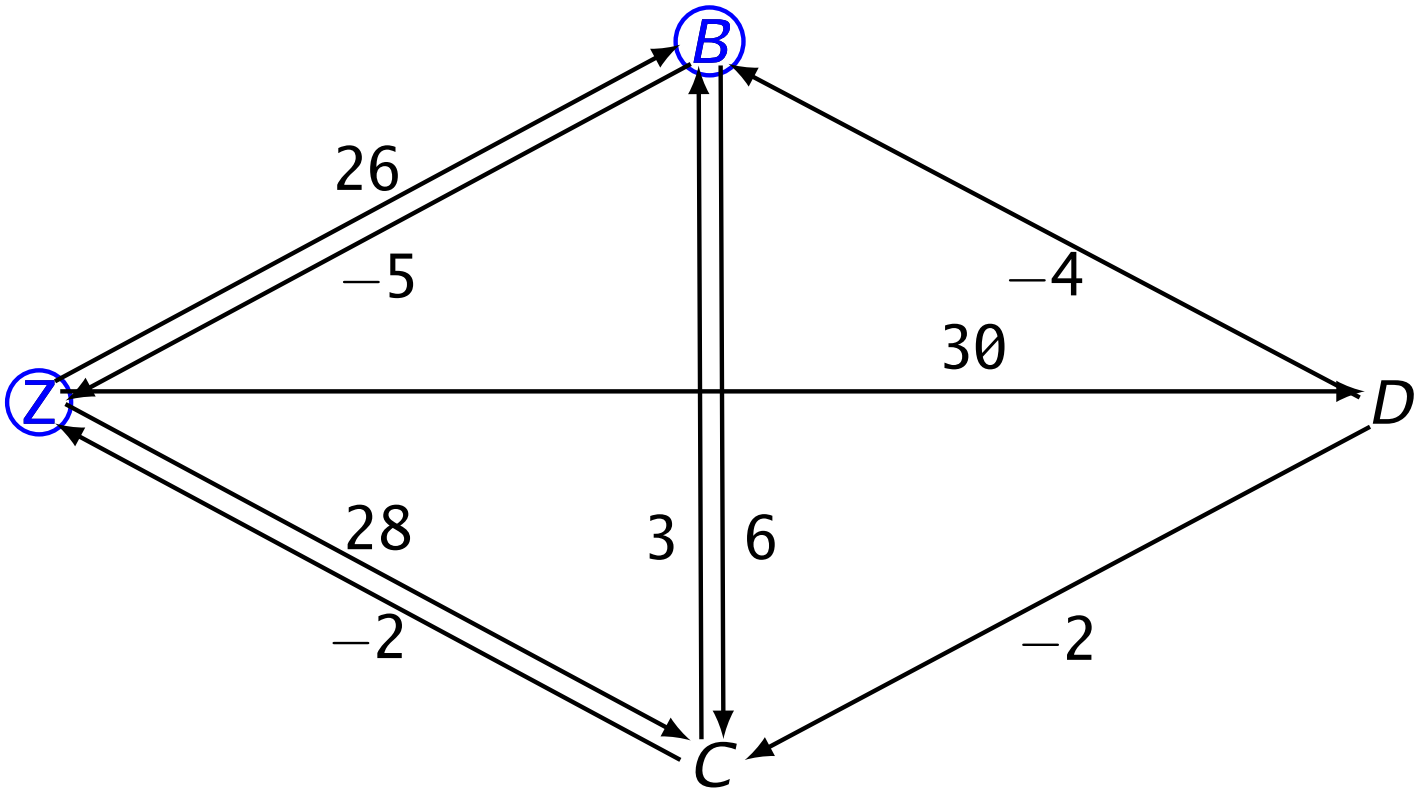
Propagate $B = 12$ to neighbors



$\mathcal{X} = \{Z, B\}$, $t = 12$, $\mathbf{E} = \{C\}$, $C \in [12, 18]$, $D \in [16, 30]$

