Homework Assignment 3

Due **Friday 6-02-04**

1. Show that the following functions, defined on the non-negative integers **N**, are primitive recursive. Give a formal definition using the basic functions, composition and primitive recursion. You may assume that addition and multiplication are primitive recursive in (c), (e), (f) and (g).

(*a*) predecessor $pd(x)$, where $pd(0) = 0$;

(*1 point*)

(*b*) subtraction $x \dot{-} n$, where $x \dot{-} n = 0$ is $x < n$;

(*1 point*)

Hint: By recursion on n, using the predecessor function.

(*c*) the absolute value $|x - y|$ of the difference between x and y; this is $x - y$ if $y < x$ and $y - x$ otherwise;

(*1 points*)

(*d*) the signature function $sg(x)$, which returns 0 if $x = 0$ and 1, otherwise; the countersignature function $\overline{sg}(x)$ which returns 1 if $x = 0$ and 0 otherwise;

(*2 points*)

(*e*) the remainder $rm(a, b)$ of the division of a by b;

(*2 points*)

Hint: $rm(0, b) = 0$; $rm(n + 1) = (rm(n, b) + 1) \cdot sg(|b - (rm(n, b) + 1)|)$.

(*f*) the quotient $[a/b]$ of the division of a by b;

(*2 points*)

Hint: $[0/b] = 0$; $[n + 1/b] = [n/b] + \overline{sg}(|b - (rm(n, b) + 1)|)$.

(*g*) the coding function $J(m, n) = m + \Sigma_{i \leq m+n} i$.

(*2 points*)

2. Prove the following facts:

(i) For $x > 1$ and $y > 2$, $x \cdot y > x + y$.
Hint: Use induction on x.

(*2 points*)

(ii) $rm(a, b) < b$.

(*2 points*)