TITLE: (Co)reflective subcategories revisited

ABSTRACT: We will show that if $\mathcal{C}$ is an additive category with split idempotents that has pseudokernels and pseudocokernels, then a full subcategory $\mathcal{B}$ is coreflective if, and only if, it is precovering, closed under direct summands and each morphism in $\mathcal{B}$ has a pseudokernel in $\mathcal{C}$ that belongs to $\mathcal{B}$. We will then show how this result applies to pretriangulated categories, in particular to stable categories, and to subcategories of the form $\mathrm{Pres}(\mathcal{U})$, when $\mathcal{U}$ is a set of objects in an ambient abelian category $\mathcal{A}$. As byproducts, we will generalize Gabriel-De la Penha theorem to module categories over small preadditive categories and show that, given any Grothendieck category $\mathcal{G}$, the direct limit closure of its subcategory $\mathrm{fp}(\mathcal{G})$ of finitely presented objects is always coreflective.