

## T-structures of Happel-Reiten-Smalø whose hearts are module categories

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Given an associative ring  $R$  and a torsion pair  $\mathfrak{t} = (\mathcal{T}, \mathcal{F})$  in the category of left  $R$ -modules, the heart  $\mathcal{H}_{\mathfrak{t}}$  of the t-structure associated to  $\mathfrak{t}$ , is an abelian category (see [BBD]). In [HKM], the authors introduced what is called an HKM torsion pair in  $R\text{-Mod}$  and proved that  $\mathcal{H}_{\mathfrak{t}}$  is a module category, for this type of torsion pair. The study of when  $\mathcal{H}_{\mathfrak{t}}$  is a module category was also initiated in [CGM] and continued in [CMT], where the authors gave necessary and sufficient conditions, when  $\mathfrak{t}$  is faithful, for  $\mathcal{H}_{\mathfrak{t}}$  to be a module category.

We study when the heart  $\mathcal{H}_{\mathfrak{t}}$  is equivalent to a module category, for any torsion pair  $\mathfrak{t}$  in  $R\text{-Mod}$ . We give necessary and sufficient conditions for  $\mathcal{H}_{\mathfrak{t}}$  to be a module category. Consequently, we give a criterion for a torsion pair to be HKM torsion pair. We concentrate on the case of a hereditary torsion pair and give examples of hereditary torsion pairs whose heart is a module category, for several types of rings.

## References

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