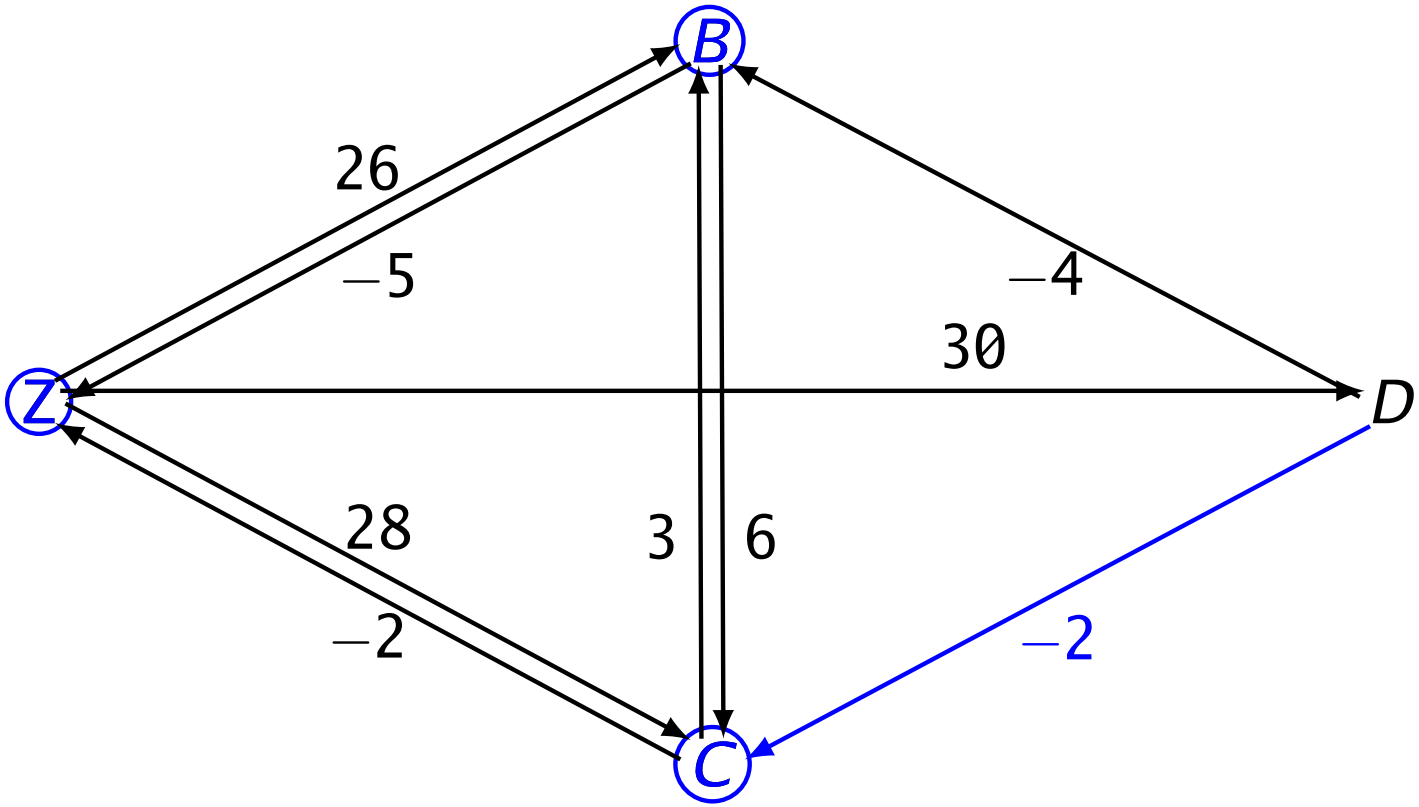
Simple Temporal Network
Dispatching the STN (ctd.)

$\mathcal{X} = \{Z, B\}$, $t = 12$, $\mathbf{E} = \{C\}$, $C \in [12, 18]$, $D \in [16, 30]$



Let t advance to 16, pick C from $\mathbf{E}$, set $C = 16$.

