

## Homework Assignment 2

Due **Friday 30-01-04**

1. Let  $A$  be a set and  $\wp(A)$  the set of all subsets of  $A$ . Show that there is no bijection  $f : A \rightarrow \wp(A)$ .

*Hint:* Suppose  $f : A \rightarrow \wp(A)$  be a bijection. Consider the set

$$X = \{y \in A \mid y \notin f(y)\};$$

show that there is an element  $x$  such that  $f(x) = X$  and consider both cases  $x \in f(x)$  and  $x \notin f(x)$ .

2. Let  $A$  be a finite set of cardinality  $n$ . By induction on  $n$ , show that  $\wp(A)$  the set of all subsets of  $A$  has cardinality  $2^n$ .

3. Prove Euclid's theorem: There are infinitely many prime numbers.

*Hint:* Suppose  $p_1, \dots, p_k$  were all the prime numbers and consider the number  $p_1 \cdot \dots \cdot p_k + 1$ . Can it be divided by some  $p_i$ ?