Therefore $ADEB$ is a parallelogram;

therefore AB is equal to DE, and AD to BE. [I. 34]

But AB is equal to AD;

therefore the four straight lines BA, AD, DE, EB are equal to one another;

therefore the parallelogram $ADEB$ is equilateral.

I say next that it is also right-angled.

For, since the straight line AD falls upon the parallels AB, DE,

the angles BAD, ADE are equal to two right angles. [I. 29]

But the angle BAD is right;

therefore the angle ADE is also right.

And in parallelogrammic areas the opposite sides and angles are equal to one another; [I. 34]

therefore each of the opposite angles ABE, BED is also right.

Therefore $ADEB$ is right-angled.

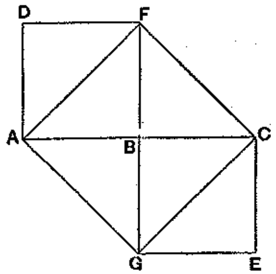
And it was also proved equilateral.

Therefore it is a square; and it is described on the straight line AB.

Q. E. F.

1, 3, 30. Proclus (p. 423, 18 sqq.) notes the difference between the word *construct* (συστήσασθαι) applied by Euclid to the construction of a *triangle* (and, he might have added, of an *angle*) and the words *describe on* (ἀναγράφειν ἀπό) used of drawing a square on a given straight line as one side. The *triangle* (or *angle*) is, so to say, pieced together, while the describing of a square on a given straight line is the making of a figure "from" *one* side, and corresponds to the multiplication of the number representing the side by itself.

Proclus (pp. 424—5) proves that, *if squares are described on equal straight lines, the squares are equal*; and, *conversely*, that, *if two squares are equal, the straight lines are equal on which they are described.* The first proposition is immediately obvious if we divide the squares into two triangles by drawing a diagonal in each. The converse is proved as follows.



Place the two equal squares AF, CG so that AB, BC are in a straight line. Then, since the angles are right, FB, BG will also be in a straight line. Join AF, FC, CG, GA.

Now, since the squares are equal, the triangles ABF, CBG are equal.

Add to each the triangle FBC; therefore the triangles AFC, GFC are equal, and hence they must be in the same parallels.

Therefore AG, CF are parallel.

Also, since each of the alternate angles AFG, FGC is half a right angle,

AF, CG are parallel.

Hence $AFCG$ is a parallelogram; and AF, CG are equal.

Thus the triangles ABF, CBG have two angles and one side respectively equal;

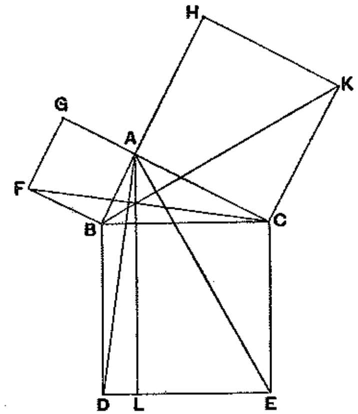
therefore AB is equal to BC, and BF to BG.

Proposition 47.

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

Let ABC be a right-angled triangle having the angle BAC right;

I say that the square on BC is equal to the squares on BA, AC.



For let there be described on BC the square $BDEC$, and on BA, AC the squares GB, HC; [I. 46]

through A let AL be drawn parallel to either BD or CE, and let AD, FC be joined.

Then, since each of the angles BAC, BAG is right, it follows that with a straight line BA, and at the point A on it, the two straight lines AC, AG not lying on the same side make the adjacent angles equal to two right angles;

therefore CA is in a straight line with AG. [I. 14]

For the same reason

BA is also in a straight line with AH.

And, since the angle DBC is equal to the angle FBA: for each is right:

let the angle ABC be added to each;

therefore the whole angle DBA is equal to the whole angle FBC. [C. N. 2]

And, since *DB* is equal to *BC*, and *FB* to *BA*,
the two sides *AB*, *BD* are equal to the two sides *FB*, *BC* respectively,
and the angle *ABD* is equal to the angle *FBC*;
therefore the base *AD* is equal to the base *FC*,
and the triangle *ABD* is equal to the triangle *FBC*. [I. 4]

Now the parallelogram *BL* is double of the triangle *ABD*, for they have the same base *BD* and are in the same parallels *BD*, *AL*. [I. 41]

And the square *GB* is double of the triangle *FBC*, for they again have the same base *FB* and are in the same parallels *FB*, *GC*. [I. 41]

[But the doubles of equals are equal to one another.]

Therefore the parallelogram *BL* is also equal to the square *GB*.

Similarly, if *AE*, *BK* be joined, the parallelogram *CL* can also be proved equal to the square *HC*;

therefore the whole square *BDEC* is equal to the two squares *GB*, *HC*. [*C. N.* 2]

And the square *BDEC* is described on *BC*,
and the squares *GB*, *HC* on *BA*, *AC*.

Therefore the square on the side *BC* is equal to the squares on the sides *BA*, *AC*.

Therefore etc. Q. E. D.