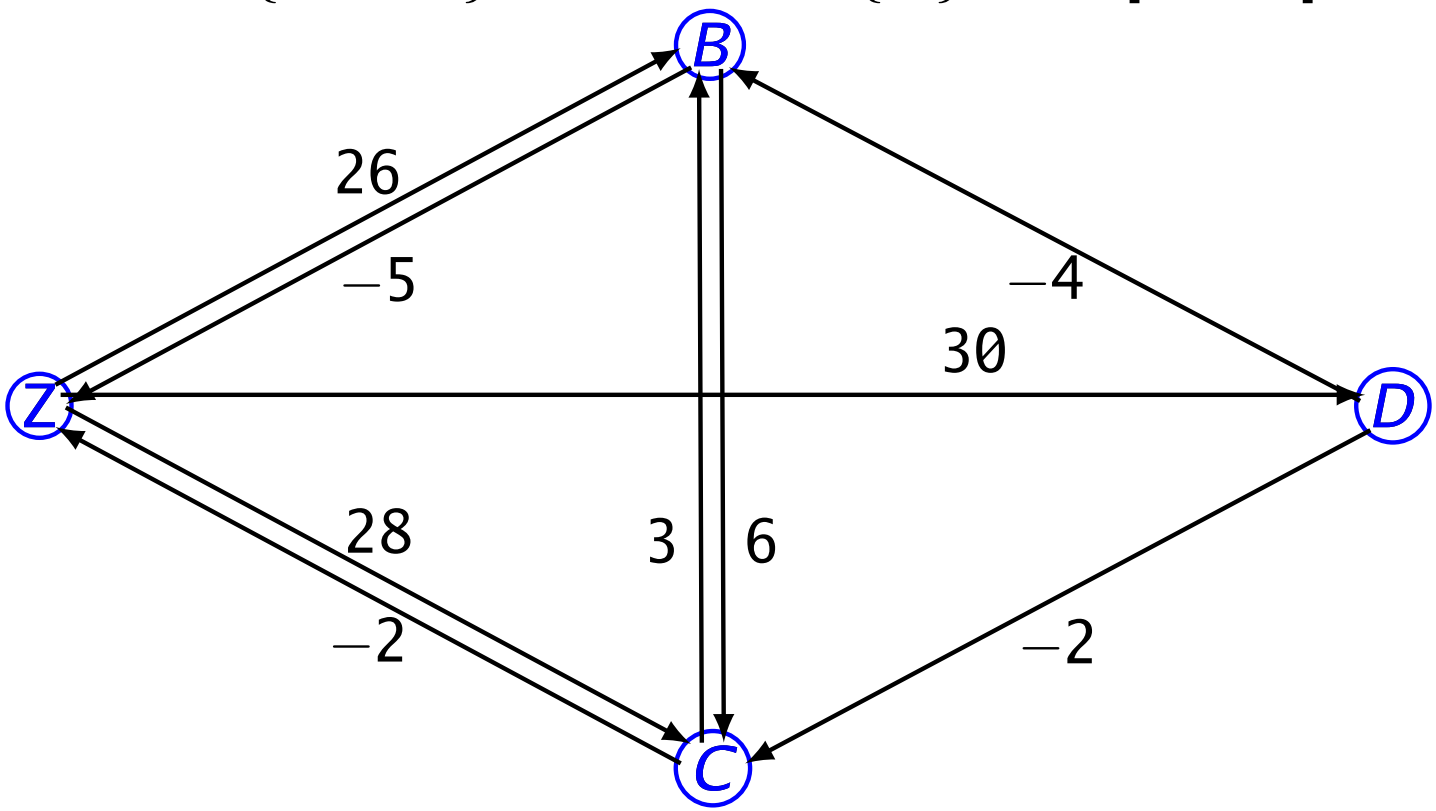
Propagate $C = 16$ to C's only remaining neighbor, D.



$\mathcal{X} = \{Z, B, C\}$, $t = 16$, $\mathbf{E} = \{D\}$, $D \in [18, 30]$

Simple Temporal Network
Dispatching the STN (ctd.)

