The inversion height of of the free field is infinite

This is a joint work with Dolors Herbera.

Suppose that R is a domain embedded in a division ring E. Let $Q_0 = R$, and, for $n \geq 0$, let Q_{n+1} be the subring of E generated by Q_n and the inverses of the nonzero elements of Q_n. Then $D = \bigcup_{n=0}^{\infty} Q_n$ is the division ring of fractions of R inside E, that is, the division ring rationally generated by R.

The *inversion height of* R is the number of nested inversions needed to express the elements of D from the elements of R, i.e. the infimum of the natural numbers n such that Q_n is a division ring.

Let X be any set of cardinality at least two and k any field.

In 1949, B.H. Neumann conjectured that the inversion height of the free (group) k-algebra on the set X inside a Mal'cev-Neumann series ring is of infinite inversion height.

It was proved by C. Reutenauer in 1996 that the conjecture holds true when X is an infinite set. We show it when X is a finite set (of at least two elements) by reducing the problem to the situation verified by C. Reutenauer.