1. **the square on,** τὸ ἀπὸ...τετράγωνον, the word ἀναγραφέν or ἀναγεγραμμένον being understood.

subtending the right angle. Here ὑποτεινούσης, "subtending," is used with the simple accusative (τὴν ὀρθὴν γωνίαν) instead of being followed by ὑπό and the accusative, which seems to be the original and more orthodox construction. Cf. I. 18, note.

33. **the two sides AB, BD....** Euclid actually writes "*DB*, *BA*," and therefore the equal sides in the two triangles are not mentioned in corresponding order, though he adheres to the words ἑκατέρα ἑκατέρα "respectively." Here *DB* is equal to *BC* and *BA* to *FB*.

44. **[But the doubles of equals are equal to one another.]** Heiberg brackets these words as an interpolation, since it quotes a *Common Notion* which is itself interpolated. Cf. notes on I. 37, p. 332, and on interpolated *Common Notions*, pp. 223—4.

"If we listen," says Proclus (p. 426, 6 sqq.), "to those who wish to recount ancient history, we may find some of them referring this theorem to Pythagoras and saying that he sacrificed an ox in honour of his discovery. But for my part, while I admire those who first observed the truth of this theorem, I marvel more at the writer of the Elements, not only because he made it fast (κατεδήσατο) by a most lucid demonstration, but because he compelled assent to the still more general theorem by the irrefragable arguments of science in the sixth Book. For in that Book he proves generally that, in right-angled triangles, the figure on the side subtending the right angle is equal to the similar and similarly situated figures described on the sides about the right angle."

In addition, Plutarch (in the passages quoted above in the note on I. 44), Diogenes Laertius (VIII. 12) and Athenaeus (X. 13) agree in attributing this proposition to Pythagoras. It is easy to point out, as does G. Junge ("Wann haben die Griechen das Irrationale entdeckt?" in *Novae Symbolae Joachimicae*, Halle a. S., 1907, pp. 221—264), that these are late witnesses, and that the Greek literature which we possess belonging to the first five centuries after Pythagoras contains no statement specifying this or any other particular great geometrical discovery as due to him. Yet the distich of Apollodorus the "calculator," whose date (though it cannot be fixed) is at least earlier than that of Plutarch and presumably of Cicero, is quite definite as to the existence of *one* "famous proposition" discovered by Pythagoras, whatever it was. Nor does Cicero, in commenting apparently on the verses (*De nat. deor.* III. c. 36, § 88), seem to dispute the fact of the geometrical discovery, but only the story of the sacrifice. Junge naturally emphasises the apparent uncertainty in the statements of Plutarch and Proclus. But, as I read the passages of Plutarch, I see nothing in them inconsistent with the supposition that Plutarch unhesitatingly accepted as discoveries of Pythagoras *both* the theorem of the square of the hypotenuse and the problem of the application of an area, and the only doubt he felt was as to which of the two discoveries was the more appropriate occasion for the supposed sacrifice. There is also other evidence not without bearing on the question. The theorem is closely connected with the whole of the matter of Eucl. Book II., in which one of the most prominent features is the use of the *gnomon*. Now the gnomon was a well-understood term with the Pythagoreans (cf. the fragment of Philolaus quoted on p. 141 of Boeckh's *Philolaos des Pythagoreers Lehren*, 1819). Aristotle also (*Physics* III. 4, 203 a 10—15) clearly attributes to the Pythagoreans the placing of odd numbers as *gnomons* round successive squares beginning with 1, thereby forming new squares, while in another place (*Categ.* 14, 15 a 30) the word *gnomon* occurs in the same (obviously familiar) sense: "e.g. a square, when a gnomon is placed round it, is increased in size but is not altered in form." The inference must therefore be that practically the whole doctrine of Book II. is Pythagorean. Again Heron (? 3rd cent. A.D.), like Proclus, credits Pythagoras with a general rule for forming right-angled triangles with rational whole numbers for sides. Lastly, the "summary" of Proclus appears to credit Pythagoras with the discovery of the theory, or study, of irrationals (τὴν τῶν ἀλόγων πραγματείαν). But it is now more or less agreed that the reading here should be, not τῶν ἀλόγων, but τῶν ἀναλόγων, or rather τῶν ἀνὰ λόγον ("of proportionals"), and that the author intended to attribute to Pythagoras a theory of *proportion*, i.e. the (arithmetical) theory of proportion applicable only to commensurable magnitudes, as distinct from the theory of Eucl. Book V., which was due to Eudoxus. It is not however disputed that the *Pythagoreans* discovered the irrational (cf. the scholium No. 1 to Book X.). Now everything goes to show that this discovery of the irrational was made with reference to $\sqrt{2}$, the ratio of the diagonal of a square to its side. It is clear that this presupposes the knowledge that I. 47 is true of an isosceles right-angled triangle; and the fact that some triangles of which it had been discovered to be true were *rational* right-angled triangles was doubtless what suggested the inquiry whether the ratio between the lengths of the diagonal and the side of a square could also be expressed in whole numbers. On the whole, therefore, I see no sufficient reason to question the tradition that, *so far as Greek geometry is concerned* (the possible priority of the discovery of the same proposition in India will be considered later), Pythagoras