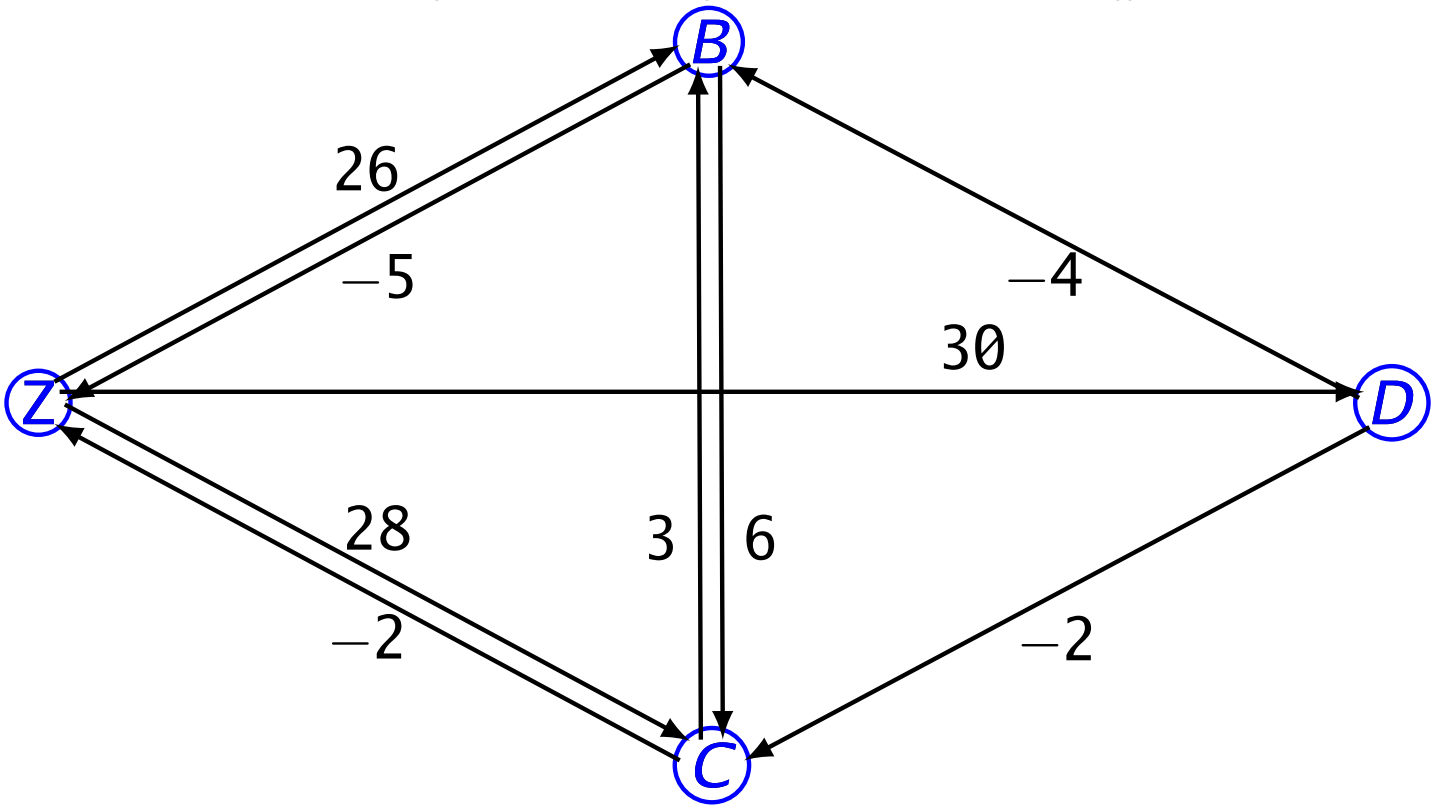
$\mathcal{X} = \{Z, B, C\}$, $t = 16$, $\mathbf{E} = \{D\}$, $D \in [18, 30]$



Let t advance to 25, pick D from $\mathbf{E}$, set $D = 25$.

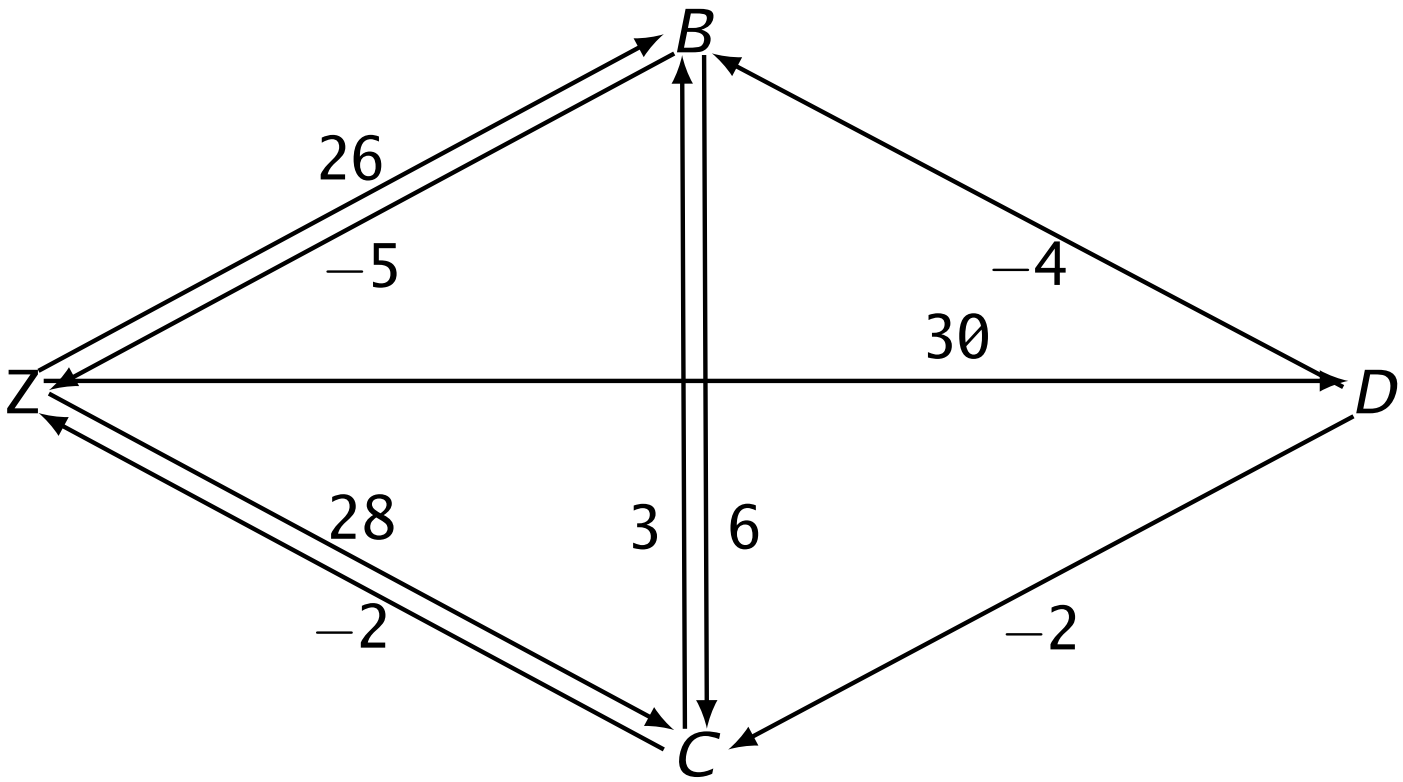
Simple Temporal Network
Dispatching the STN (ctd.)

$\mathcal{X} = \{Z, B, C, D\}$, $t = 25$, $\mathbf{E} = \{\}$



Solution: $Z = 0, B = 12, C = 16, D = 25$.

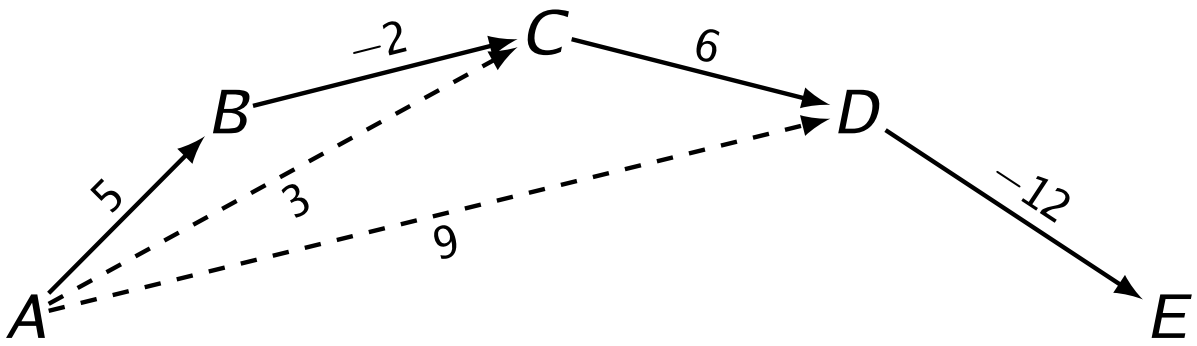
Easy to check that $Z = 0, C = 20, B = 23, D = 28$ can also be generated by the dispatcher.



Simple Temporal Network
New View of Dispatchability*

(1) A path $\mathcal{P}$ has the prefix/postfix (PP) property if:

- Every proper prefix of $\mathcal{P}$ has non–negative length, and
- Every proper postfix of $\mathcal{P}$ has negative length.



* [Morris, 2014]

Simple Temporal Network
New View of Dispatchability*

(1) A path $\mathcal{P}$ has the prefix/postfix (PP) property if:

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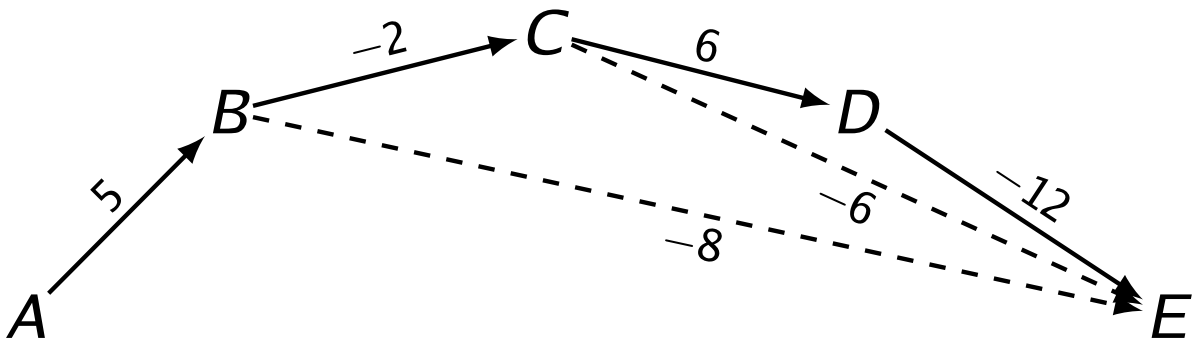
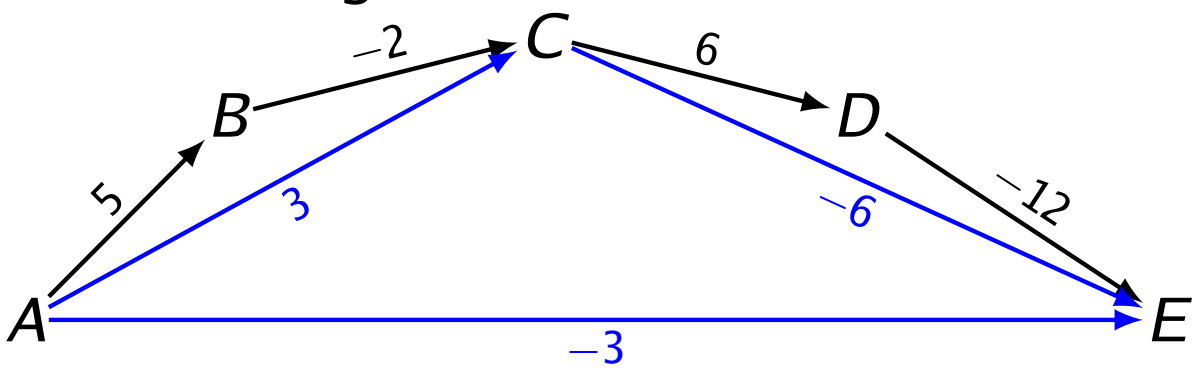


* [Morris, 2014]

(2) An STN is PP–complete if for each shortest path from any X to any Y that has the prefix/postfix property, there is an edge from X to Y with the same length.



(3) **A consistent and PP–complete STN is dispatchable.**

* [Morris, 2014]

Further graphical analyses of the dispatchability of STNs has been presented recently [Morris, 2016].

Additional Research on STNs

- Temporal Decoupling Problem (TDP) [Hunsberger, 2002, Jr. and Durfee, 2013, Mountakis et al., 2017]
- APSP algorithms on *chordal graphs* [Xu and Choueiry, 2003]
- Enforcing partial path consistency [Planken, 2008]
- Incorporating STNs in multi–agent auctions [Hunsberger and Grosz, 2000]
- For further info:
  `http://www.cs.vassar.edu/~hunsberg/__papers__/`
  (See pages 22–24, 27–28, 32, 53–70, 76–79 of 2005 AAMAS tutorial.)

STN Summary

- STNs have been used to provide flexible planning and scheduling systems for more than a decade.
- Efficient algorithms for checking consistency, incrementally updating the APSP matrix, and managing execution in real time for maximum flexibility.
- However, STNs cannot represent uncertainty (e.g., actions with uncertain durations) or conditional constraints (e.g., only do X if test result is negative).

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